



MATHSQUEST
MANUAL FOR THE
TI-Nspire CAS
CALCULATOR

5TH EDITION

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Introduction

Maths Quest Manual for the TI-Nspire CAS Calculator is a comprehensive, step-by-step guide to using the TI-Nspire CAS calculator. It is designed to assist students and teachers to integrate Computer Algebra Systems (CAS) into their learning and teaching of Mathematics. It is intended to be used by students and teachers to supplement the Maths Quest series for VCE.

Features

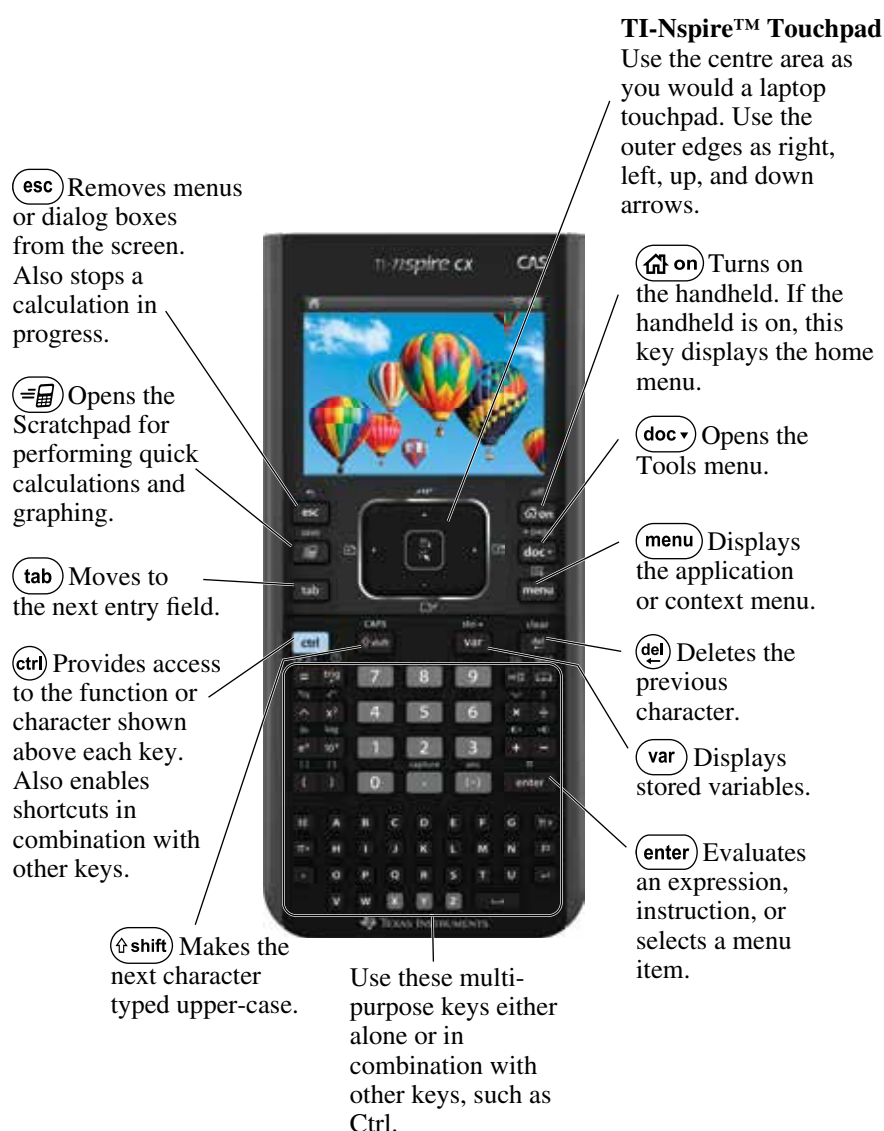
- Full colour screen shots
- Each chapter is divided into ‘how to’ sections that provide clear, step-by-step instructions for the user.
- Easy-to-follow keystrokes and screen dumps accompanied by explicit explanations are provided.

Note: This book has been written for the TI-Nspire CAS Operating System Software version 4.

CHAPTER 1


Navigating the TI-Nspire

To begin using the TI-Nspire, look carefully at the diagram below and note the important features. The features highlighted are the most commonly used features and will be referred to throughout this book.



Note: The operating system used in preparing this book is version 4. The instructions described in this book are for the TI-Nspire CX CAS calculator and all instructions are given for default settings. Please note that the operating system works on the TouchPad and ClickPad versions as well – but not in colour. To update your operating system go to www.education.ti.com and follow the links.

1.1 How to change System Settings

1. When the TI-Nspire is first turned on, it starts with the Home screen as shown. You can return to this screen by pressing the key with the house icon . Settings contains tools that allow the user to change the settings on the calculator. Use the arrow keys on the Touchpad to move to select Settings, or press:




- 5: Settings .

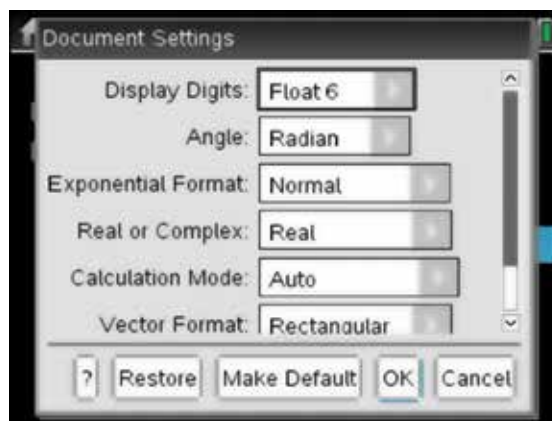


2. Now press:

- 2: Document Settings .



3. Use the tab key  to move down through the settings. To move back to a previous setting, use the shift key  followed by the tab key . Tab to Calculation Mode.




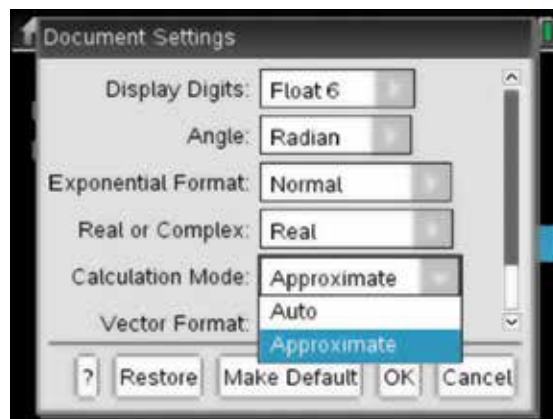
-
4. To set the calculator to Approximate mode, press:

- **CLICK** .

Use the down arrow ▼ on the Touchpad to scroll to the Approximate option. Then press:

- **ENTER** .

Pressing the enter key  again ensures that you have locked in the new setting. Other settings can be changed in a similar way.





1.2 How to use the Notes application




1. One of the features of the TI-Nspire is a Notes application that allows the user to write and save text and mathematical statements. It also allows the user to share notes with other TI-Nspire users and to download prepared notes from the computer.

The Notes application can be found on the Home screen. Use the arrow keys on the Touchpad to select the Notes icon, then press:

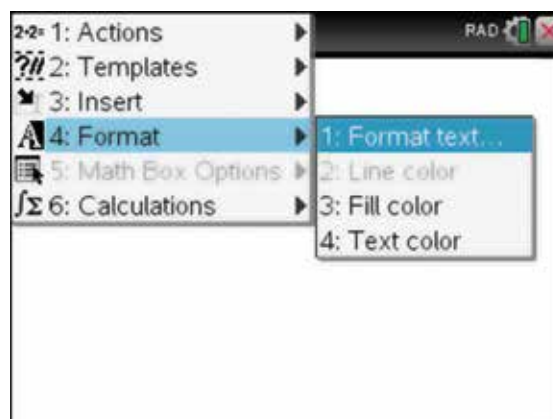
- **ENTER** .







2. You can type text using the letter keys. The shift key  will insert uppercase letters. It is also possible to use common formats such as bold, italics, underline subscripts or superscripts. To do this, highlight the required text, hold down the shift key  and use the arrow keys on the Touchpad, then press:

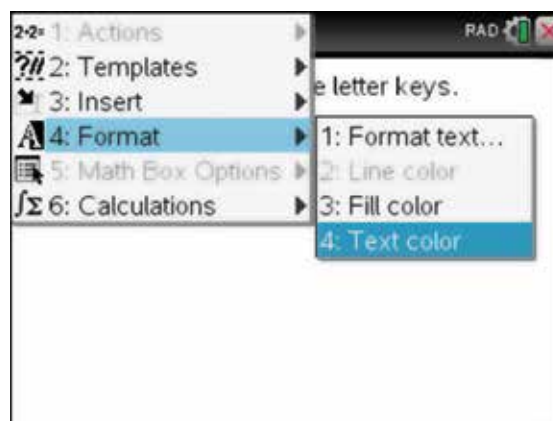
- **MENU** .
- **4: Format** .
- **1: Format text ...** .

Options to format the text include bold, italics and more.



-
3. It is also possible to colour text in a notes application. Colour can now be used in Graphs, Geometry, Lists and Spreadsheets, Data and Statistics and in the Notes application. Colour is not available on the Calculator application. To colour the text, highlight the text as before, then press:

- **ctrl** 
- **MENU** 
- **4: Format** 
- **4: Text color** .



-
4. Using the arrow keys, on the Touchpad, choose a colour for the highlighted text.

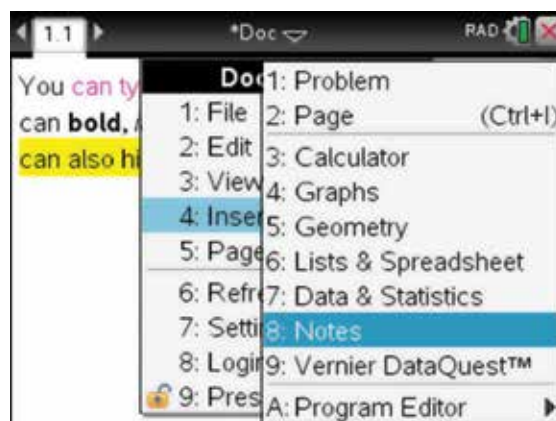


-
5. It is also possible to colour the background, highlight the text, repeat the above and use the fill colour.



6. To insert another Notes page into this document, press:

- **doc** (doc ▾)
- **4: Insert** (4)
- **8: Notes** (8).

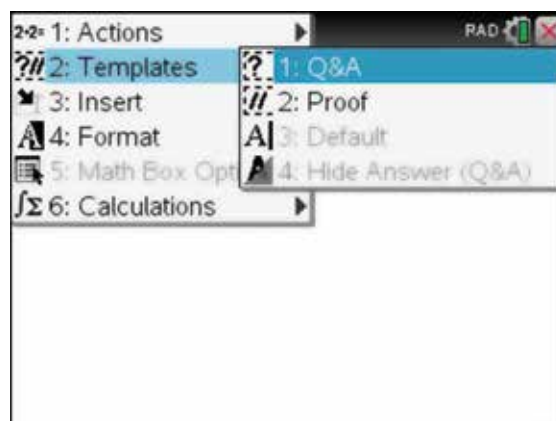


7. You can use this page to perform calculations, pose questions and write proofs. For example, let us pose the following question in this new Notes page:

Simplify $2a^2 \times 5a^3$.

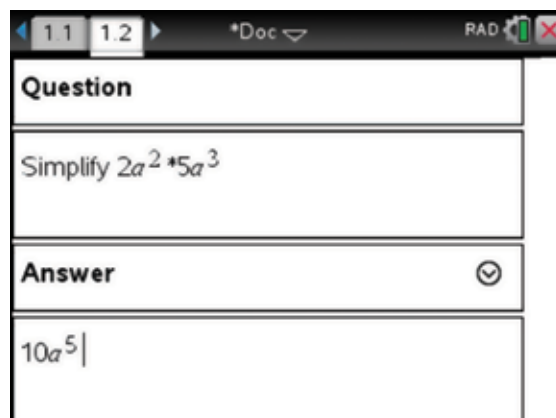
To do this we require the Q&A template. To access this, press:

- **MENU** (menu)
- **2: Templates** (2)
- **1: Q&A** (1).



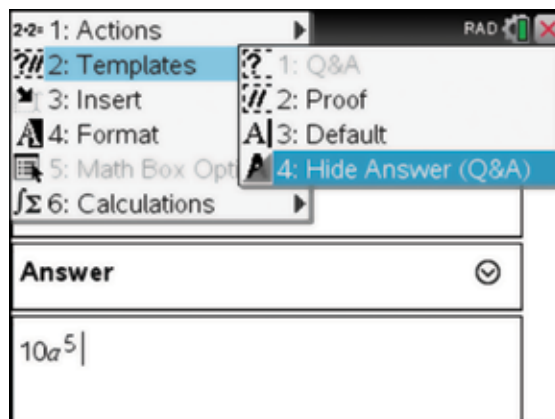
8. Type the question in the Question field. Then use the down arrow ▼ on the Touchpad to move to the Answer field and type the answer.

Note: To type a power, press the hat key (^). Use the tab key (tab) to exit the superscript mode.



9. The answer can be hidden from view. To hide the answer, press:

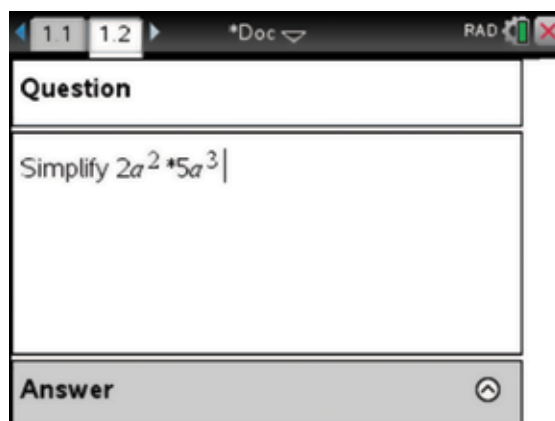
- MENU 
- 2: Templates 
- 4: Hide Answer .



10. The answer is now hidden. The answer can be recalled at any stage. To show the answer, press:

- MENU 
- 2: Templates 
- 4: Hide Answer .

Note: In addition to what has been shown, the Notes application provides shortcuts to add and evaluate mathematical expressions.



1.3 How to navigate between documents and pages

1. To insert a Calculator page into this document, press:

- HOME 





With the Calculator icon selected as shown, press:

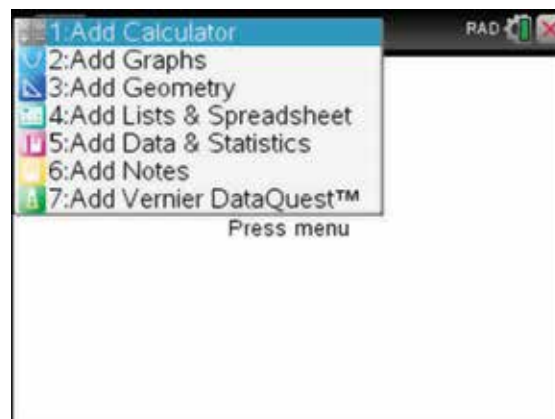
- ENTER 

Note: You can delete this document and start again or save it for another time. More information about deleting and saving documents is provided in section 1.4.







2. Now press:

- **doc** 
- **4: Insert** 
- **1: Problem** 
- **1: Add Calculator** 



3. Notice the 1.2, 1.3 and 2.1 in the tabs in the top left corner of the screen. The first two refer to pages 2 and 3 of problem 1. The current page is indicated by the light background; this is page 1 in problem 2 of the current document. To toggle between these pages, press:



- **ctrl** 
 - **left arrow** 
- or
- **ctrl** 
 - **right arrow** 

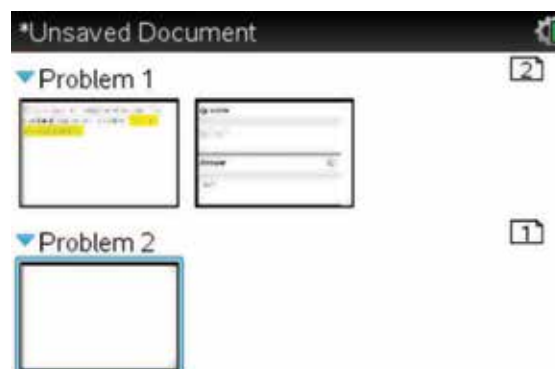
or simply click on the arrows next to the tabs.






4. Press:

- **ctrl** 
- **up arrow** 

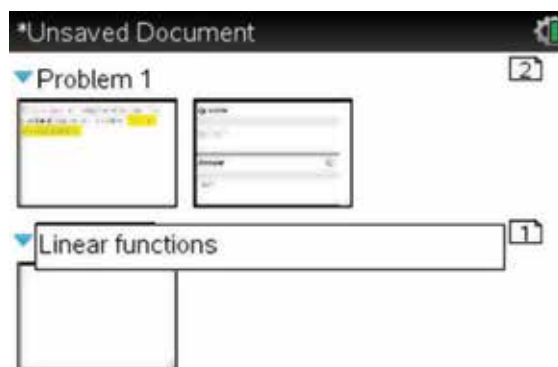
This brings up the page sorter view. From here we can see the current page with a thicker blue border around it. We could delete the page from the document using , or copy the page or reorder the pages. This can be very useful if a document has many pages. When a page is selected, pressing the enter key  makes the page active.



5. Problems can be named by selecting the problem name as shown, then pressing:



- **ctrl** 
- **MENU** 
- **7: Rename** .


Each document can have at least one and up to 30 problems. Each problem can have at least one and up to 50 pages. Each page can have up to four different work sections; these can be grouped as any combination of the different applications.



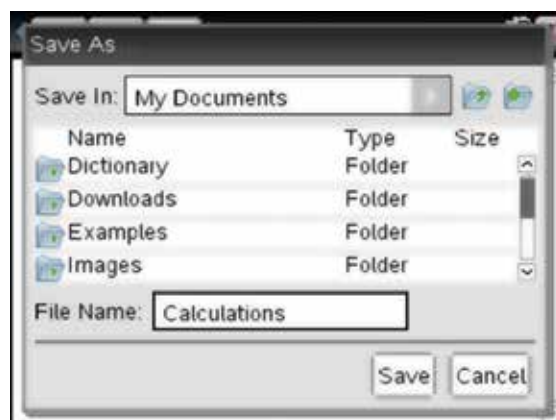
1.4 How to save and delete TNS files

1. We have not as yet saved our document, this is indicated by the *Doc at the top of the screen. To save a document, press:



- **ctrl** 
- .

The following screen appears. You can press tab to move to a folder in which to save your document. There may be some saved files or folders already there, so name your document something you will remember easily. We will name this file Calculations. Type 'Calculations' in the File Name dialog box. Tab  to Save and press the enter key

.



2. *Calculations* is now saved as a TNS file. We can see the file name at the top of the screen. Also, there is no * next to the file name. If you make any changes to the document, the * will appear in front of the file name. This indicates that you have made changes but have not resaved your document. To resave the document at any time, press:

- **ctrl** 
- **S** .

Note: These shortcut key commands, along with many others, are similar to those used in Microsoft Office.



3. To close the file, press:

- **doc** **(doc)**
- **1: File** **(1)**
- **3: Close** **(3)**

or simply click on the red **X** close box in the top right-hand corner of the screen.

To find this document at some later stage, press:

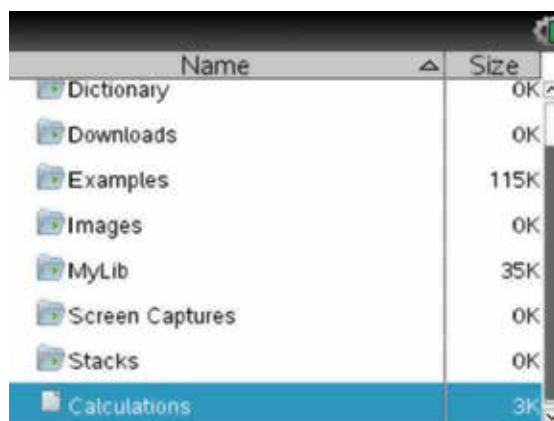
- **HOME** **(home)**
- **2: My Docs** **(2)**.



4. Scroll down to your saved document: Calculations.

Pressing the enter key **(enter)** would recover and reload the document.

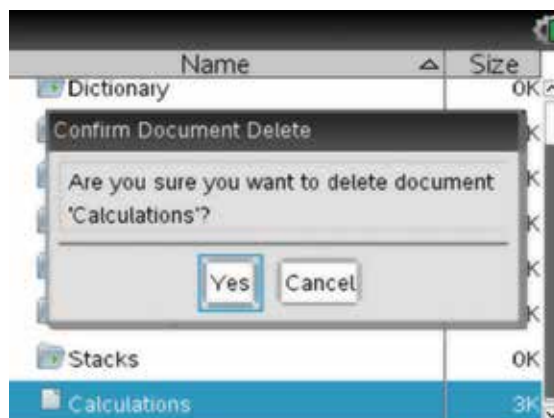
Throughout this book we will often not save the files, so you will see *Unsaved at the top of the screens. Saving the file would create further instructions. However, when we are dealing with a series of related calculations and working on a long involved problem, we will save the file and give instructions on how to save the file.



5. To delete a document or to free up memory, press:



- **delete** **(del)**.

The screen shown appears. Since Yes is selected, pressing the enter key **(enter)** will delete the file and the file name will disappear from the file listing.



1.5 How to perform basic calculations

1. To perform simple calculations, we can use the Scratchpad. This is just a work area for calculations that do not need to be saved in a document. To open the Scratchpad and use a Calculator page, press:


- **HOME**  **on**
- **A: Calculate** 

or

- 

The Scratchpad can only be used as a Calculator or Graphs page.

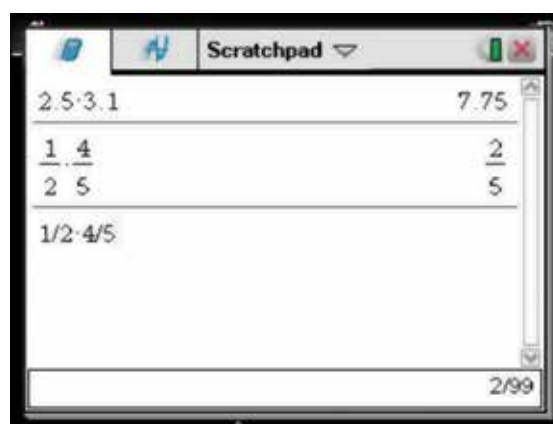


2. Type the following examples as shown, pressing the enter key  to get the answers.



(a) 2.5×3.1

(b) $\frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$


This reveals an important fact: fractions need to be entered correctly.

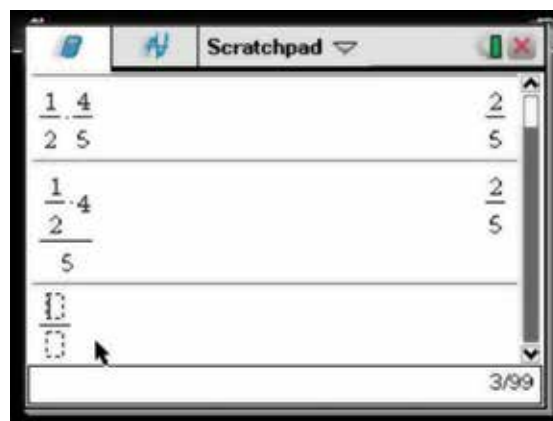


3. Alternatively, you could use the fraction template. To do this, press:

- **ctrl** 
- \div 

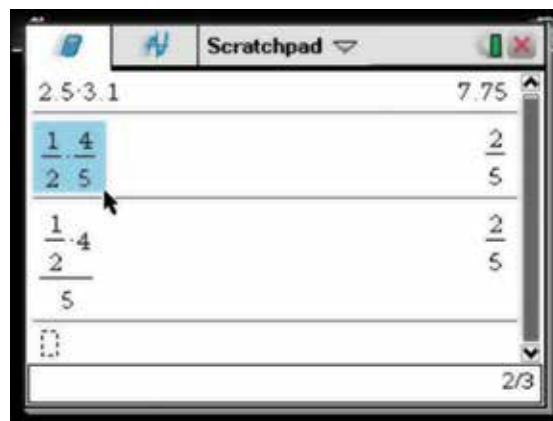
Try the above examples again using this template.




Use the tab key  to move between fields.



4. If you find you have made a mistake and, in fact, wished to enter $\frac{1}{2} \times \frac{16}{5} =$, use the arrow keys on the Touchpad to move back through the working and highlight an earlier entry you wish to alter. Then press:

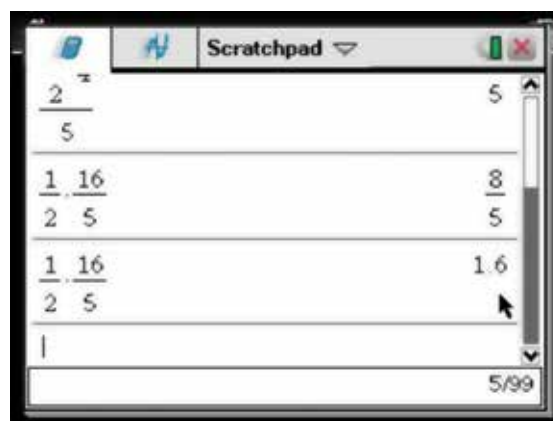
- **ENTER** .






5. Use the back arrow  and the delete key  to delete numbers, and then type in new numbers. Use the tab key  to move between fields.

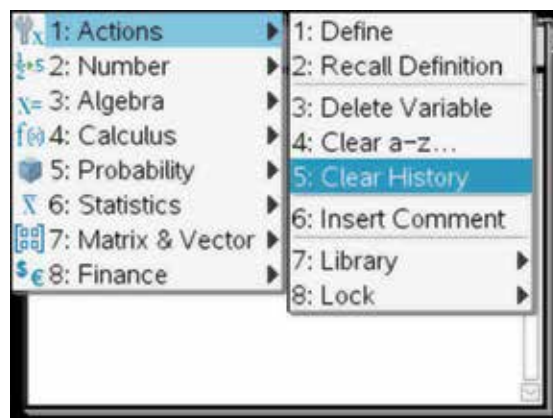
Note: To convert the exact answer to its decimal equivalent (or approximation) as shown, press:

- **ctrl** 
- **ENTER** .



6. If your screen is getting too congested, it can be cleared. To do this press:

- **MENU** 
- **1: Actions** 
- **5: Clear History** .



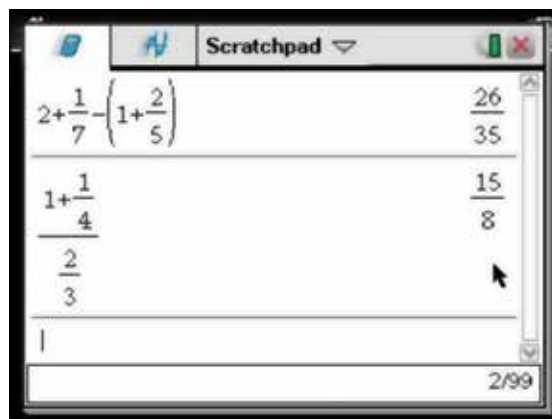
7. Mixed numbers must be entered using brackets.

For example, $2\frac{1}{7}$ must be entered as $\left(2+\frac{1}{7}\right)$.

Now try these.

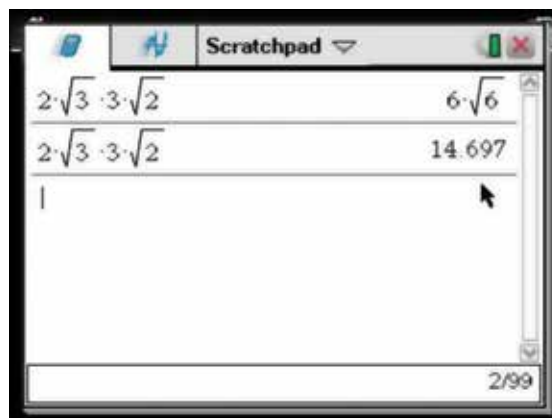
(a) $2\frac{1}{7} - 1\frac{2}{5} =$

(b) $1\frac{1}{4} \div \frac{2}{3} =$



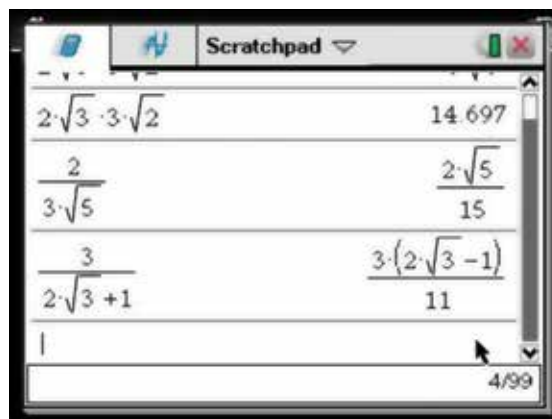
1.6 How to use the Mathematical Expressions palette

1. If the Calculation Mode in the General Settings is set to Auto, the calculator will produce answers in either exact or approximate form, depending on how data is entered. For example, $2\sqrt{3} \times 3\sqrt{2}$ will produce the answer $6\sqrt{6}$ as shown. The approximate value is also shown. In exact form, answers will always be given exactly. In approximate form, answers will always be given as decimal approximations. The number of decimal places displayed is determined from the Display Digits in the General Settings; for this example they are set to Fix 3, that is 3 decimal places.



2. Sometimes the calculator returns surprising results such as those shown.

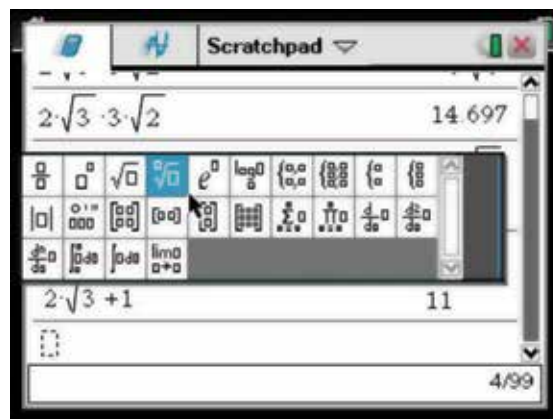
Note: The calculator will automatically rationalise the denominator of a surd.



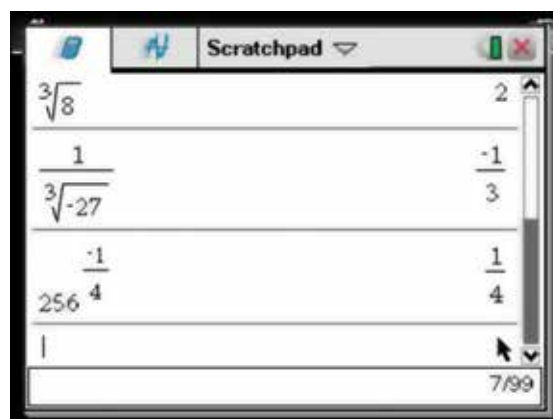
- Numbers can be entered using the Mathematical Expressions palette. To do this, press:



The Mathematical Expressions palette will appear as shown. Use the arrow keys on the Touchpad to select the required mathematical notation.



- You can now use conventional mathematical notation as required. Some examples are shown.

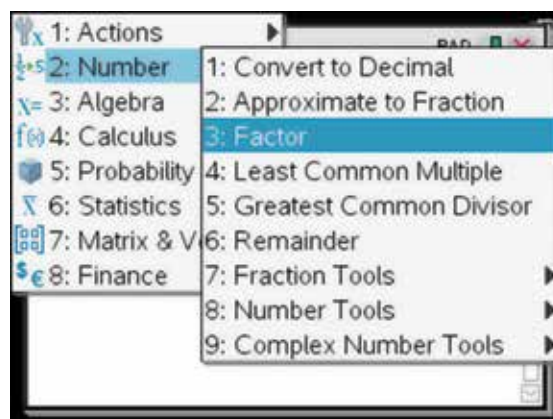


1.7 How to perform common numerical tasks

- Some other interesting features of the calculator are shown in the screen opposite.

For example, to find the prime factors of a number, press:

- MENU**
- 2: Number**
- 3: Factor** .

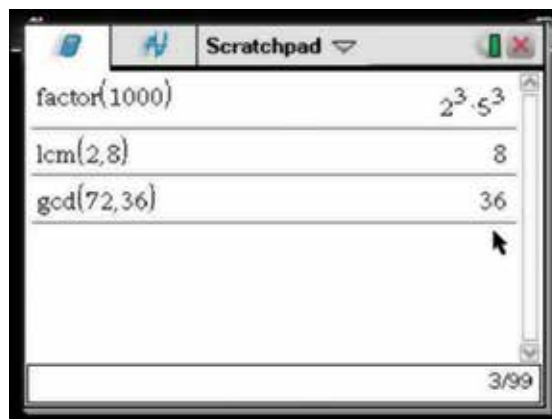


-
2. Complete the entry line as:

`factor(1000).`

Then press the enter key **enter**.

Examples of finding the Least Common Multiple (lcm) and the Greatest Common Divisor (gcd) are also shown.



1.8 How to transfer files between calculators

You can send documents and the operating system (OS) files to another calculator. If a file with the same name is already stored in the receiving calculator, the file will be renamed automatically. For example, if the file was FileName and the receiving calculator already has a file called FileName, then the incoming file will be renamed FileName (2).

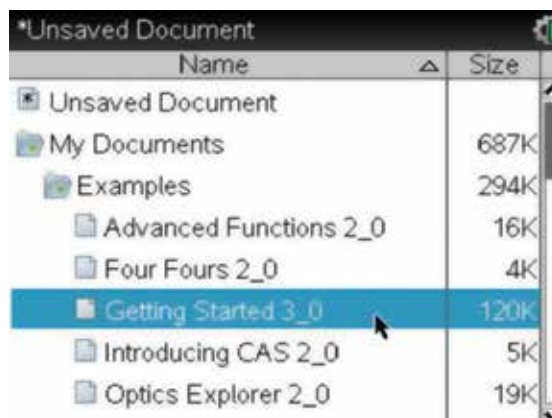
1. To send a file between calculators, first you must connect the calculators with the connector cable.

To locate the file to send, open My Docs by pressing:




- **HOME** **on**
- **2: My Docs** **2**.



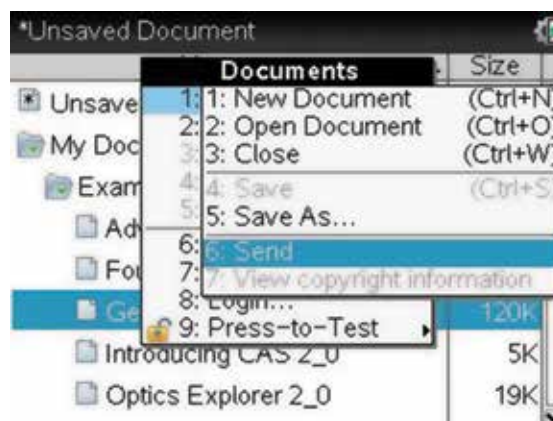
2. Select the file (or folder) you wish to send by using the arrows on the Touchpad.






3. To send a file from My Docs, press:

- **doc** 
- **1: File** 
- **6: Send** .

While the file is being transferred, a progress bar will be displayed. A message will appear when the transfer is complete. No action is required on the receiving calculator.







4. To see the current version of the operating system on your handheld calculator, along with battery levels and available memory space, press:

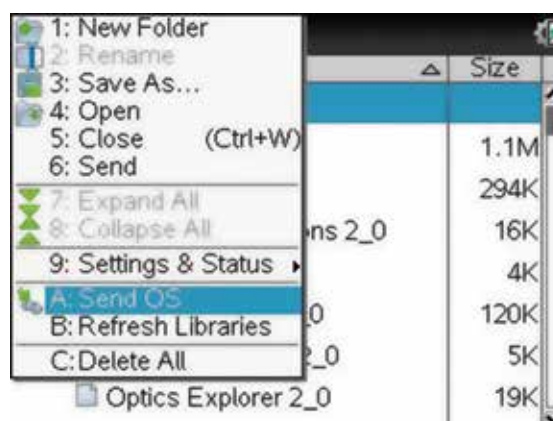
- **HOME** 
- **5: Settings** 
- **4: Status** .



5. To send the operating system from one handheld to another (to update the operating system on the receiving handheld calculator no action is required), press:

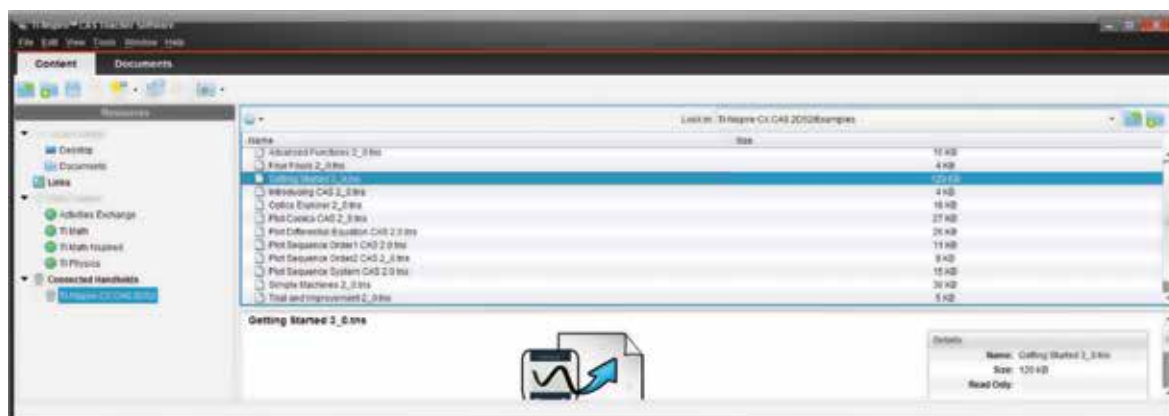
- **HOME** 
- **2: My Docs** 
- **MENU** 
- **A: Send OS** .

This may take a few minutes. Do not disconnect the two handheld calculators until prompted to. The files on the receiving handheld calculator will remain intact. Make sure you send operating systems to a handheld that is compatible, for example, you cannot send a CAS operating system to a non-CAS handheld.



1.9 How to transfer files between calculators and computers

Many useful TNS files are located at <http://education.ti.com/educationportal/sites/US/homePage/index.html> and can be downloaded to a handheld calculator free of charge. These files can be transferred between computers and calculators. To do this, you could use the TI-Nspire Computer Link Software, which will need to be installed on your computer, or use the TI-Nspire Student Software emulator. To send a file between a computer and a calculator, connect the handheld calculator to the computer with the USB cable. Open the TI-Nspire Teacher or Student Software and select your handheld calculator. Now select the Content tab on the screen. Open the folder in which the desired TNS files have been saved. Drag this file or folder to the TI-Nspire handheld. This will copy the file onto the handheld calculator. A new folder with the current date as its name will be in the My Documents folder with the transferred files.





CHAPTER 2

Variables and expressions

The instructions that follow will assist you in working with variables and expressions, which are common features of many mathematical processes.

2.1 How to simplify algebraic expressions

1. It is often necessary to open a new document. To do this, press:

- **ctrl** 
- **N** .

Select:

No (if you do not want to save the current document).

Then press ENTER .



2. To work with variables, open a new Calculator page. To do this, press:

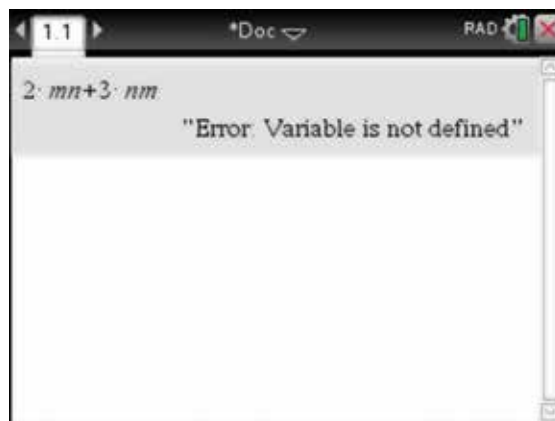
- **1: Add Calculator** .



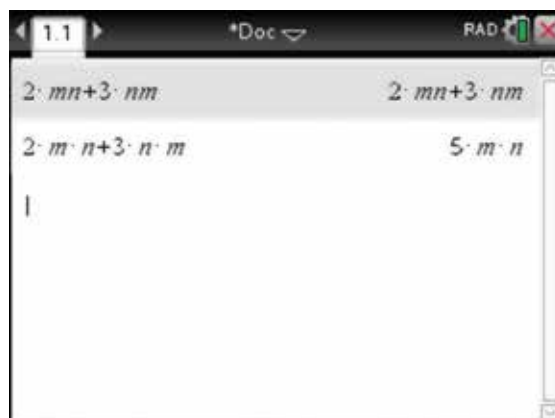
3. Complete the entry line as:

$$2mn + 3nm.$$

Note: The result shown occurs because the two variables were entered without the multiplication operation between them.



4. This is the answer you should expect.



5. Try simplifying the following.

(a) $9n + 4 + 3n + 7$

(b) $5y^2 + 3y + 2y^2 - 7y$

(c) $6p \times 9hn$

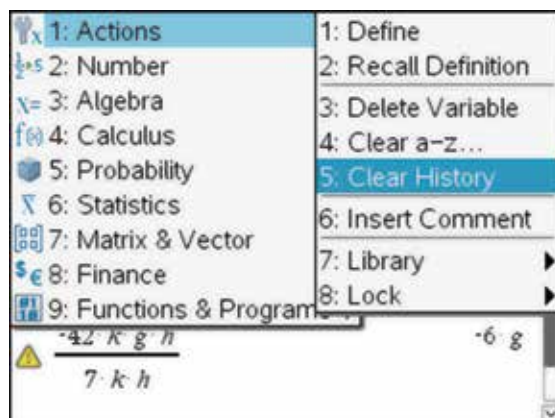
(d) $\frac{-42kgh}{7kh}$

Again, note the importance of including the multiplication sign between variables.



6. If the screen appears congested, it can be cleared by completing the following steps. Press:

- MENU **menu**
- 1: Actions **1**
- 5: Clear History **5**.






2.2 How to substitute values for pronumerals

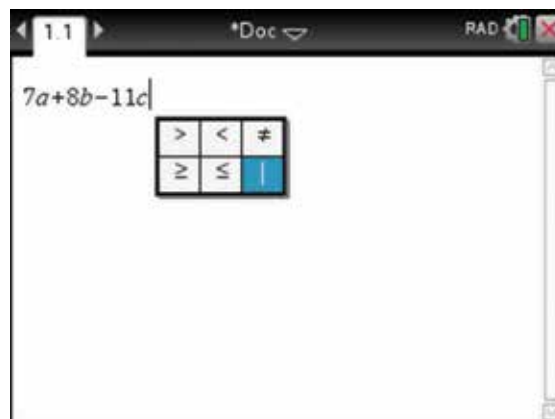
1. Answer the following question.


Find $7a + 8b - 11c$ given $a = 2$, $b = 3$ and $c = 5$.

To do this, on a Calculator page, complete the entry line as:

$7a + 8b - 11c$ | $a = 2$ and $b = 3$ and $c = 5$.

To get the given or with symbol |, press the ctrl key  and then  to bring up the palette shown. Use the Touchpad to select the | symbol. Type 'and', or it can be found in the catalog .





2. Then press enter .

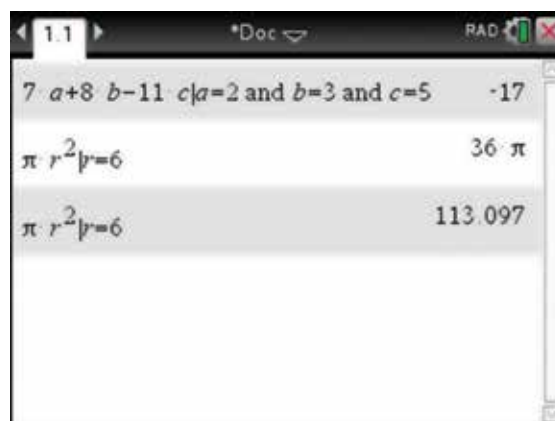


3. Answer the following problem using the same process as above.

The area A of a circle of radius r is given by $A = \pi r^2$. Find the area of a circle of radius 6, giving your answer correct to 4 decimal places.

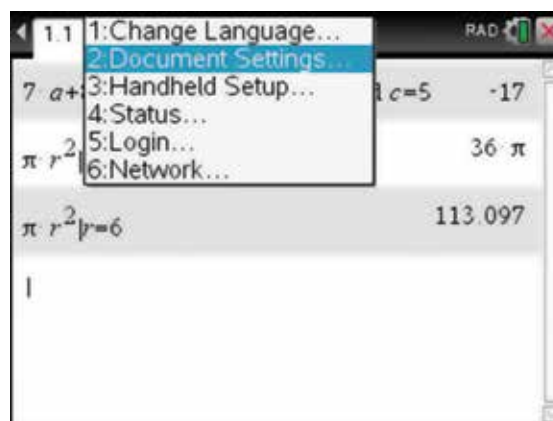
Note that the  key is located at the bottom left of the calculator, next to the alpha keys and . When the Calculation Mode in the General Settings is set to Auto, a decimal approximations can be accessed by pressing:

- ctrl 
- ENTER .

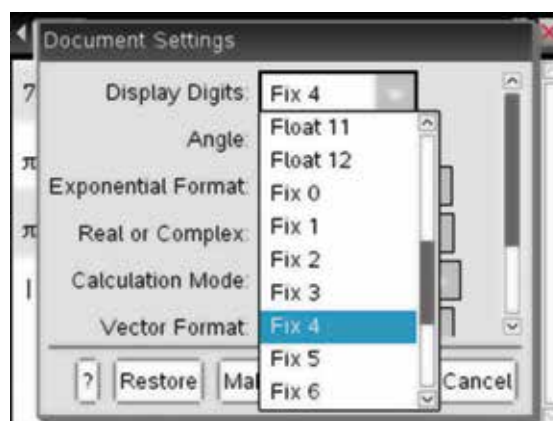


4. To set or change the number of decimal places for the current document and all new documents, using the cursor, click on the Document Settings icon located next to the close box. Then press:

- **2: Document Settings** **(2)**.

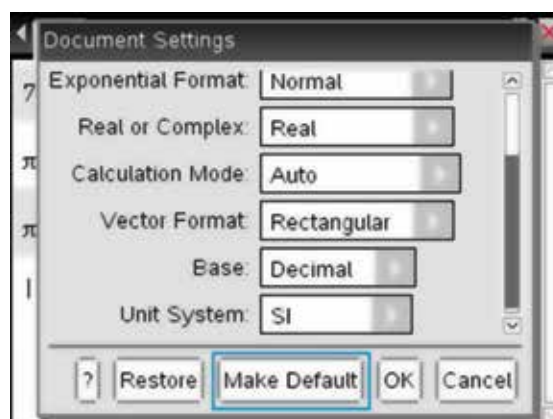


5. Press the down arrow ▼ on the Display Digits box and then select the number of decimal places required. For example, Fix 4 means 4 decimal places.



6. To apply these settings to all new documents, including the Scratchpad, press the tab key **(tab)** a number of times until Make Default is selected. Then press:

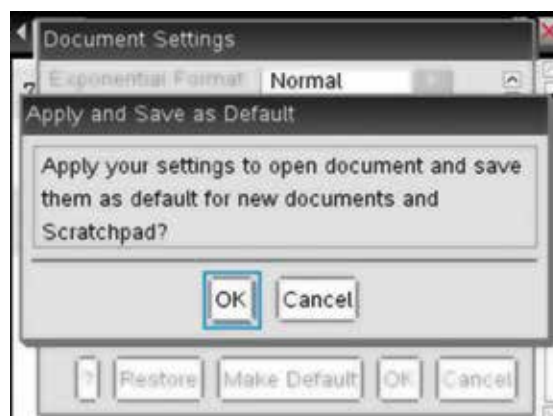
- **ENTER** **(enter)**.



7. The following screen appears. Press:

- **ENTER** **(enter)**.

You will then be returned to the Calculator page, which was the last page on which you were working.





2.3 How to substitute values using function notation

1. Consider the following problem.

If $y = x^4 - \frac{4}{5}x^3 + 3x^2 - \frac{16}{9}$, find y when $x = -3$.

Let us define y as $f1(x)$. For this problem, we will use the Scratchpad. To open a Scratchpad, press:

- **HOME**  **on**
- **A: Calculate** 

or press:




- .

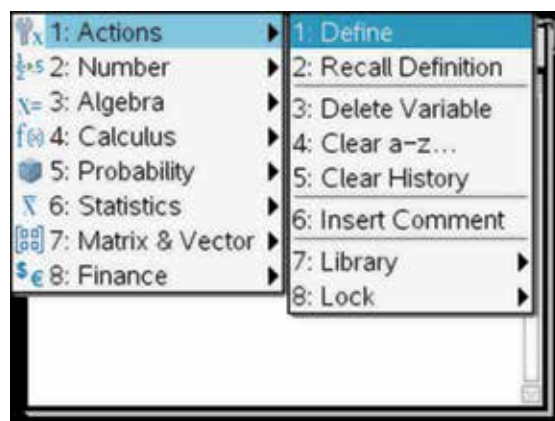


2. An empty Scratchpad will appear. The Scratchpad can be used only as a Calculator or Graphs page and can be used for quick calculations. A subset of the menus normally available for the Calculator and Graphs screens is available in the Scratchpad.



3. To define the function, press:

- **MENU** 
- **1: Actions** 
- **1: Define** .

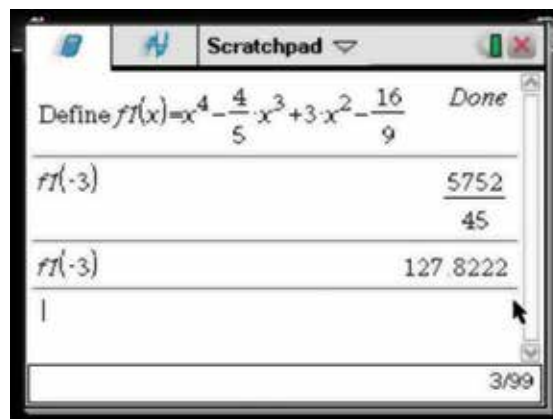


- Complete the entry line as:

Define $f1(x) = x^4 - \frac{4}{5}x^3 + 3x^2 - \frac{16}{9}$.

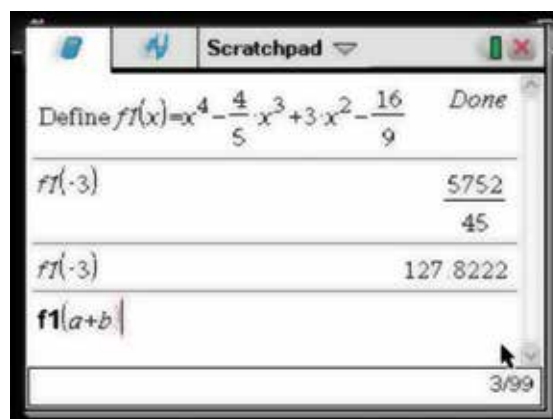
Then press the enter key **(enter)**.

Then type $f1(-3)$ and press the enter key **(enter)**.



- Try finding $f1(a+b)$.

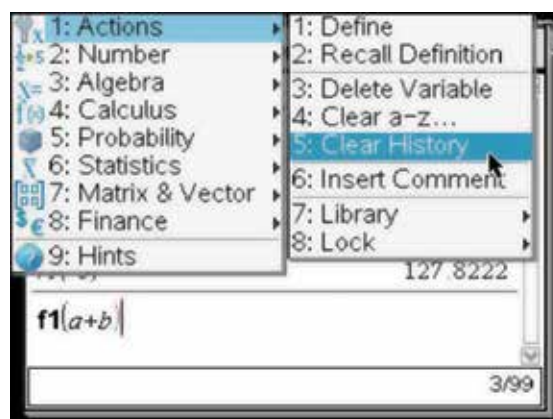
Note: When entering a function that has been defined already, the calculator denotes the defined function in bold to serve as a reminder that it has been defined.






2.4 How to expand algebraic expressions

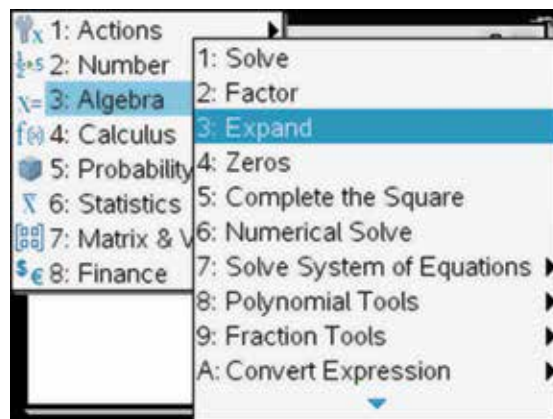
- To clear the Scratchpad page or a Calculator page, press:

- MENU** **(menu)**
- 1: Actions** **(1)**
- 5: Clear History** **(5)**.



2. To expand expressions, press:

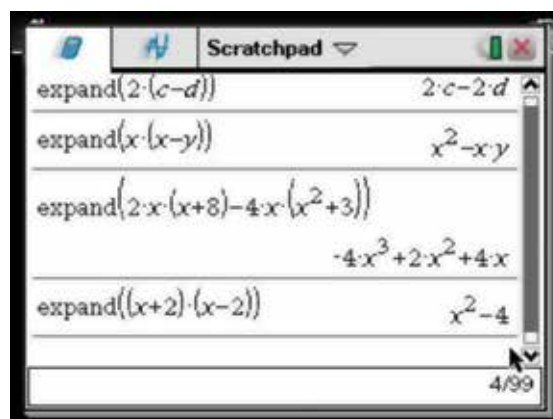
- MENU 
- 3: Algebra 
- 3: Expand .



3. Use the steps outlined above to expand the following.




- (a) $2(c-d)$
- (b) $x(x-y)$
- (c) $2x(x+8)-4x(x^2+3)$
- (d) $(x+2)(x-2)$

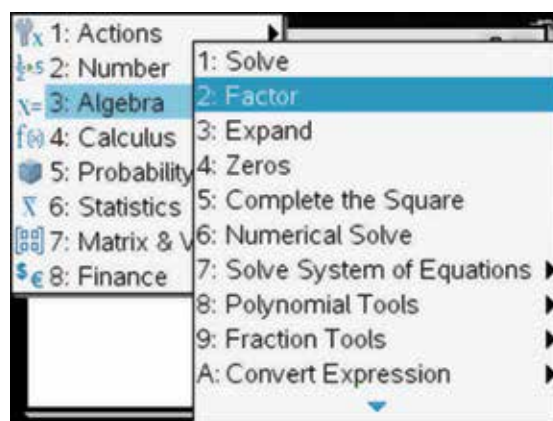
Note: Make a habit of including the multiplication sign rather than leaving it out.



2.5 How to factorise algebraic expressions using Highest Common Factor (HCF)

1. Clear the Scratchpad page. To factorise algebraic expressions using Highest Common Factor (HCF), press:

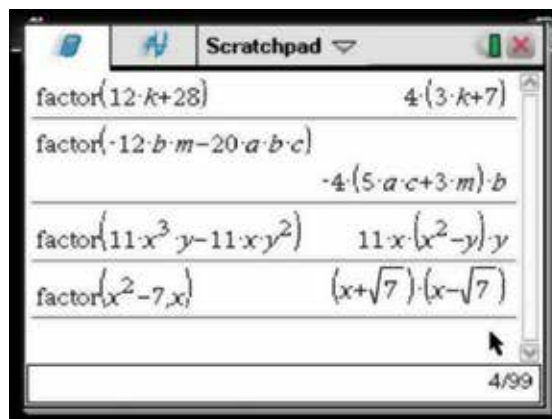
- MENU 
- 3: Algebra 
- 2: Factor .



2. Follow the steps above and complete the entry lines as shown to factorise the following expressions.

- (a) $12k + 28$
- (b) $-12bm - 20abc$
- (c) $11x^3y - 11xy^2$
- (d) $x^2 - 7$

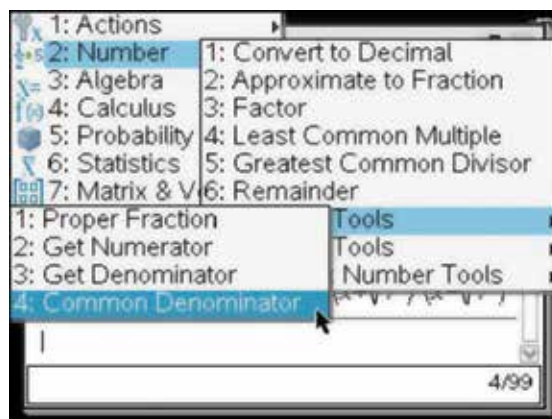
Note: If the factors are irrational, such as those in (d), you must complete the command as shown: factor($x^2 - 7, x$).



2.6 How to add and subtract algebraic fractions

1. To add algebraic fractions, it is necessary to find a common denominator. To do this, press:

- **MENU** (menu)
- **2: Number** (2)
- **7: Fraction Tools** (7)
- **4: Common Denominator** (4).

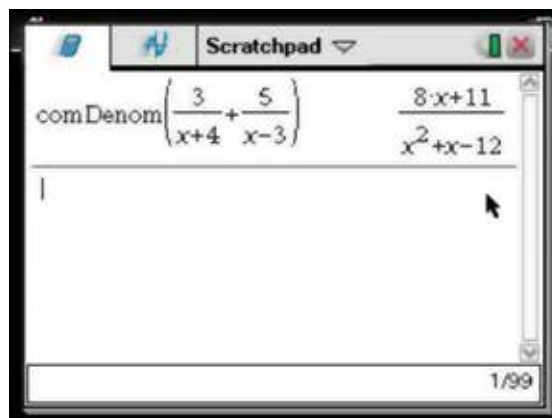


2. Using the steps outlined above, complete the entry line as:

$$\text{comDenom}\left(\frac{3}{x+4} + \frac{5}{x-3}\right).$$

Then press the enter key (enter).

Note: You will need to use the fraction template by pressing the ctrl key (ctrl) and then \div .



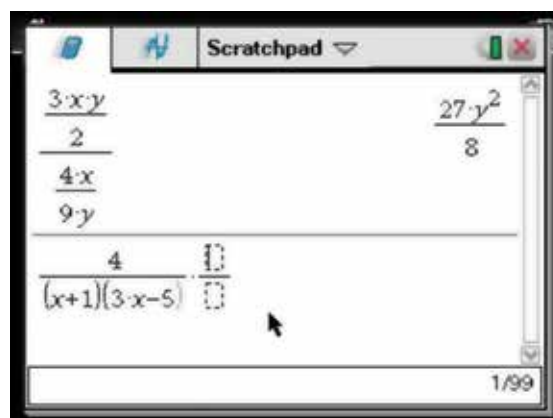
2.7 How to multiply and divide algebraic fractions

- To multiply or divide algebraic fractions, press the ctrl key (ctrl) and \div to get the fraction template, and type the fractions directly onto the screen as shown.

(a) $\frac{3xy}{2} \div \frac{4x}{9y}$

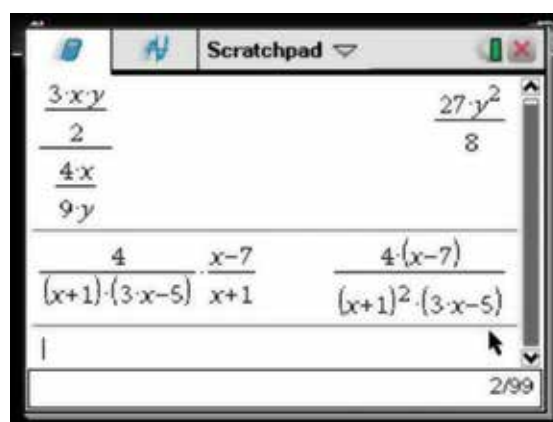
(b) $\frac{4}{(x+1)(3x-5)} \times \frac{x-7}{x+1}$

Note: Include the multiplication operator between the variables.



2. Notes

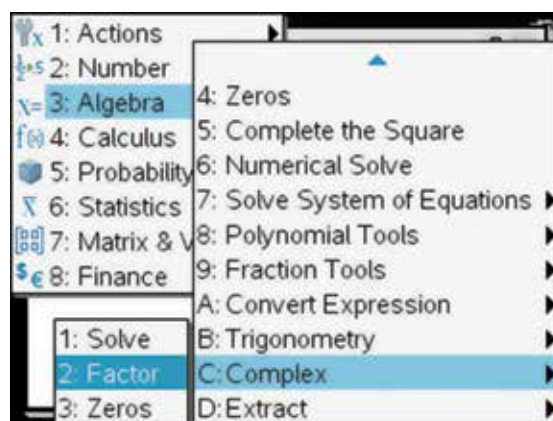
- Pressing the tab key (tab) will bring the cursor to the appropriate line of working.
- Include the multiplication operator between variables and brackets.



2.8 How to factorise over the complex number field

- To factorise over the complex number field, clear the Scratchpad page and press:

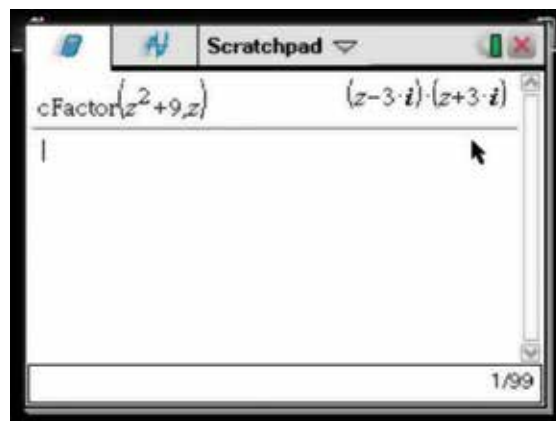
- MENU** (menu)
- 3: Algebra** (3)
- C: Complex** (C)
- 2: Factor** (2).



-
2. Complete the entry line as:

$\text{cFactor}(z^2 + 9, z)$.

Then press the enter key **enter** .



CHAPTER 3

Linear functions

The study of linear relationships is integral to all studies of mathematics and forms the platform on which the understanding of the behaviour of functions is based.

3.1 How to plot linear graphs

1. To open a new document, press:

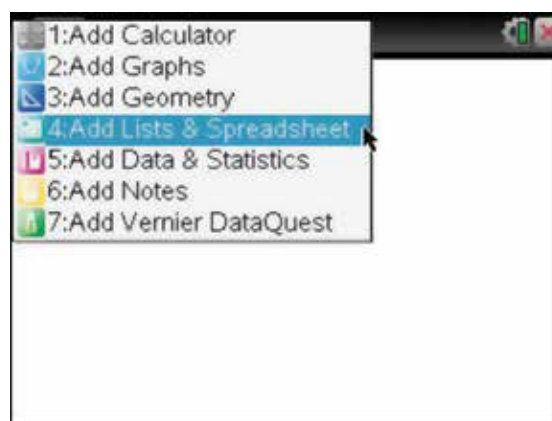
- **HOME**  **on**
- **1: New Doc** .

You will be asked if you wish to save the previous unsaved document. If you do not wish to do so, select:

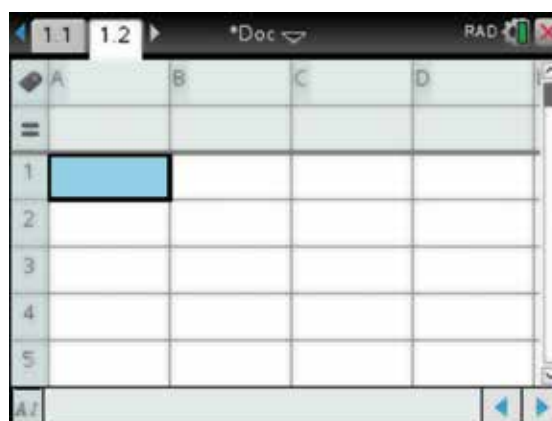
- **No.**

To enter data, add a Lists & Spreadsheet page by pressing:

- **4: Add Lists & Spreadsheet** .



2. The page shown will appear. Note that the cursor is over cell A1. You can use the arrow keys on the Touchpad to move between cells.



3. To plot the points for the linear graph with the rule $y = 3x + 2$, follow the steps outlined below. Enter the x -values in column A. Start with -10 in cell A1, -9 in cell A2, etc. all the way down to 10 in cell A21. There are more efficient ways to do this; these will be described in later chapters. Be sure to press enter **enter** after each value. To label column A, use the up arrow **▲** to scroll back to the very top cell of the column, next to the letter A. You should see by the dark line that appears around it that the cell is selected. Call this column $xvalue$ by typing these letters, then press enter **enter**.

	A xvalue	B	C	D
1	-10			
2	-9			
3	-8			
4	-7			
5	-6			

4. Similarly, to label column B, use the right arrow **▶** to move into cell B. Call this column $yvalue$, then press enter **enter**. The grey header cell below B is now highlighted. Complete the entry line as:

$$= 3 \times xvalue + 2.$$

Note: The formula appears at the base of your screen, which allows you to edit it at any time.

	A xvalue	B yvalue	C	D
1	-10			
2	-9			
3	-8			
4	-7			
5	-6			

Entry line: $yvalue = 3 \cdot xvalue + 2$

5. Press **enter** and the values appear in the column.

	A xvalue	B yvalue	C	D
1	-10	-28		
2	-9	-25		
3	-8	-22		
4	-7	-19		
5	-6	-16		

Entry line: $B1 = -28$

6. To produce a Quick Graph on this page, for these points, complete the following steps. Press:


- **MENU** **menu**
- **3: Data** **3**
- **9: Quick Graph** **9**.

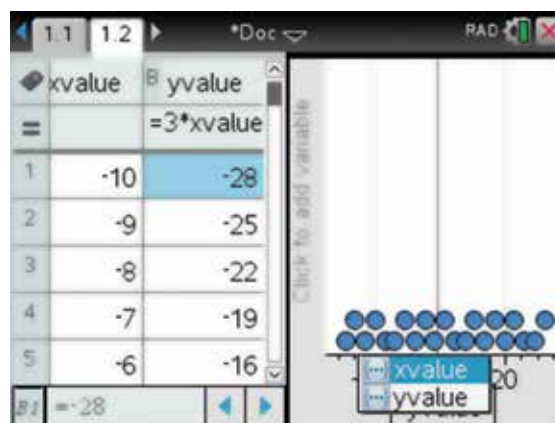
Note: A Quick Graph will be shown in a split screen for easy reference. Instructions for a full-screen colour graph will follow.


	A	B	C	D
2	-9			
3	-8			
4	-7			
5	-6			

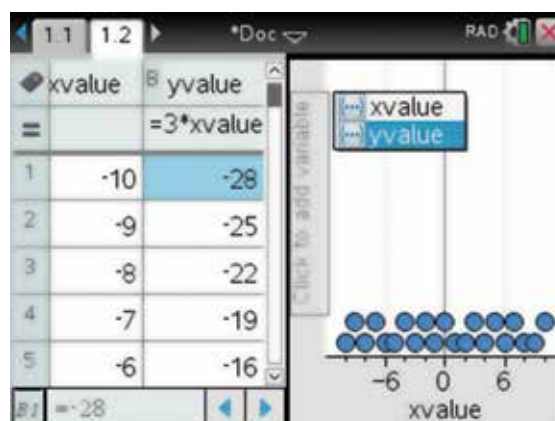
Entry line: $B1 = -28$

Menu path: 1: Actions, 2: Insert, 3: Data, 9: Quick Graph

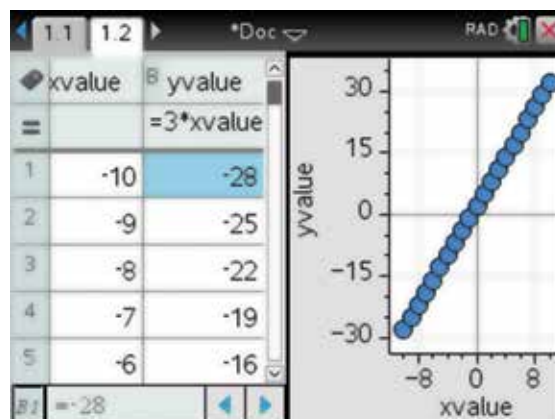
7. To move to the horizontal axis, press the tab key **(tab)**. Then select xvalue by pressing the click key .



8. To move to the vertical axis, press the tab key **(tab)**. Then select yvalue and press the click key .

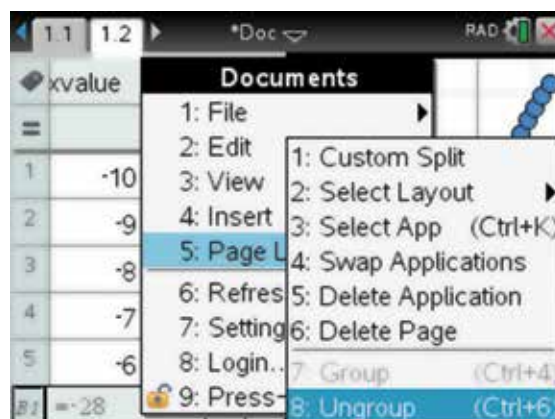


9. Then press the click key  or the enter key **(enter)**. This will draw a Quick Graph for the table of values.



10. To see the graph in a full screen, press:

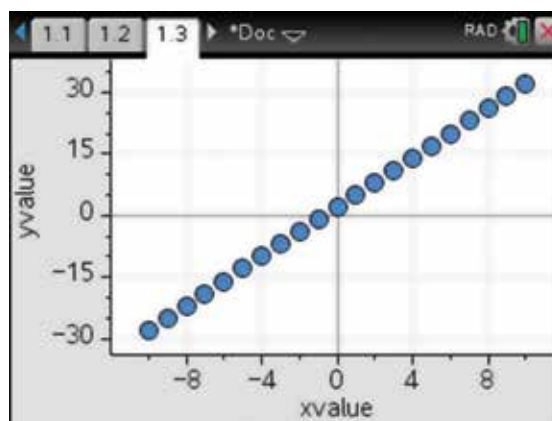
- doc **(doc)**
- 5: Page Layout **(5)**
- 8: Ungroup **(8)**.



11. Then to get to the graphs screen page, press:



- **ctrl** **(ctrl)**
- **►**.

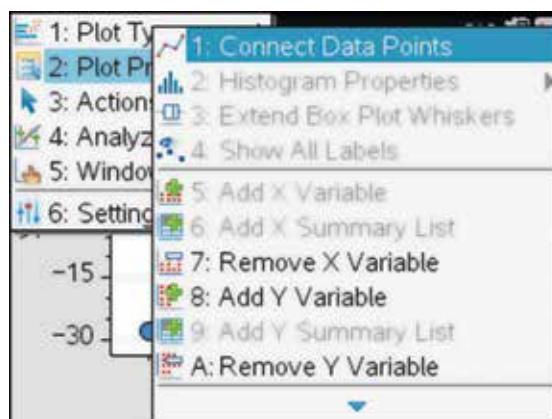
The graph appears as shown.



12. To join the points with a straight line, complete the steps below. Press:

- **MENU** **(menu)**
- **2: Plot Properties** **(2)**
- **1: Connect Data Points** **(1)**.

This will give the required linear graph of $y = 3x + 2$.
To show the coordinates of a particular point, move the pointer  to one of the points; it will change into an open hand  and the coordinates will be displayed.

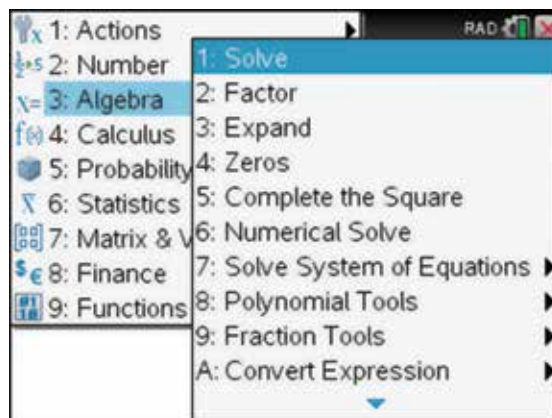


3.2 How to rearrange linear equations to make y the subject

1. The Solve command can be used to rearrange equations, particularly linear equations, in order to express the equation in the form $y = mx + c$.

For example, to find the y-intercept and gradient for the linear equation $4x + 5y = 3$, add a new Calculator page in the current document by pressing:

- **ctrl** **(1)** or **ctrl** **(doc)**
- **1: Add Calculator** **(1)**
- **MENU** **(menu)**
- **3: Algebra** **(3)**
- **1: Solve** **(1)**.



- Complete the entry line as:




solve (4x+5y=3,y).


Then press:

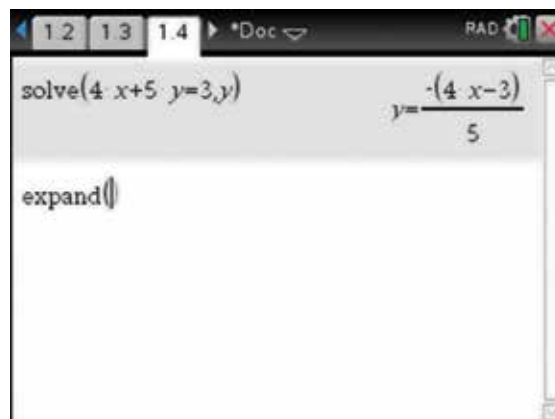
- **ENTER** .

The expand command will separate the two terms.

To find the expand command, press:

- **MENU** 
- **3: Algebra** 
- **3: Expand** .



Use the Touchpad  to highlight the equation in the previous line as shown.

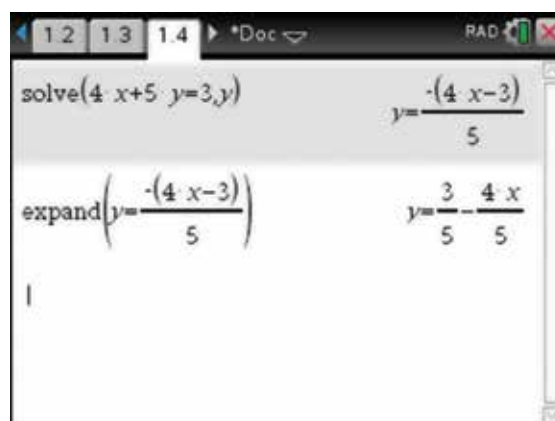


- Then press:

- **ENTER** .


This will paste the equation into position after expand(and will save retyping the equation. The gradient is $m = -\frac{4}{5}$ and the y-intercept is $c = \frac{3}{5}$.



The numbers 1.2 and 1.3 in the top left-hand corner refer to problem 1 and pages 2 and 3 in the current unsaved document. You are working on the page denoted by the tab 1.3, which has a white background as shown. You can toggle back to the previous page by using ctrl  and the left arrow  on the Touchpad, or simply pressing the blue arrows using the cursor.



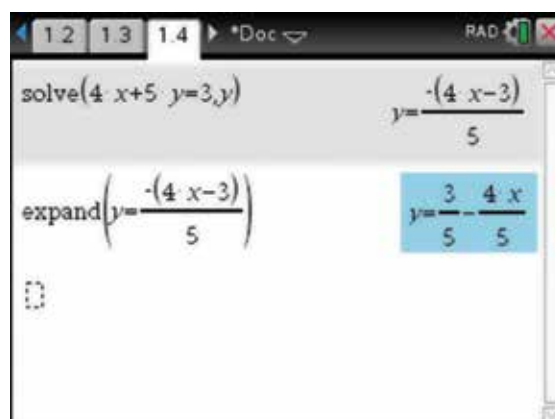
3.3 How to sketch linear graphs by pasting the rearranged equation into the graphing window

- Now we want to graph the straight line in section 3.2 and look at its features.

To copy the equation $y = \frac{3}{5} - \frac{4x}{5}$, use the up arrow  to highlight the equation, then press:

- **ctrl** 
- **C:** .

Note: This is similar to the Ctrl+C (copy) command in Microsoft Word.



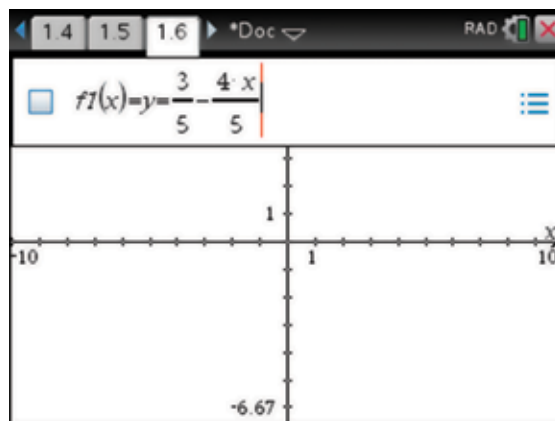
2. You will need a graphing window in which to draw the graph. To add a Graphs page to the current document, press:

- **ctrl** (ctrl)
- **I** (I)
- **2: Add Graphs** (2).

To paste the relevant equation into the function entry line next to $f1(x) =$, press:

- **ctrl** (ctrl)
- **V:** (V).

The 'y =' is still present and will need to be deleted.



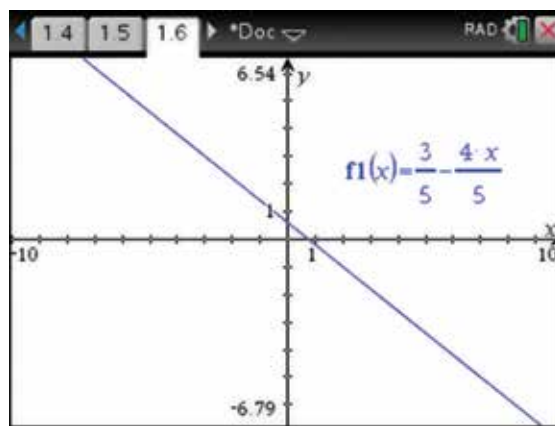
3. Press the left arrow (left arrow) to position the cursor, then press delete (del) twice to delete the 'y ='.

You should now have $f1(x) = \frac{3}{5} - \frac{4x}{5}$.

To draw the function, press:

- **ENTER** (enter).

The rule for the function $f1(x)$ appears on the graph screen in the same colour as the graph; it can be deleted or moved around the screen if required.



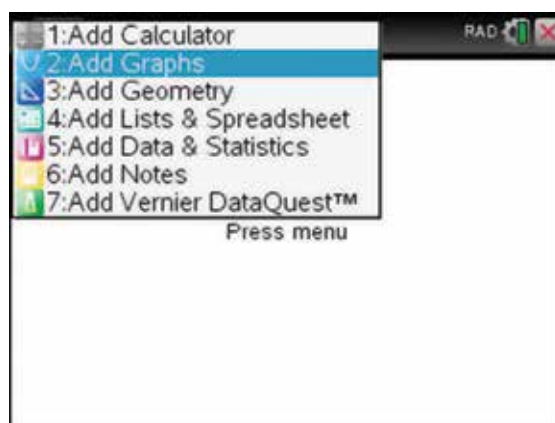
3.4 How to sketch linear graphs in the graphing window

1. To create a graph in a new document, press:

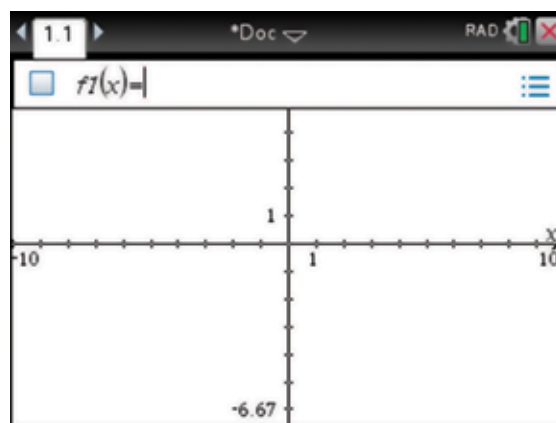
- **HOME** (home)
- **1: New Doc** (1).

You will be asked if you wish to save the previous unsaved document. If you do not wish to do so, select:

- **No**
- **2: Add Graphs** (2).



2. Your work area now contains the x -axis and the y -axis for a graph, the function entry line and specific tools for graphs. The scale on the axes as shown is the default scale. Instructions for altering this will follow.

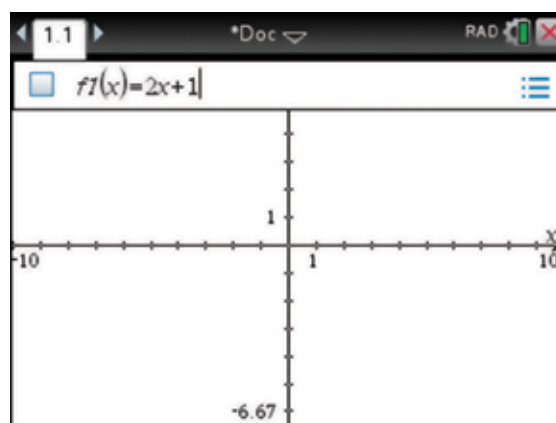


3. To draw the graph of $f(x) = 2x + 1$, complete the function entry line as:

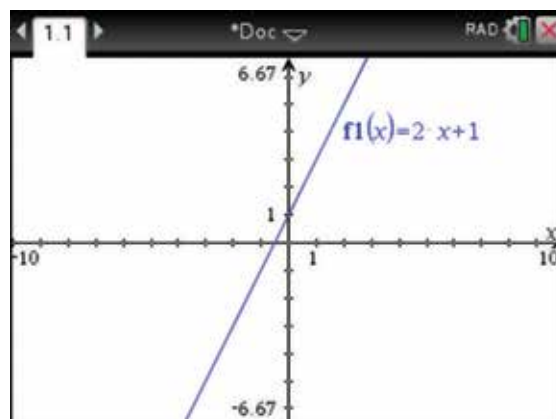
$$f1(x) = 2x + 1.$$



Then press:

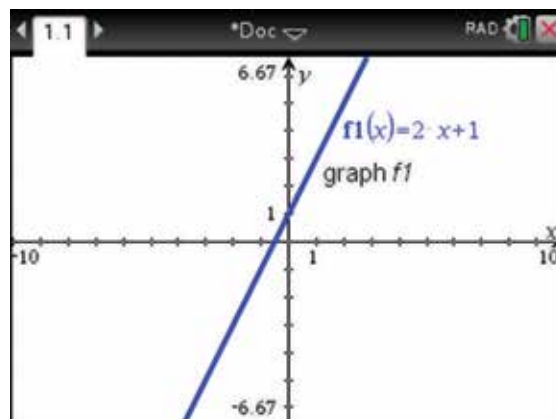
- **ENTER** (enter).



4. The graph will appear with the rule.

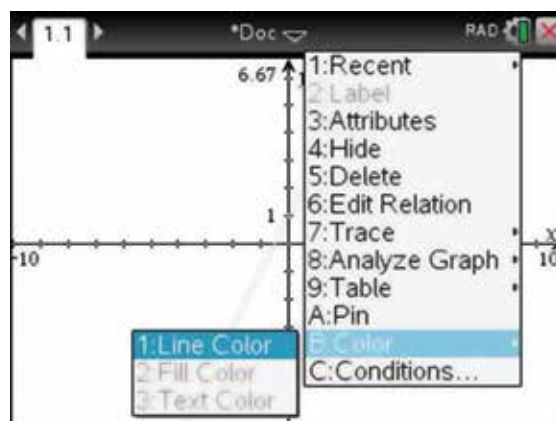


5. To change the colour of the graph, move the pointer  so that it is over the graph, when it turns into  and the label 'graph f1' appears.



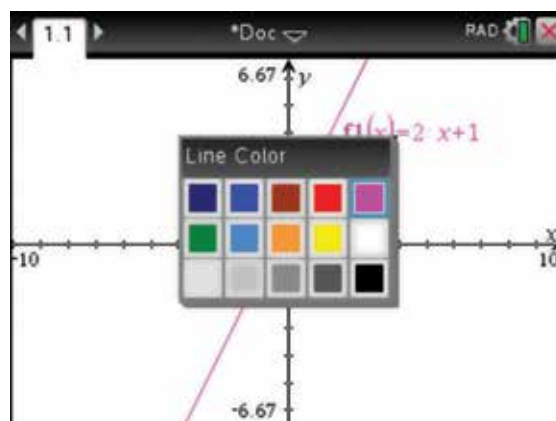
6. Press:

- **ctrl** (ctrl)
- **MENU** (menu)
- **B:Color** (B)
- **1: Line Color** (1).



7. Choose a colour and press enter (enter).

The label for the graph appears in the same colour as the graph. If we have several graphs on the one graph page, this is useful to identify which graph is which.



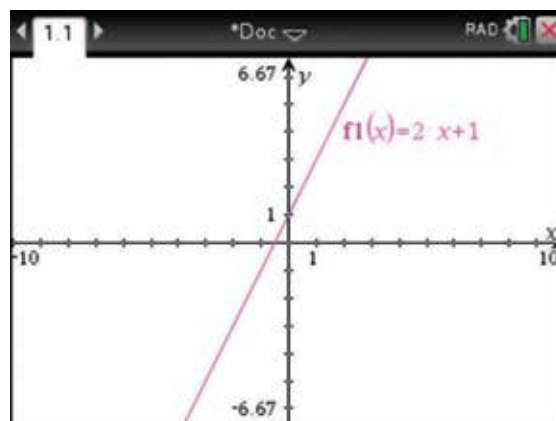
8. To return to the function line, press:

- **tab** (tab).

To return to the graph area, press:

- **esc** (esc).

Note: Pressing the ctrl key (ctrl) and (G) toggles between hiding and displaying the function entry line.

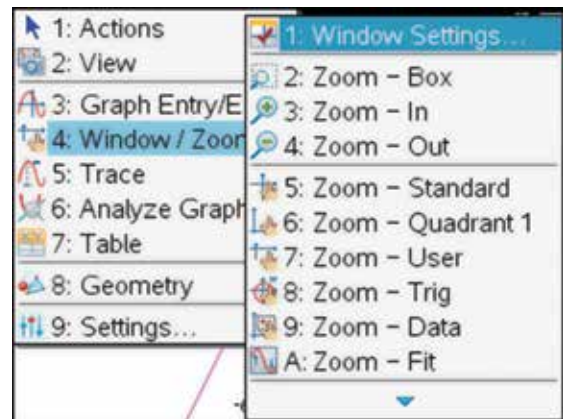


3.5 How to alter the scale settings for graphs

1. To alter the scale of the axes for the graph in section 3.4, complete the following steps.

Press:

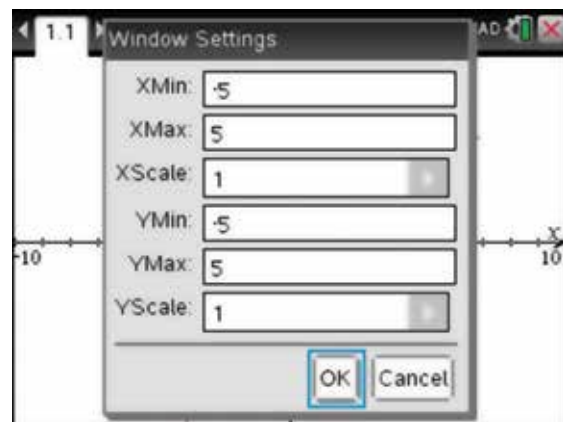
- **MENU** (menu)
- **4: Window / Zoom** (4)
- **1: Window Settings** (1).



2. A Window Settings box appears, showing the default graphing scale. You can now change values to the desired settings. Enter the values as shown. Use the tab key (tab) when moving between fields. To return to a previous field, press the shift key (shift) followed by the tab key (tab). When satisfied with the window settings, select:

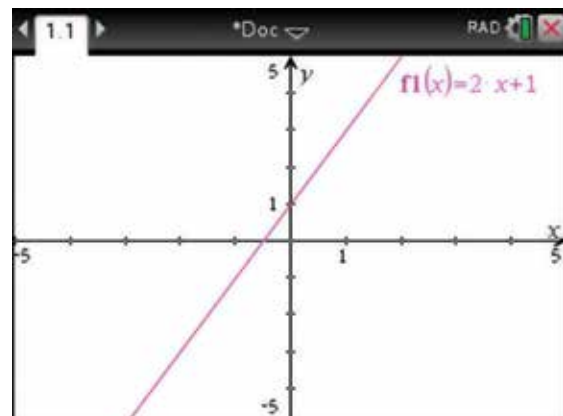
- **OK**
- **ENTER** (enter).

The scales on the x -axis and the y -axis can be left in Auto mode as shown or they can be altered as required.






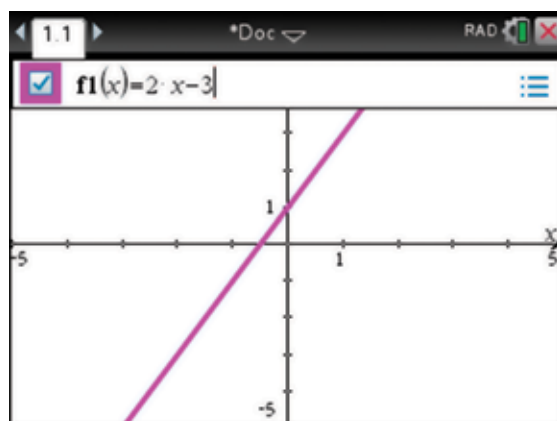
3. The graphing page appears with the new scale.

It is possible to change the scale using a variety of methods. More of these will be explored in later chapters.



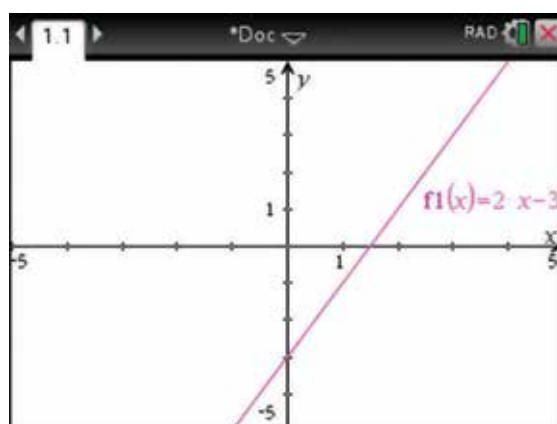
3.6 How to edit the equation of a graph on the Cartesian plane




1. You can edit the equation of the graph on the Cartesian plane in section 3.5, and the sketch of the graph will change automatically. To do this, use the Touchpad to place the pointer  over the equation. The open hand  will appear. Press the click button  firmly twice. The function entry line will appear. You can now edit the equation. In this case, change +1 to -3.



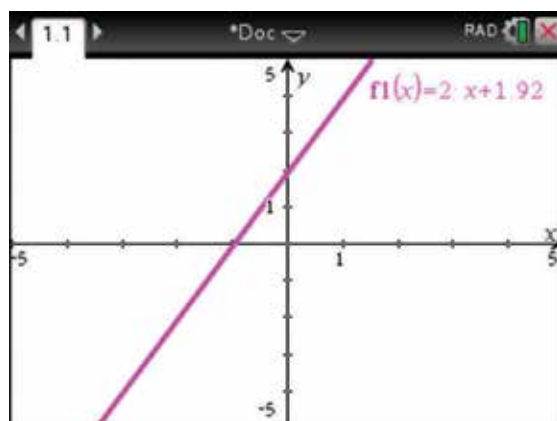
2. To view the new graph, press:

- ENTER .







3. Another way to change the rule of the graph is to move the cursor close to the graph until  and 'graph f1' appears. Then press and hold the click button  until the closed hand  appears.



Then use the Touchpad to move the graph up, down, left or right, and watch the effect on the equation. In this case, the gradient of the graph remains unchanged but the y-intercept can be changed.



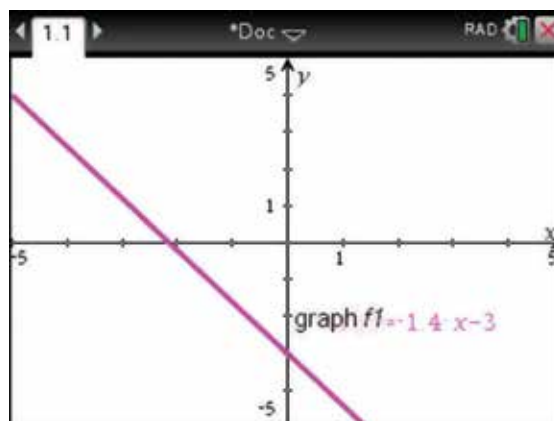
- To release grabbing the graph, press:

- **esc** .

To see changes to the gradient of the graph, use the Touchpad to move the pointer  along the graph until the pointer changes into the form shown . Then press and hold the click  button until the closed hand  appears.

Note: The symbol to rotate the line  appears when you hover over the line near the endpoints of the line, while the symbol  appears if you hover over the line near its middle.




Use the Touchpad to rotate the line. Watch as the gradient changes but the y-intercept remains unchanged.

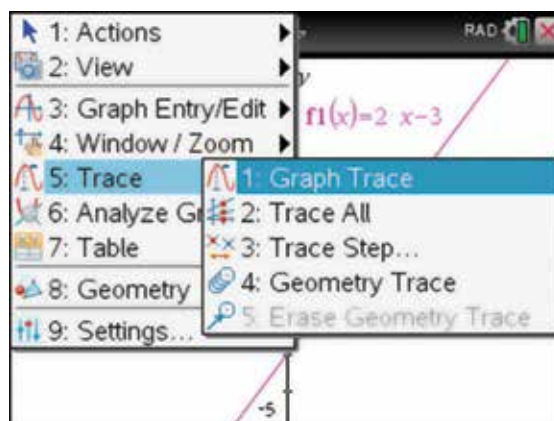


3.7 How to find the x-intercept of a graph

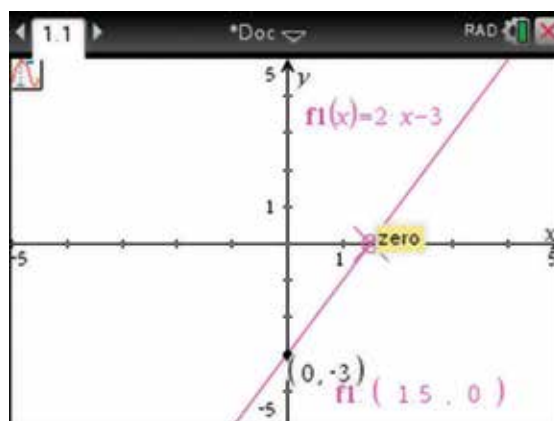
- To find the x-intercept of the linear graph $f_1(x) = 2x - 3$, complete the following steps.

Press:

- **MENU** 
- **5: Trace** 
- **1: Graph Trace** .



- Use the Touchpad to trace the required position on the x-axis. The label 'zero' appears in a box when the x-intercept is reached, and the coordinates (1.5, 0) are shown.



3.8 How to graph linear inequations

1. Begin with a new document and add a Graphs page.

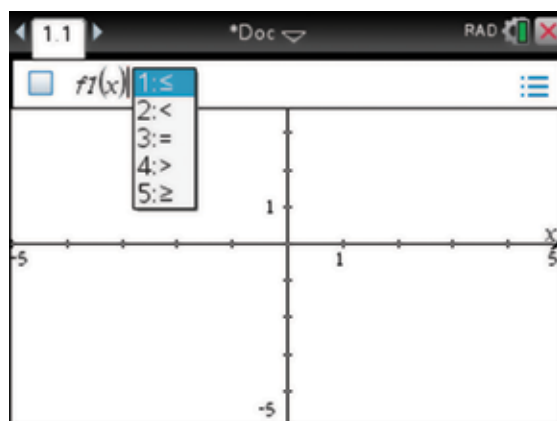
To graph $y \leq 2x$, complete the following steps.

Go to the function entry line. Press the delete key **del** to delete $f1(x) =$; then press:

- 1: **1** or ENTER **enter**

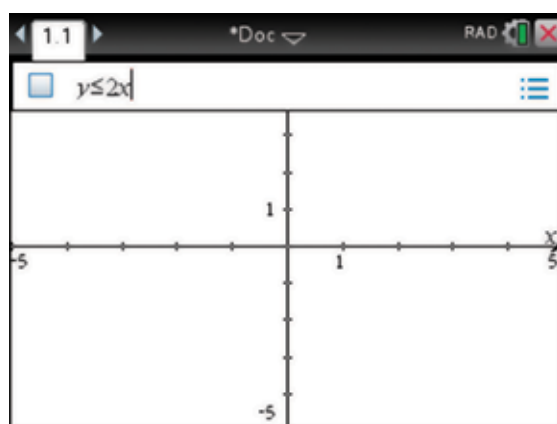
to insert the less than or equal sign, then type $2x$.

As soon as an inequality symbol is used, you are no longer graphing a function, so your rule appears as $y \leq 2x$.

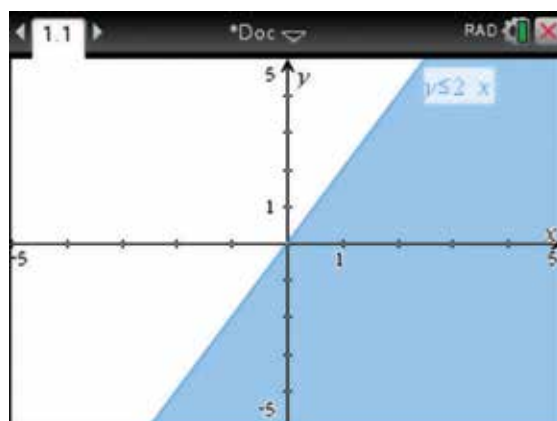


2. To view the graph of the inequality, press:

- ENTER **enter**.



3. The shaded coloured required region is the required region.





CHAPTER 4

Solving equations

The ability to solve a range of equations is essential in the study of mathematics at all levels.




4.1 How to solve linear equations

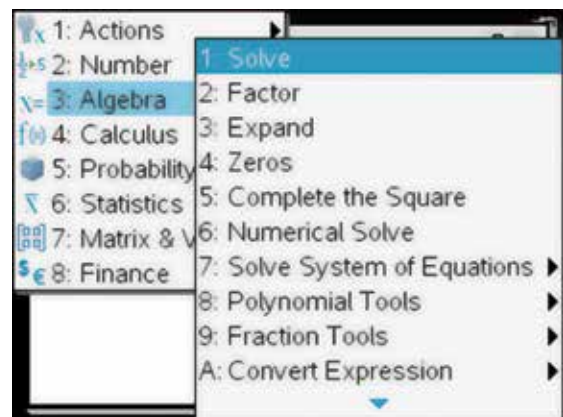
1. Equations can be solved using the Scratchpad. To get to this screen, press:

- HOME  on
- A: Calculate .



2. To solve an equation such as $6x + 46 = 75$, press:

- MENU 
- 3: Algebra  3
- 1: Solve  1.



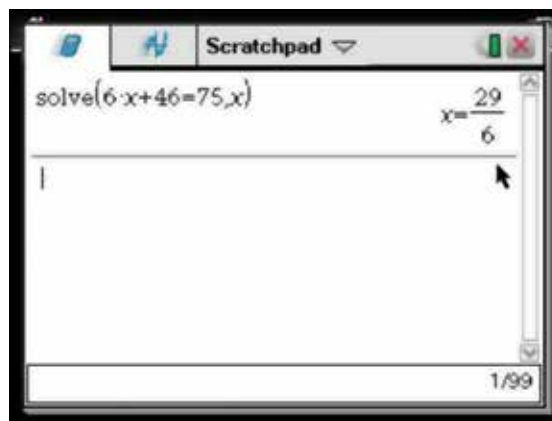
3. Complete the entry line as:

solve($6x + 46 = 75, x$).

This means 'solve the equation with respect to x '.





Then press:

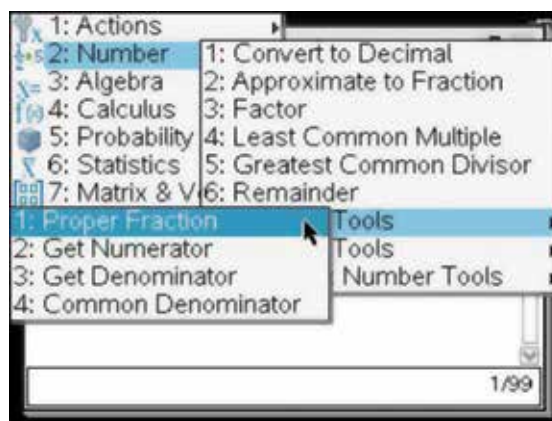
- ENTER .




4. The result is given as an improper fraction.

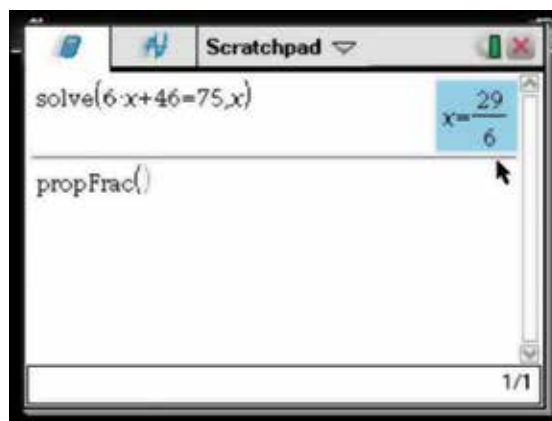
To change to a proper fraction, complete the following steps. Press:

- MENU 
- 2: Number 
- 7: Fraction Tools 
- 1: Proper Fraction .





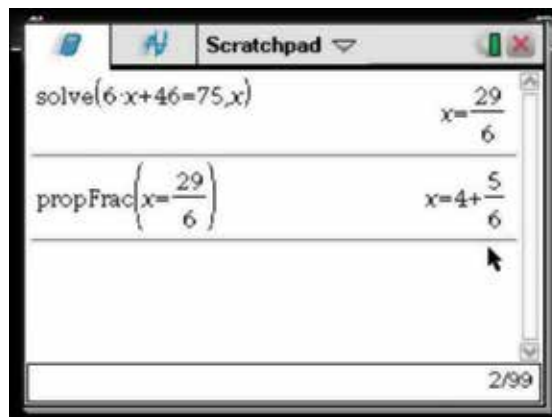
5. `propFrac()` will appear on the screen.

Use the up arrow  on the Touchpad to locate and highlight the previous answer.



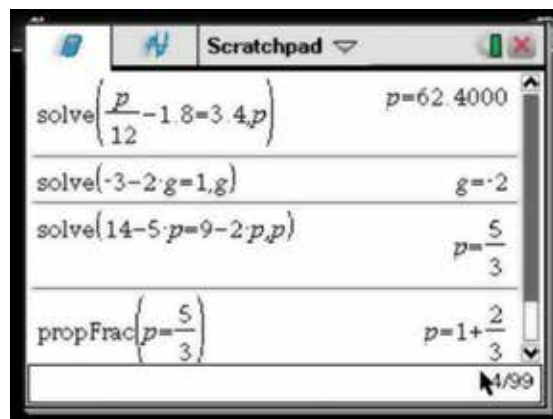
6. Then press:

- ENTER 
- ENTER .



7. Try to solve each of the following for the pronumeral shown in brackets.

- (a) $\frac{p}{12} - 1.8 = 3.4, (p)$
 (b) $-3 - 2g = 1, (g)$
 (c) $14 - 5p = 9 - 2p, (p)$



4.2 How to rearrange formulas

1. For example, to make T the subject in the equation for simple interest, $I = PRT$, press:

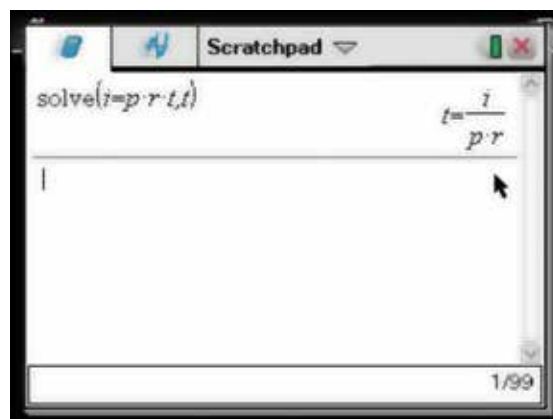
- MENU
- 3: Algebra
- 1: Solve

Then complete the entry line as:

solve ($i = p \times r \times t, t$).

Note that you must include the multiplication sign between variables. Then press:

- ENTER

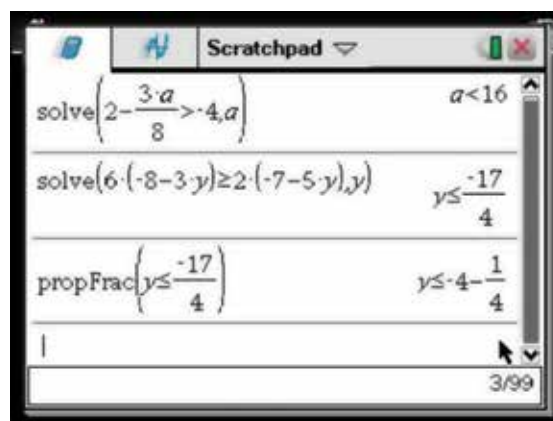


4.3 How to solve linear inequations

1. Press the ctrl key and to access the palette of inequality signs. Follow the steps given earlier for solving, and then complete the entry lines for the following inequations. Solve:

- (a) $2 - \frac{3a}{8} > -4$
 (b) $6(-8 - 3y) \geq 2(-7 - 5y)$

The \leq and \geq symbols can also be obtained by using the symbol template. Press the ctrl key and the catalog key , and then select the appropriate symbols using the Touchpad.



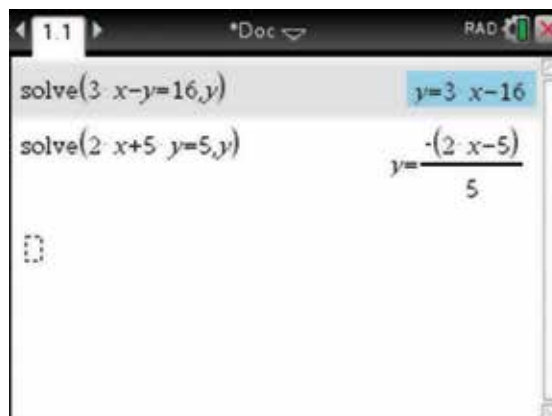
4.4 How to solve simultaneous equations graphically

Solve the following pair of simultaneous equations by sketching both graphs on the one set of axes and

$$\begin{aligned} 3x - y &= 16 \\ 2x + 5y &= 5 \end{aligned}$$

1. Open a new document and add a Calculator page (see section 2.1). Then rearrange both equations to make y the subject on the calculator screen by following the steps outlined in section 4.2. Complete as shown in the screen opposite. To copy and paste the rearranged equation into the Graph page, use the up arrow \blacktriangle on the Touchpad to highlight the first rearranged equation. Then press:

- **ctrl** ctrl
- **C** C .



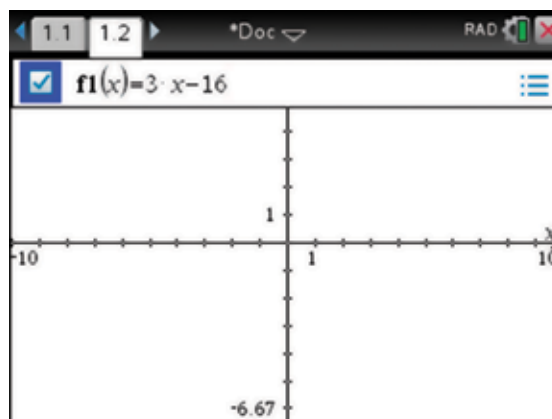
2. Now press:

- **HOME** home .

Select the Graph icon to add a Graphs page to the document. Then press:

- **ENTER** enter
- **ctrl** ctrl
- **V** V .

The equation will be pasted into the function entry line next to $f1(x)$. Use the left arrow \blacktriangleleft on the Touchpad and then press the delete key del twice to delete ' $y =$ '. Then press the enter key enter .

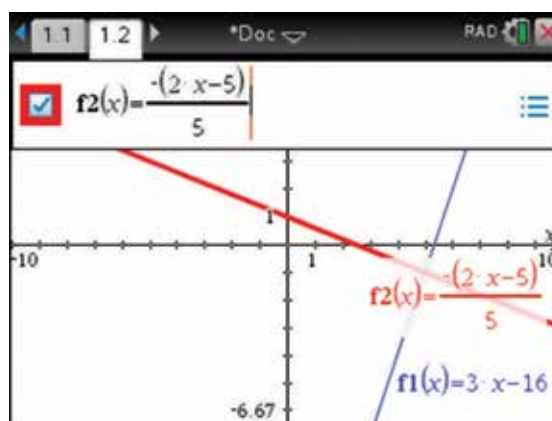


3. To return to the Calculator page to get the second equation, press:

- **ctrl** ctrl
- **left arrow** \blacktriangleleft

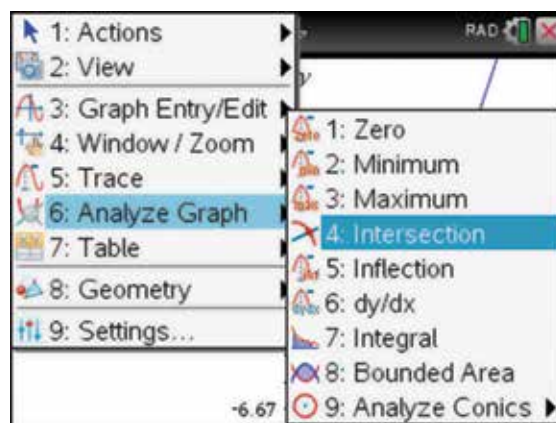
Repeat the earlier process for copying and pasting the second equation. Ensure that you paste the second equation next to $f2(x)$ and delete the ' $y =$ ' as before.


To get back to page 1.2, press the ctrl key ctrl and then the right arrow \blacktriangleright . To get to the entry line for $f2(x)$, press the tab key tab .



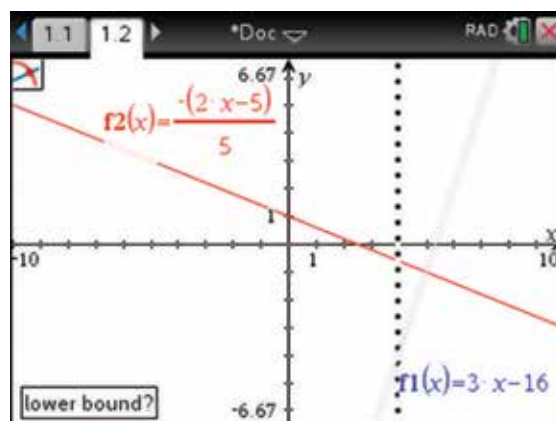
4. To find the point of intersection, complete the following steps. Press:


- **MENU** **menu**
- **6: Analyze Graph** **6**
- **4: Intersection** **4**.



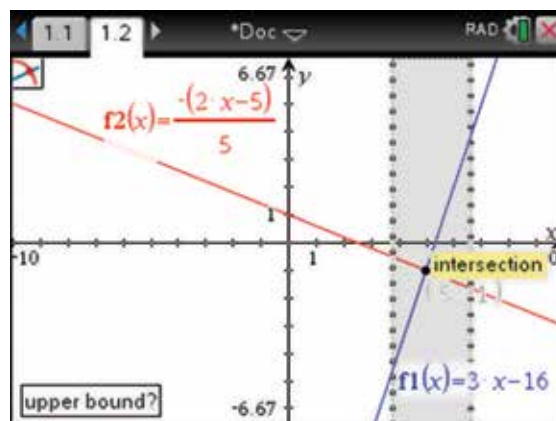
5. Use the Touchpad to move the hand with the finger  to the left of the point of intersection. Press:

- **ENTER** **enter**.



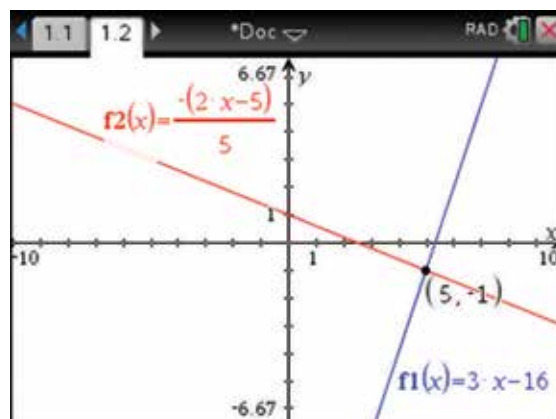
6. Use the Touchpad to move the hand with the finger  to the right of the point of intersection. Press:

- **ENTER** **enter**.






7. The point of intersection (5, -1) is displayed.

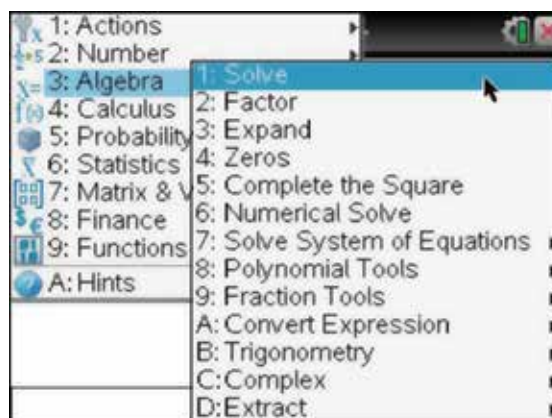
This graphical method will give solutions only as decimal approximations.



4.5 How to solve simultaneous equations algebraically


1. It is possible to solve simultaneous equations algebraically on a Calculator page. For example, solve the following equations simultaneously.
- $$3x - y = 16$$
- $$2x + 5y = 5$$
- Return to the Calculator page and press:


- **MENU** 
- **3: Algebra** 
- **1: Solve** .

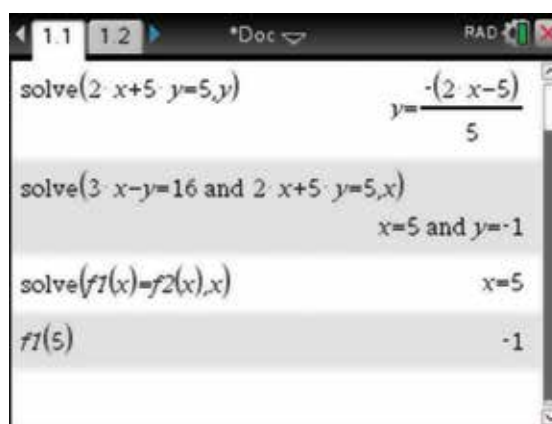


2. Then complete the entry line as:

`solve(3x - y = 16 and 2x + 5y = 5, x).`

Then press enter .

Alternatively, since $f1(x)$ and $f2(x)$ have already been defined in the graphs page, we could solve them as shown. Note that 'and' is found by pressing the catalog key , or it can be typed.

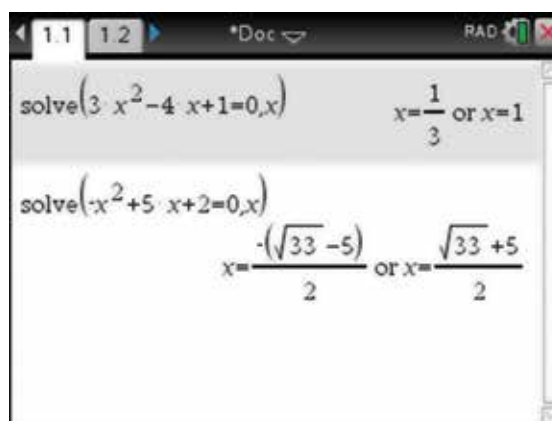


4.6 How to solve quadratic equations

1. The Solve application can be used to solve the following quadratic equations as shown:

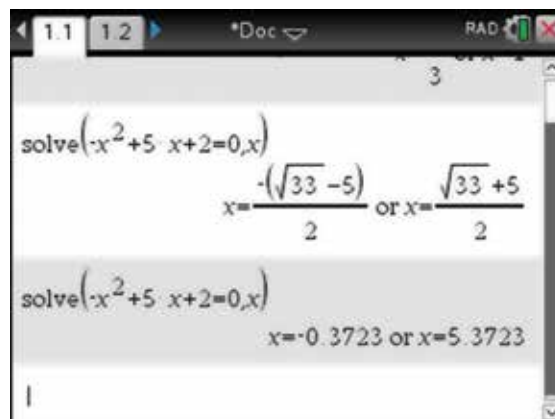
(a) $3x^2 - 4x + 1 = 0$

(b) $-x^2 + 5x + 2 = 0$.



2. If a decimal approximation is required, you can get it by pressing:

- **ctrl** 
- **ENTER** .

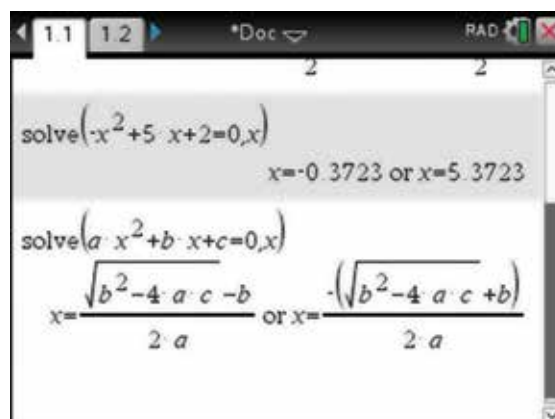


3. The Solve application can also be used to get general solutions to the quadratic equation $ax^2 + bx + c = 0$.

Complete the entry line as:

$\text{solve}(a \times x^2 + b \times x + c = 0, x)$.

Note: You must include the multiplication sign between pronumerals.



4.7 How to solve linear simultaneous equations with three unknowns





1. The following three linear simultaneous equations can be solved simultaneously for a , b and c .

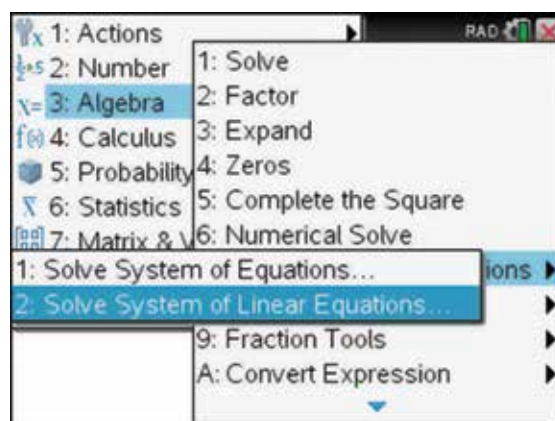
$$2a + b - 3c = 6$$

$$-4a + 3b - c = 5$$

$$a - 2b + \frac{3}{2}c = -5$$

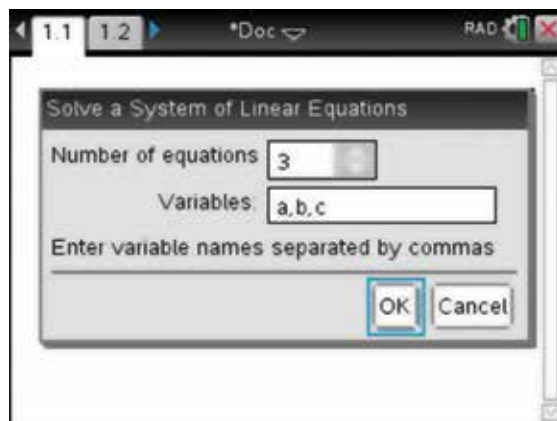
Press:

- **MENU** 
- **3: Algebra** 
- **7: Solve System of Equations** 
- **2: Solve System of Linear Equations** .



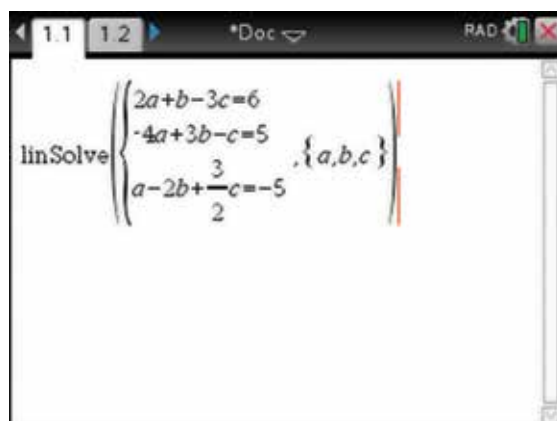
2. Complete as shown, tab **tab** to OK and press:

- **ENTER** **enter**.



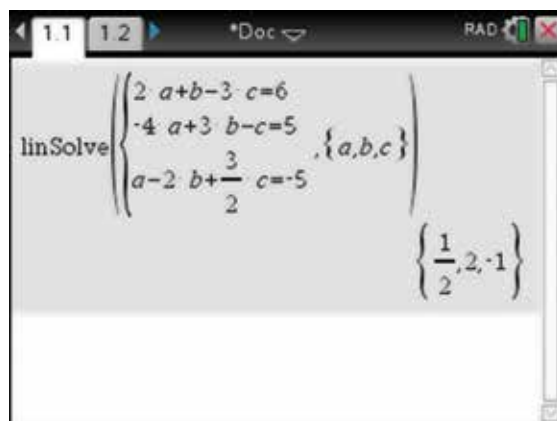
3. Complete the entry line as:

$$\text{linsolve} \left(\begin{cases} 2a+b-3c=6 \\ -4a+3b-c=5 \\ a-2b+\frac{3}{2}c=-5 \end{cases}, \{a,b,c\} \right).$$



4. Then press the enter key **enter**. The solution is

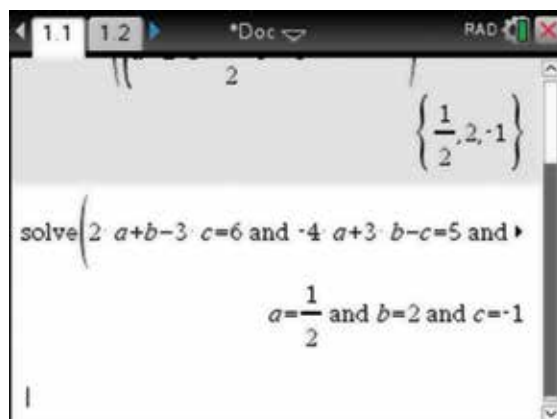
$$a = \frac{1}{2}, b = 2 \text{ and } c = -1.$$



5. Alternatively, we can solve these three linear simultaneous equations as shown.

$$\text{solve}(2a+b-3c=6 \text{ and } -4a+3b-c=5$$

$$\text{and } a-2b+\frac{3}{2}c=-5,a)$$

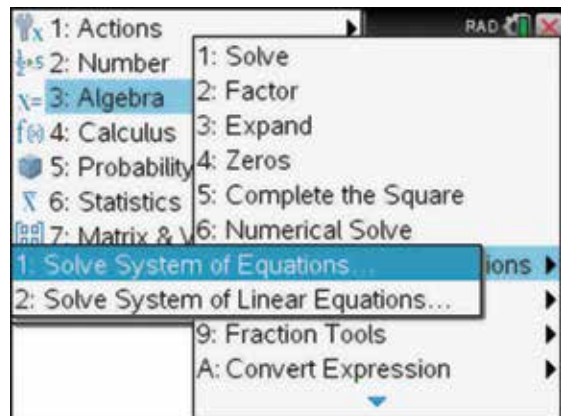


4.8 How to solve a quadratic equation and a linear equation simultaneously

1. It is possible to find the point(s) of intersection of quadratic and linear graphs. For example, solve the following non-linear equations simultaneously:
 $y = x^2 + 6x + 11$ and $y = 4 - 2x$.

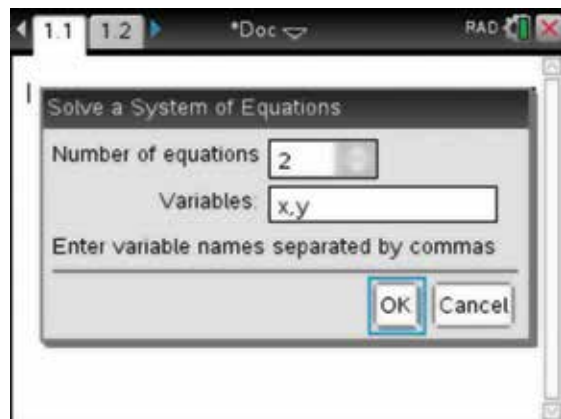
Press:

- **MENU** **menu**
- **3: Algebra** **3**
- **7: Solve System of Equations** **7**
- **1: Solve System of Equations** **1**.



2. Complete as shown, tab **tab** to OK and press:

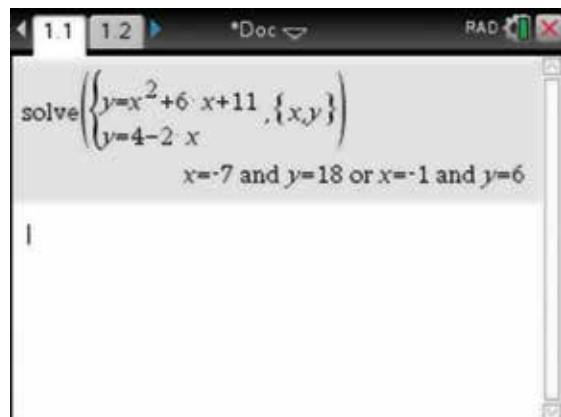
- **ENTER** **enter**.



3. Complete as shown, tab **tab** to OK and press:

- **ENTER** **enter**.

Note: There are two points of intersection, $(-7, 18)$ and $(-1, 6)$. These could also be solved graphically.



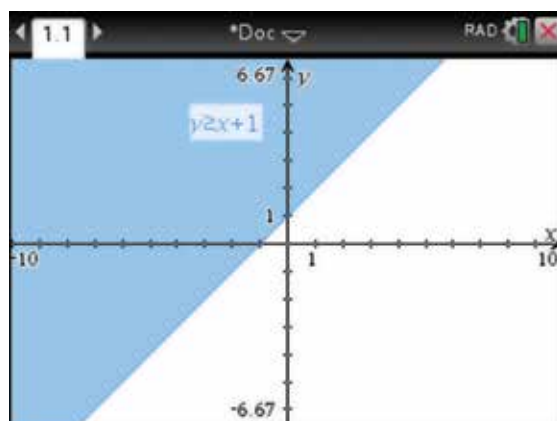
4.9 How to solve simultaneous inequations graphically

1. Open a new document and add a Graphs page (see section 3.4) to sketch the region formed by the following simultaneous inequations.

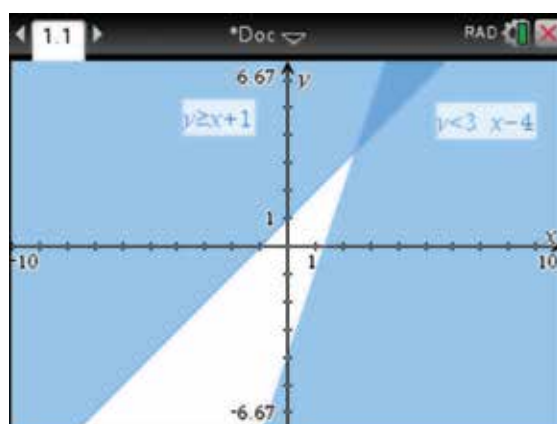
$$y \geq x + 1$$

$$3x - y > 4$$

The first blue region represents $y \geq x + 1$.



2. We need y as the subject, so you need to rearrange the second inequation to read as $y < 3x - 4$. Type this inequation into the function entry line. To get the entry line for $f_2(x)$, press the tab key (tab). The darker shading represents the required area that satisfies both inequations. The broken line represents an inequality of $<$, whereas the continuous line represents an inequality of \geq .



CHAPTER 5

Quadratic functions

The study of quadratic functions builds on students' understanding of the relationships between variables and their ability to move between graphical, symbolic and algebraic representations.

5.1 How to plot parabolas

1. To plot the graph of $y = x^2$ for values of x between -3 and 3 , complete the steps outlined below.

A spreadsheet is required. To open a new spreadsheet, press:

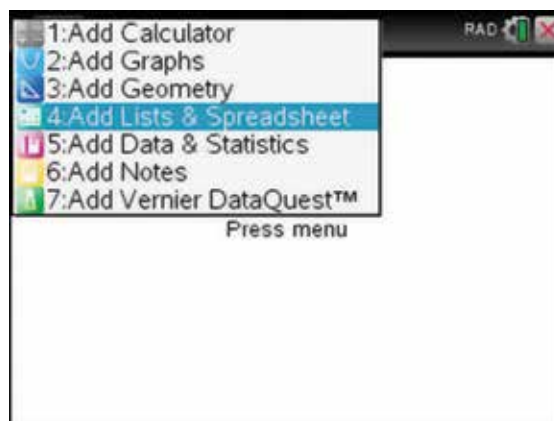
- **HOME**  **on**
- **1: New Doc** .

Select:


- **No.**

Press:

- **4: Add Lists & Spreadsheet** .



2. Enter the x -values from -3 to 3 in column A.

Remember to press the enter key  after every data entry.



3. Scroll to the top of the column. In the cell next to A, label column A as xvalue, then press the enter key **enter**.

Move across to the cell next to B. Label column B as yvalue, then press the enter key **enter**.

	xvalue	yvalue		
=				
4		0		
5		1		
6		2		
7		3		
8				

4. Move the cursor to the yvalue (B) column and highlight the grey (header) cell below the title.

To enter the formula ($y = x^2$) in this cell, complete the entry line as:

=xvalue².

	xvalue	yvalue		
=		=xvalue ²		
4		0		
5		1		
6		2		
7		3		
8				

5. To fill column B with the y-values for the rule $y = x^2$, press the enter key **enter**.

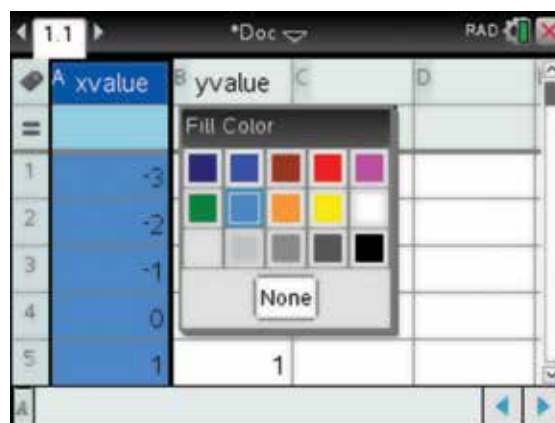
	xvalue	yvalue		
=		=xvalue ²		
1	-3	9		
2	-2	4		
3	-1	1		
4	0	0		
5	1	1		

6. Now using the Touchpad, move the cursor to the top of column A and select this column. To colour the background of the cells or the fill colour, press:

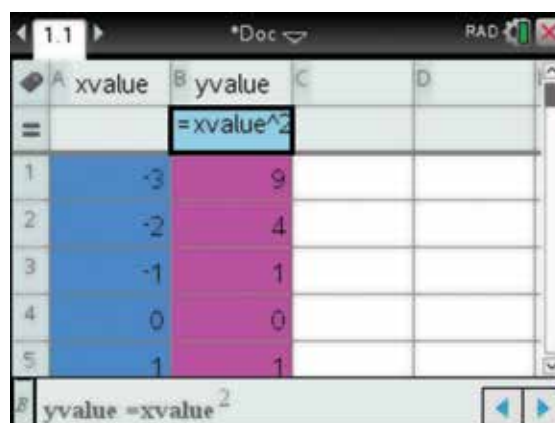
- **ctrl** **ctrl**
- **MENU** **menu**
- **9: Color** **9**
- **2: Fill Color** **2**.

	xvalue	yvalue		
=				
1	-3			
2	-2			
3	-1			
4	0			
5	1	1		

7. Use the Touchpad, to choose a fill colour for the background of the column A.

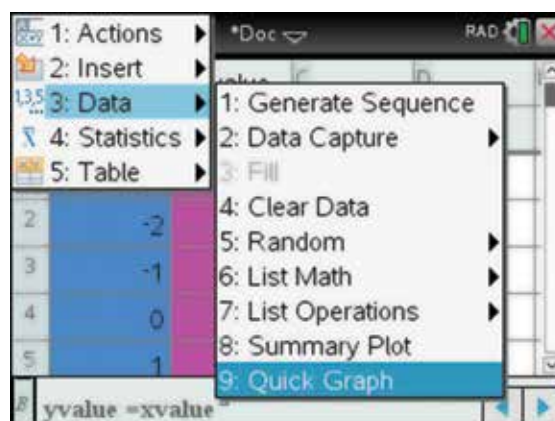


8. Repeat and choose a different fill colour for the background of column B. Leave the Text Colour as black.

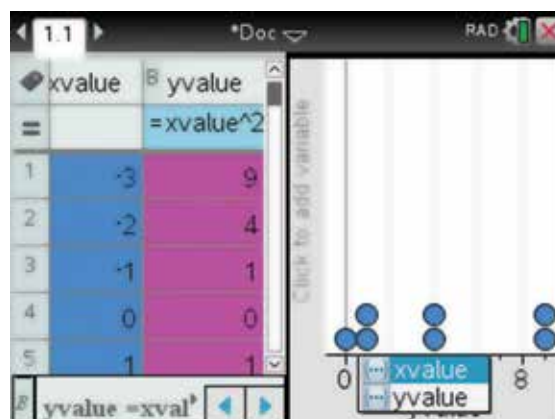



9. To plot a Quick Graph on this page, in a split screen of the coordinate points for this rule, press:

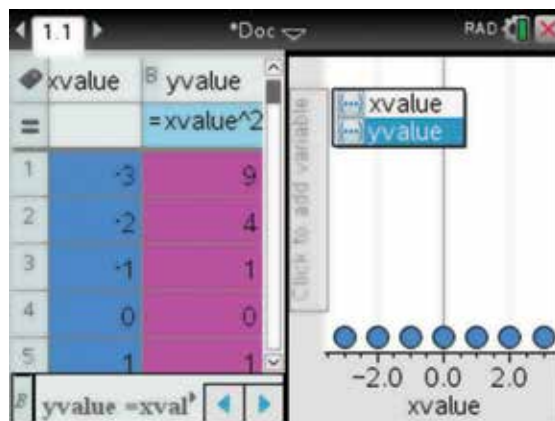
- MENU **menu**
- 3: Data **3**
- 9: Quick Graph **9**.



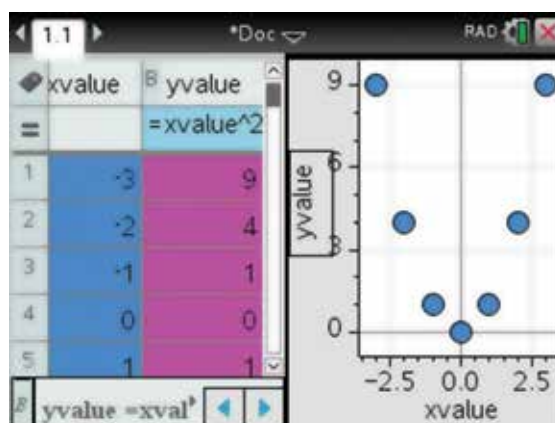
10. Press the tab key **tab** to move to the horizontal axis. This will give you a list of the names (variables) of the columns of data available in your spreadsheet. Select xvalue and press the click key **click**.



11. Press the tab key **(tab)** to move to the vertical axis.
Select yvalue and press the click key .

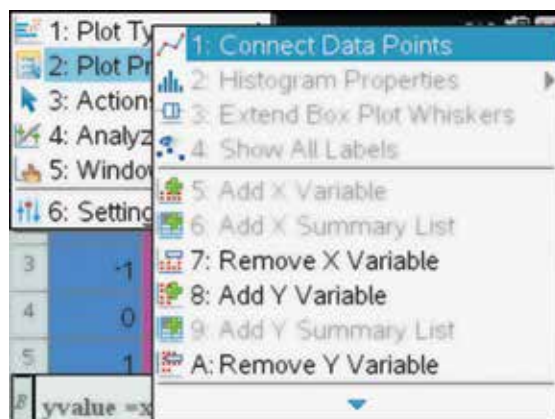




12. You now have the points plotted for the rule $y = x^2$.

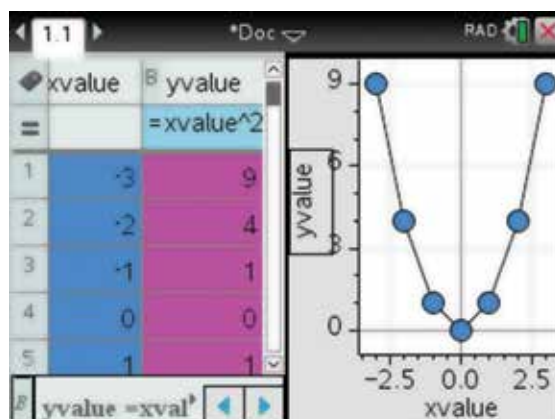


13. To connect the points on the graph, press:

- **MENU** **(menu)**
- **2: Plot Properties** **(2)**
- **1: Connect Data Points** **(1)**.



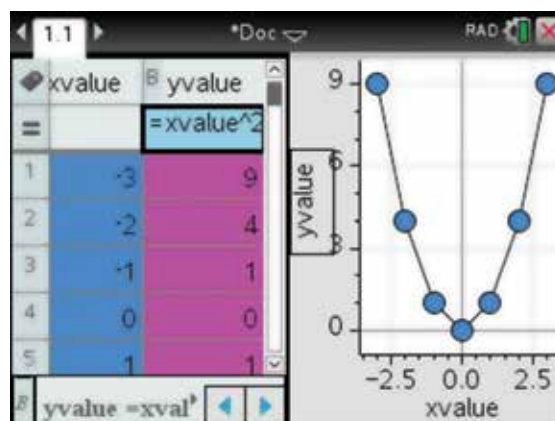
14. It is possible to show the coordinates of a particular point. To do this, move the pointer  to a data value until it changes to an open hand . This will give the coordinates of the selected data point.



15. To toggle between the lists of data and the graph, press:

- **ctrl** (ctrl)
- **tab** (tab).

The screen that is currently in use will have the outside border highlighted in black. In the screen opposite, for example, the spreadsheet is highlighted. This graph can also be drawn in a full Data & Statistics page. The instructions for doing this are provided in section 3.1.



5.2 How to apply transformations to quadratic functions

1. To start a new document and add a Graphs page, press:

- **ctrl** (ctrl)
- **N** (N).

Select:

- **No**.

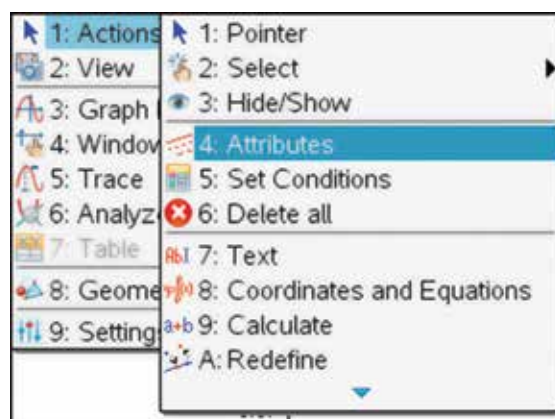
Then press:

- **ENTER** (enter)
- **2: Add Graphs** (2).

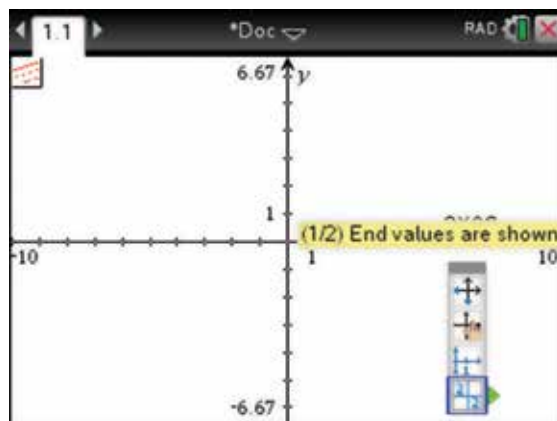


2. To change the attributes of the axes, press:

- **MENU** (menu)
- **1: Actions** (1)
- **4: Attributes** (4).



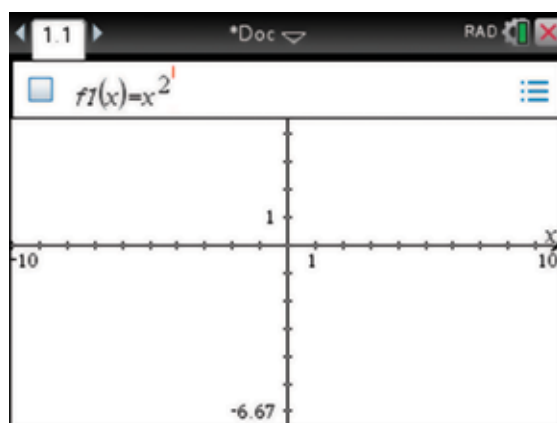
- The Attributes icon will appear in the graph work area. Move the cursor onto an axis so the word 'axes' appears, and then press the enter key **enter**. The axes attributes options bar will appear. Scroll down to '(1/2) End values are shown', and then press the enter key **enter**. Press the escape key **esc** to get rid of the Attributes icon. Using these attributes, many different selections and options are possible.



- Press the tab key **tab** to return to the function entry line. To draw the graph of $f(x) = x^2$, complete the function entry line as:

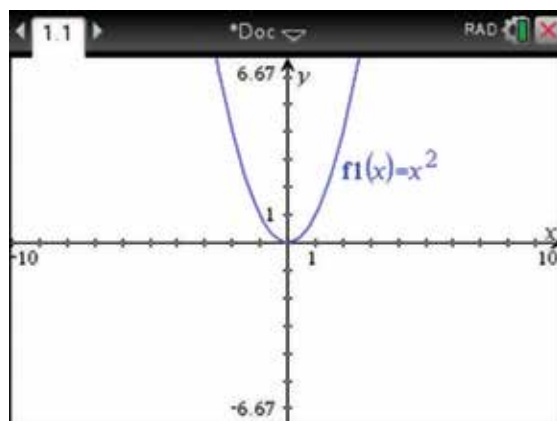
$$f1(x) = x^2$$




Then press the enter key **enter**.



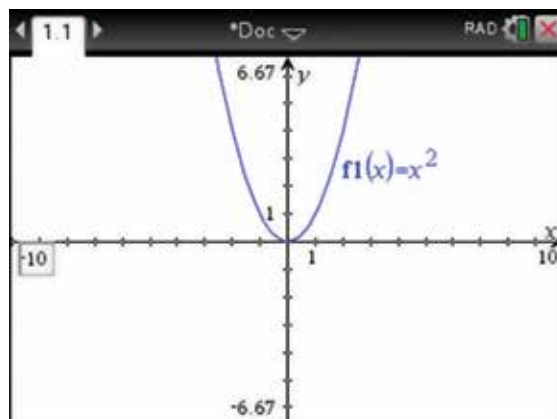
- The function entry line is no longer visible.

Note: To return to the function entry line, press the tab key **tab**.



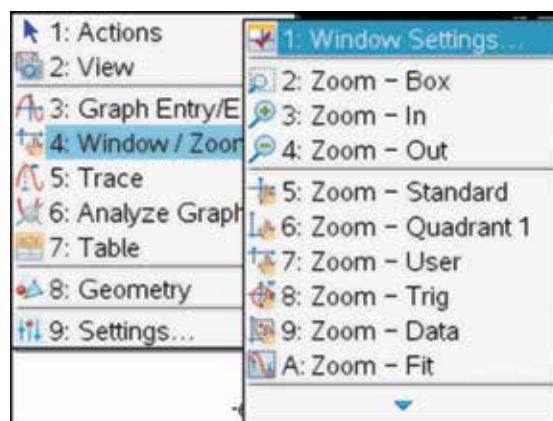
- To change the scale directly on the axes, move the pointer  to the maximum or minimum value you wish to change until the pointer changes into a hand with a pointer finger , and then double-click . This will create a text box around the value, which can then be edited.


Note: Any value displayed in the graph work area can be edited in this way.




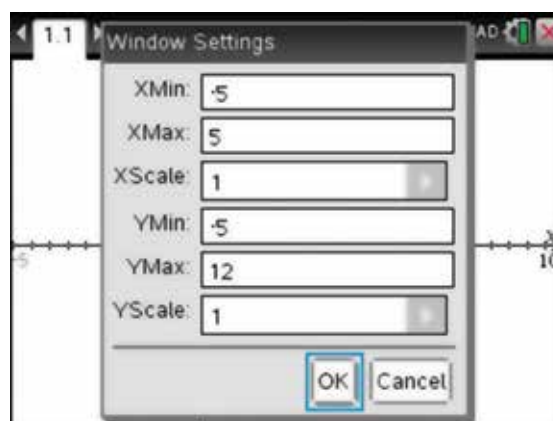
7. An alternative method for changing window settings is to press:

- MENU 
- 4: Window / Zoom 
- 1: Window Settings .



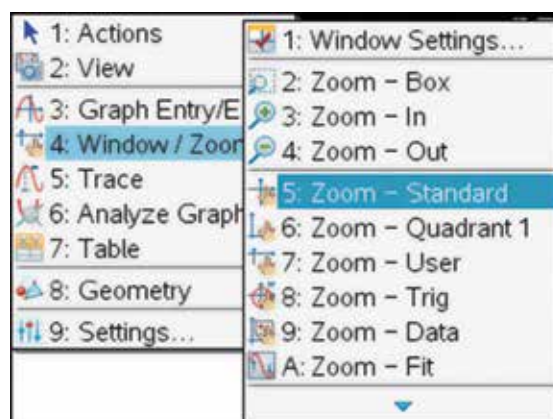
8. Use the tab key  to move between fields and select the values as shown. This allows you to change minimums, maximums and scale for all axes manually.




Select OK when satisfied, then press the enter key .

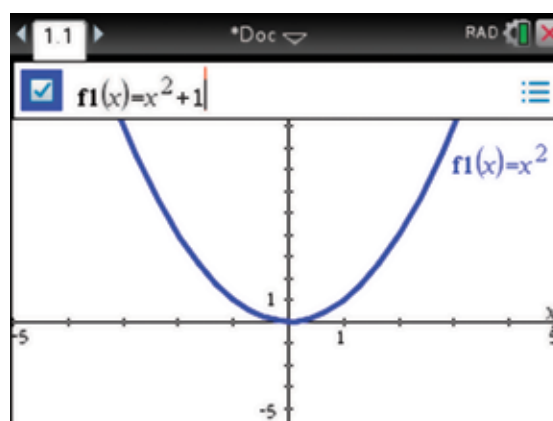


9. Alternatively, one of the Zoom options under Window Settings can be chosen; the settings will be changed automatically.

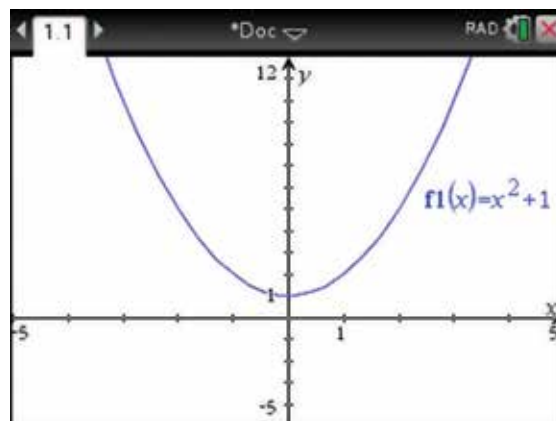
Note: For the graph in step 10, the manually applied settings from step 8 are used.





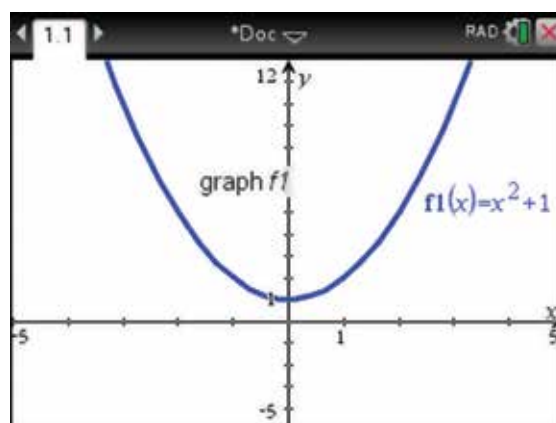
10. To change the equation of the parabola in the graph work area, move the pointer  until it hovers over the equation of the graph and the open hand can be seen, . Double-click . The function entry line will appear.




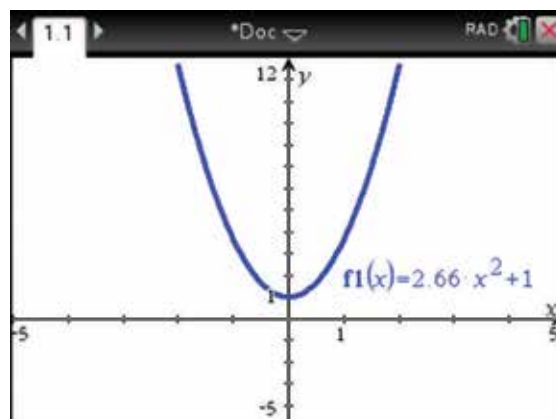
11. To view the new graph, press the enter key **enter**.





12. It is also possible to change the equation by moving the graph. To do this, move the pointer  towards the graph, the pointer changes to the dilation symbol  and 'graph f1' appears.

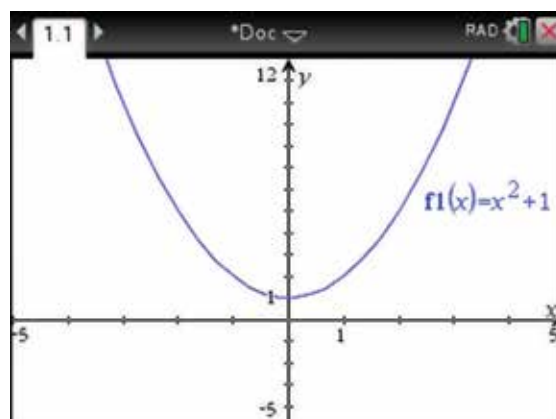


13. Then press and hold down the click key . The graph can now be dragged. Using the Touchpad, move the graph. Notice its equation is changed as it moves. You will see the coefficient of x^2 change as the graph is dilated towards or away from either axis.



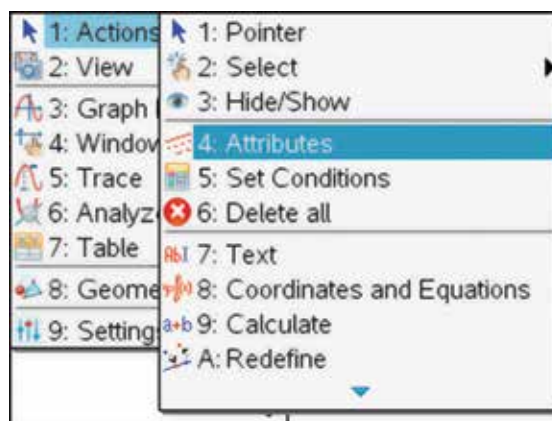
14. To undo the changes you have made to the graph, press:



- **ctrl** 
- **esc** .



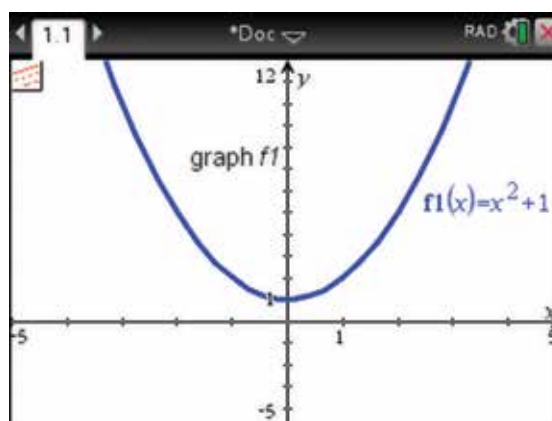
15. To change the appearance (attributes) of the graph, follow the steps below. Press:


- **MENU** **(menu)**
- **1: Actions** **(1)**
- **4: Attributes** **(4)**.

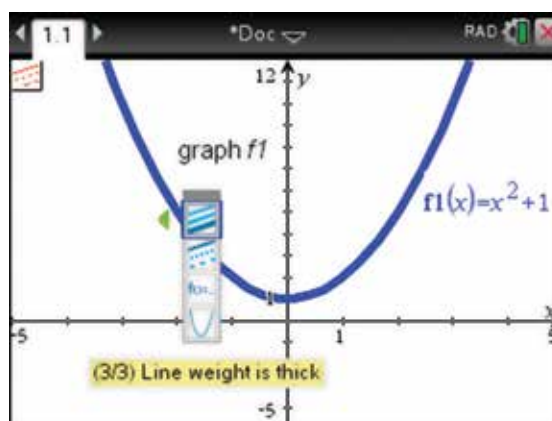





16. Move the pointer  towards the graph; 'graph f1' appears and the pointer changes to a horizontal arrow . Then press the enter key **(enter)**.

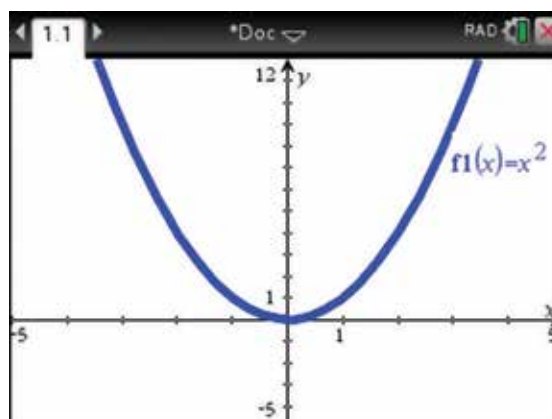
Note: The Attributes icon appears in the top left corner of the graph work area.



17. A drop-down box with options for the attributes will appear. Press the right arrow  to choose 3/3 Line weight is thick. Then press the enter key **(enter)**. There are many possibilities to choose for the attributes of a graph and also for its equation label. Press the esc key **(esc)** to get rid of the Attributes icon.



18. The basic quadratic equation is $f(x) = x^2$. To change the graph of the parabola into $f1(x) = x^2$, move the pointer  until it hovers over the equation of the graph and the label appears. The pointer will turn into an open hand . Then double-click ; the function entry line will appear. Edit it by deleting +1; to view the new graph, press the enter key **(enter)**.



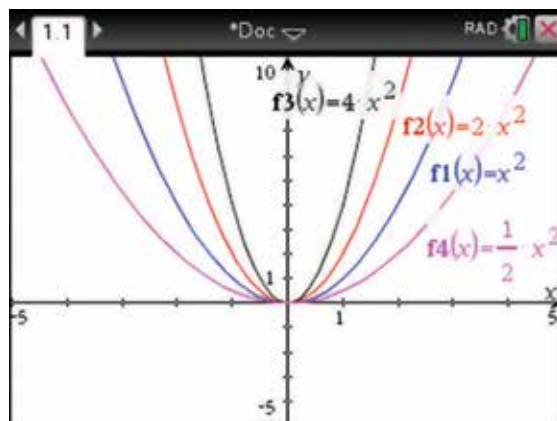
19. To study the transformations of the parabola with more than one graph on the axes, return to the function entry line by pressing:

- **tab** **(tab)**.

For example, to view the effect of changing the dilation factor from the x -axis, draw the graphs of:

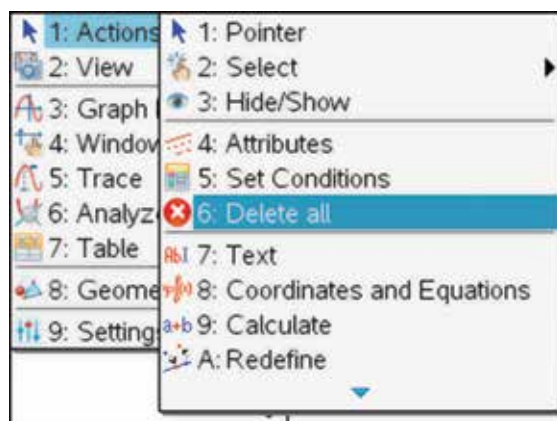
$$f2(x) = 2x^2, \quad f3(x) = 4x^2, \quad f4(x) = \frac{1}{2}x^2.$$

Press the enter key **(enter)** after typing each equation into the function entry line. Press the tab key **(tab)** to return to the function entry line. Note that the labels have been moved to an appropriate place, and that the coloured labels make it easy to identify each graph.



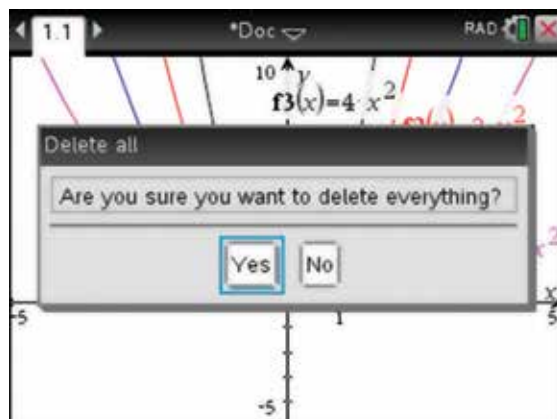
20. To delete all graphs, press:

- **MENU** **(menu)**
- **1: Actions** **(1)**
- **6: Delete all** **(6)**.



21. Select Yes; then press:

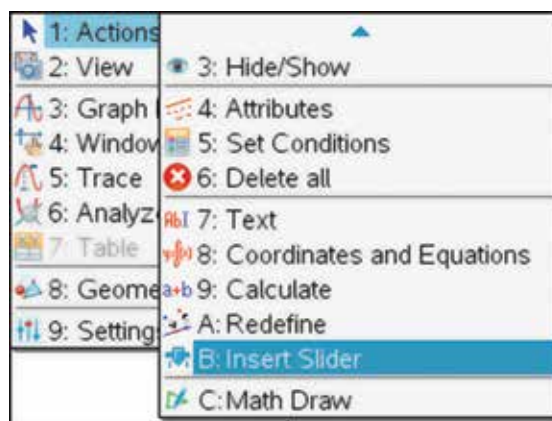
- **ENTER** **(enter)**.



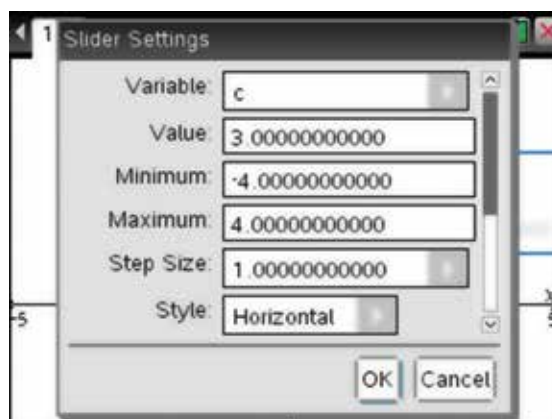
22. An alternative method to viewing transformations is to use a slider.

To insert a slider on a graph page, press:

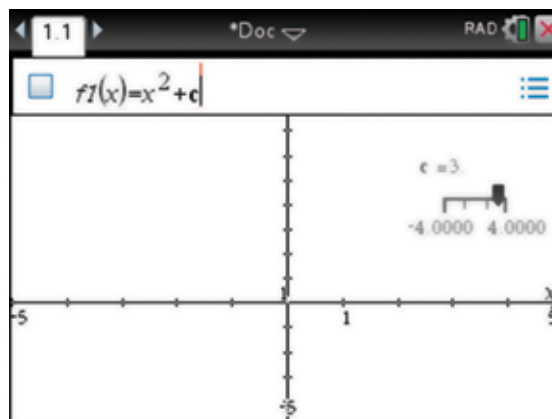
- **MENU** **menu**
- **1: Actions** **1**
- **B: Insert Slider** **B**.



23. Use the tab key **tab** to move between fields in Settings, name the slider c , and select the values as shown. You can also scroll down to set the Display Digits to zero; this ensures that integer values only will be shown. Select OK when satisfied, then press the enter key **enter**.

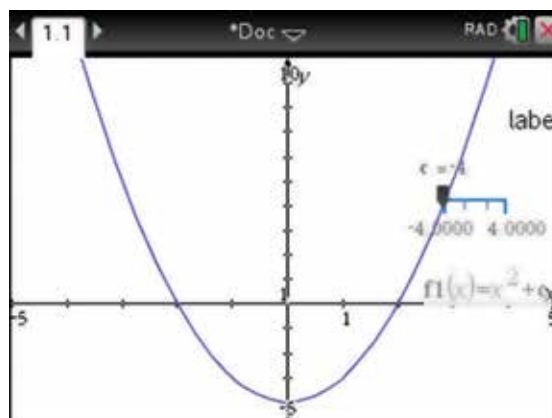


24. Press the tab key **tab** and enter the equation $f1(x) = x^2 + c$.
Press the enter key **enter** and the graph will appear.



25. Grab the slider and move it back and forth to watch the graph transform.

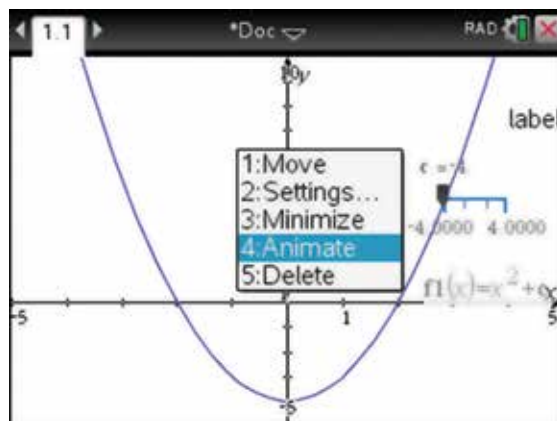
Note: The example shown here is a translation.



26. The slider can be also be animated automatically.
Press:

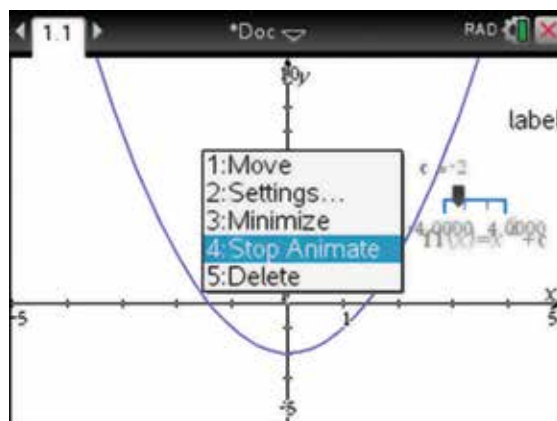
- **ctrl** (ctrl)
- **MENU** (menu)
- **4: Animate** (4).

The graph will translate vertically up and down parallel to the y-axis with the current value of c shown in the slider.



27. To stop the animation. Press:

- **ctrl** (ctrl)
- **MENU** (menu)
- **4: Stop Animate** (4).



5.3 How to find features of the parabola graphically

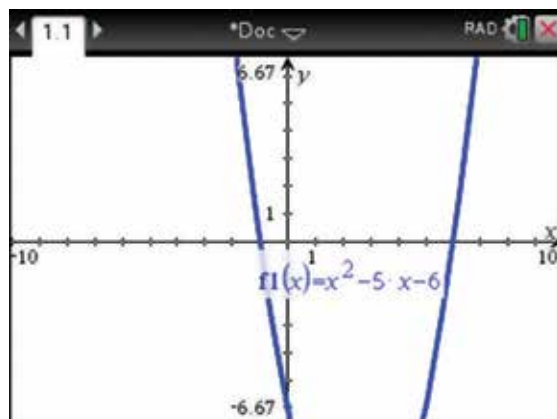
1. Open a new document and add a Graphs page (see section 5.2). Then complete the function entry line as:

$$f1(x) = x^2 - 5x - 6.$$

Press:


- **ENTER** (enter).

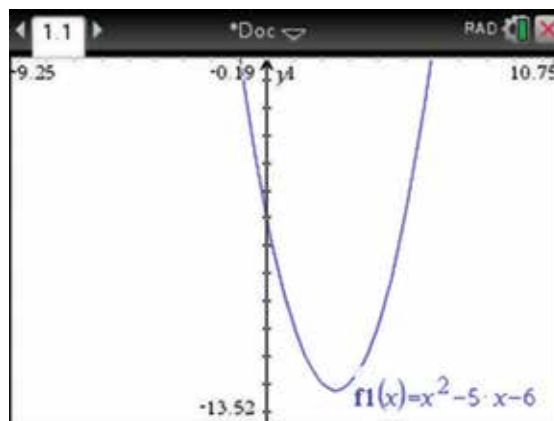
Notice that the turning point cannot be seen in the default window.



- To get a better view of the graph, move the entire graphing page by pressing:

- **ctrl** (ctrl)
- **CLICK** (click icon).

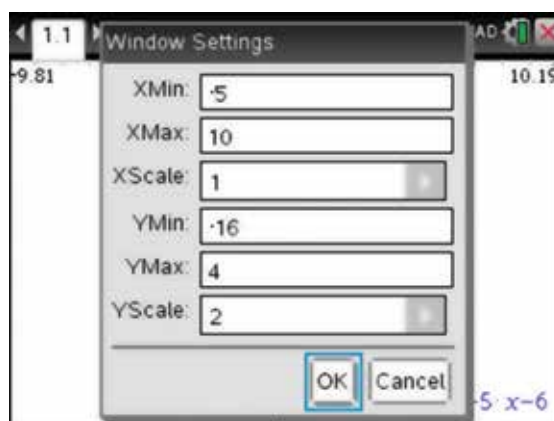
The pointer will change into a closed fist ; now the entire screen can be moved using the Touchpad. This is known as panning. Press the esc key (esc) to lock the position of the graph.



- Press:

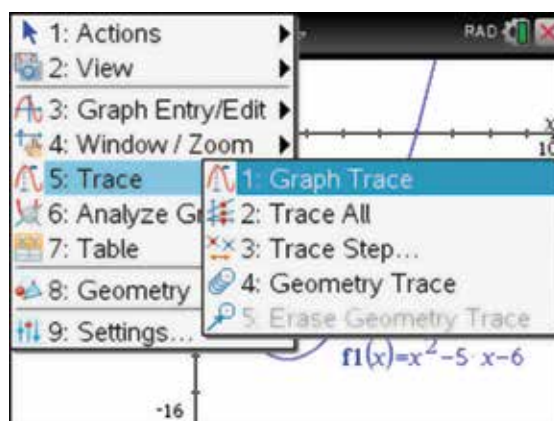
- **MENU** (menu)
- **4: Window / Zoom** (4)
- **1: Window Settings** (1).

Change the Window Settings as shown on the screen opposite.

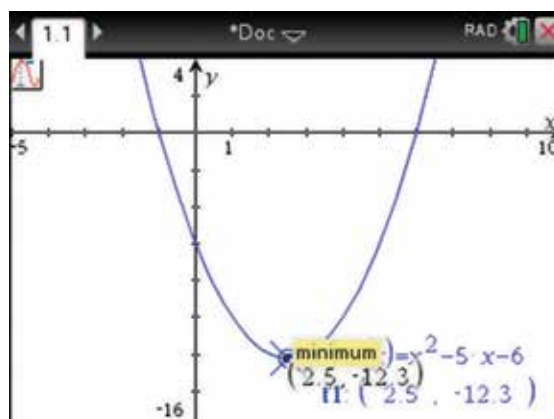


- To find the x -intercepts, the y -intercept and the turning points, complete the following steps. Press:

- **MENU** (menu)
- **5: Trace** (5)
- **1: Graph Trace** (1).

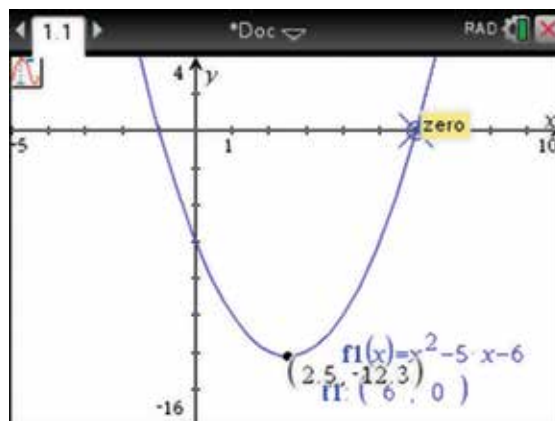


- Press the enter key (enter) to fix the coordinates of the minimum on the graph. For graphs with a maximum point, the maximum appears in a box.



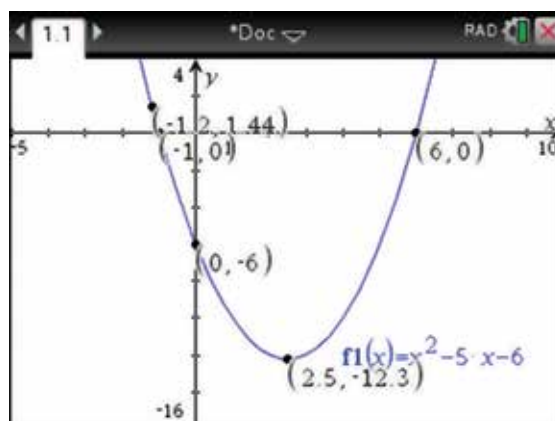
- Then use the Touchpad to trace to one of the x -intercepts. When the intercept is located, 'zero' appears in a box and the coordinates are shown.

Press the enter key **enter** to fix the coordinates on the graph.



- If the coordinates are difficult to read due to congestion on the screen, move the pointer until an open hand appears over the point, then press and hold the click key. When the closed hand appears, drag the point, or equation, using the Touchpad, to a more suitable viewing position. Repeat these steps to locate the second x -intercept.

Note: The graph automatically moves in the window as you trace along.



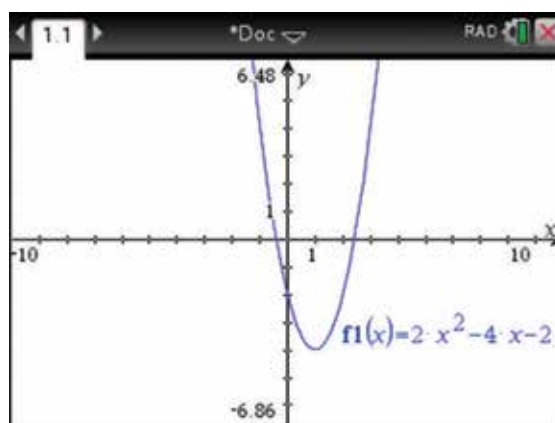
5.4 How to solve quadratic equations to determine irrational x -intercepts

- Open a new document and add a Graphs page (see section 5.2). Then complete the entry line as:

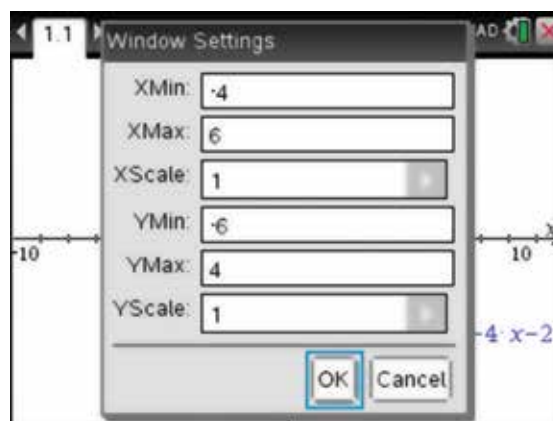
$$f1(x) = 2x^2 - 4x - 2.$$

Press:





- ENTER** **enter**.

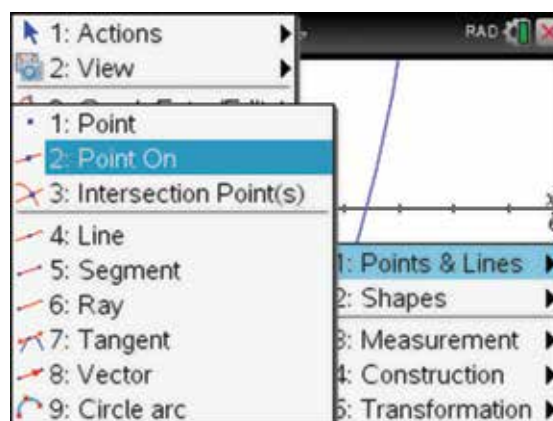


2. Change the Window Settings as shown on the screen opposite (see section 5.2).



3. An alternative method for finding x - and y -intercepts is to press:

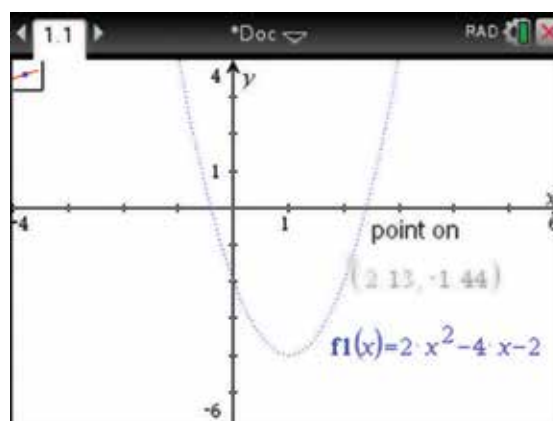
- **MENU** 
- **8: Geometry** 
- **1: Point & Lines** 
- **2: Point On** 





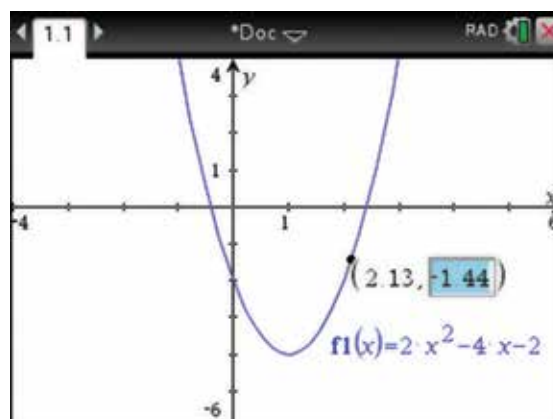
4. Move the pencil until it is over the graph near the x -intercept and 'point on' appears with the coordinates dimmed. Press:

- **CLICK** 
- **esc** 

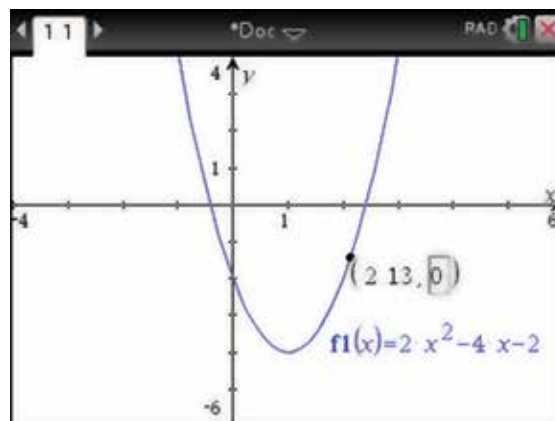
The coordinates are displayed.



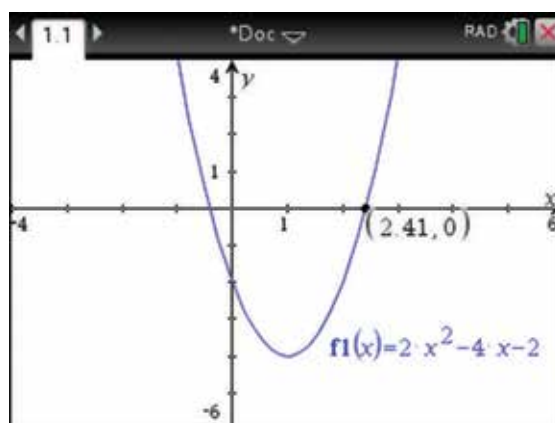
5. Move the pointer until it hovers over the y -coordinate of the point. When it changes into the open hand , with the word 'text' appearing, press the click key  twice.



6. A textbox appears and the coordinate can be edited. Change the y -coordinate to 0, then press the enter key **enter**. The point will jump to the x -coordinate of the axial intercept.

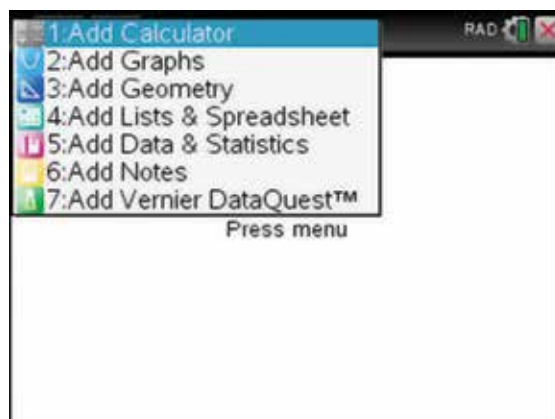


7. This method can also be used to find other significant points.



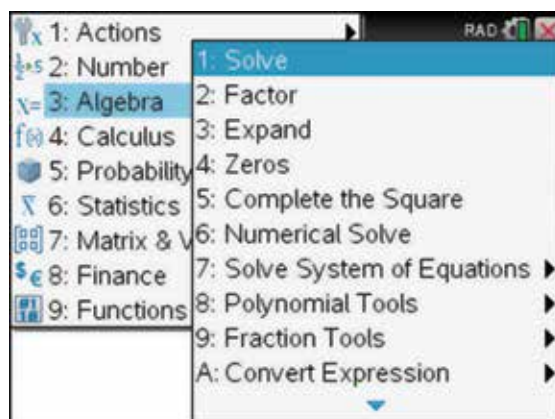
8. The x -intercepts for this function are decimal approximations. To find the exact values for these intercepts, add a Calculator page to this document by pressing:

- **ctrl** **ctrl**
- **I** **I**
- **1: Add Calculator** **1**.



9. To find the exact solutions using the Solve application, press:

- **MENU** **menu**
- **3: Algebra** **3**
- **1: Solve** **1**.



10. Since $f1(x)$ is already defined in the Graphs page, complete the entry line as:

$\text{solve}(f1(x)=0,x)$.

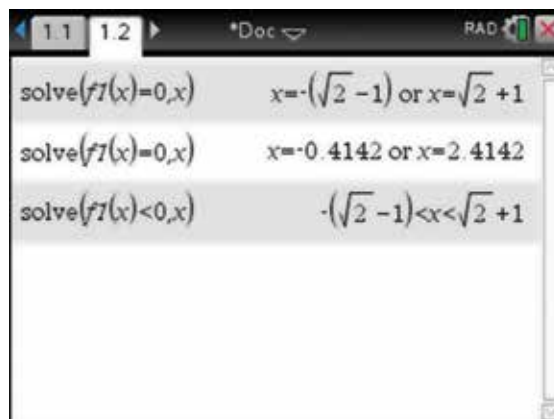
Then press the enter key **enter**. The x -intercepts in exact form are $(-\sqrt{2}+1, 0)$ and $(\sqrt{2}+1, 0)$. Press:

- **ctrl** **ctrl**
- **ENTER** **enter**

to get a decimal approximation. Note that the Display Digits from the General Settings has been set to Fix 4 to agree with the results from the Graphs page.

Inequations, such as $2x^2 - 4x - 2 < 0$, can also be solved; these are entered as:

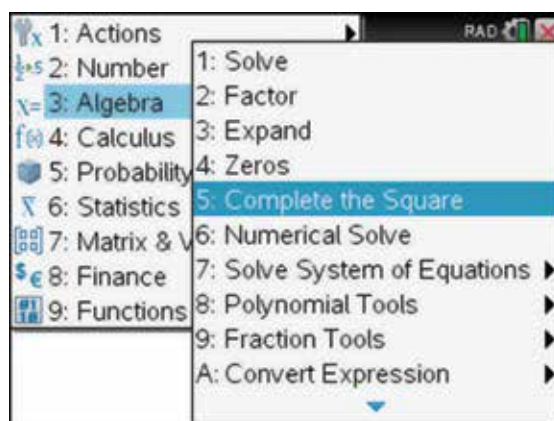
$\text{solve}(f1(x) < 0, x)$.



11. We can also find the turning point, by expressing the quadratic function in the completing the square form.

Press:

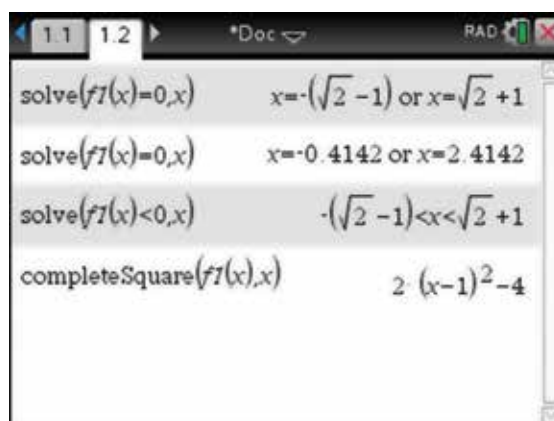
- **MENU** **menu**
- **3: Algebra** **3**
- **5: Complete the Square** **5**.



12. Complete the entry line as:

$\text{completeSquare}(f1(x), x)$.

The point $(1, -4)$ is a minimum turning point.



CHAPTER 6

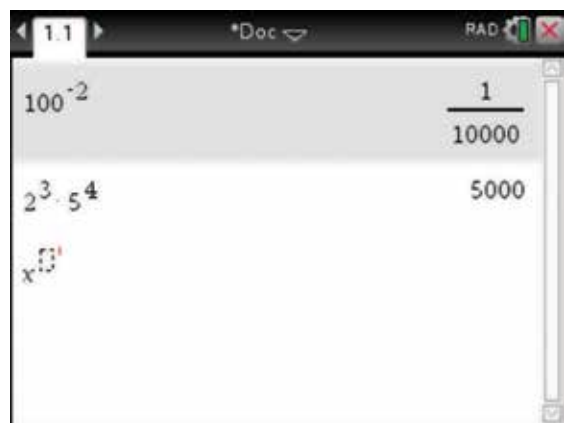
Exponential and logarithmic functions

Situations involving growth and decay can be modelled in mathematics using exponential functions. The inverse situations can be modelled using logarithmic functions.

6.1 How to simplify expressions involving indices and surds

1. To raise a number or pronumeral to a power, the symbol $^$ can be used. For example, to calculate 100^{-2} , type 100 into the entry line and then press the hat key $^$. An index box appears into which you can type -2 . Then press the enter key enter . Note that when you use the hat key $^$ to raise the number to a power, you must press the tab key tab after the power to bring the cursor into line with your next entry.

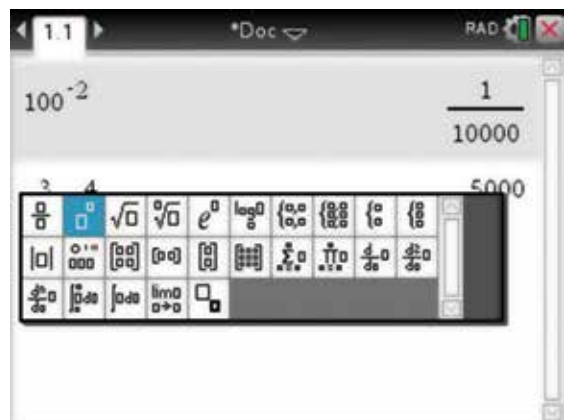
To type squared values, you can press x^2 .



2. The Maths Expression palette may also be used. To access this, press:

- maths .

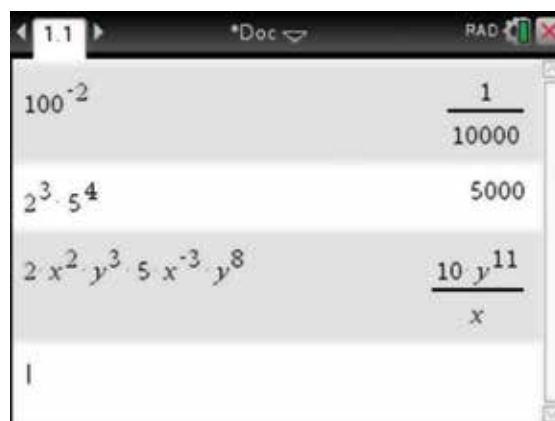
Use the Touchpad to locate the template highlighted in the screen opposite.



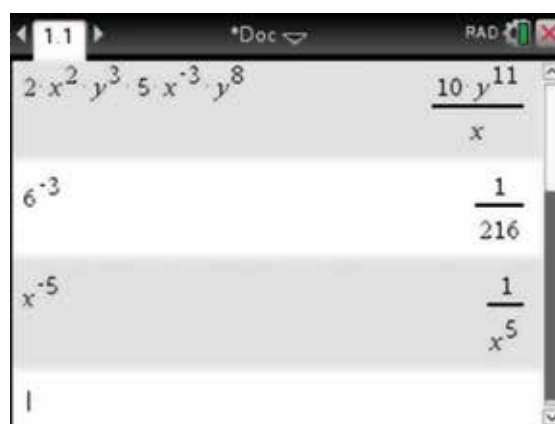
3. Use the template to simplify the following:

$$2x^2y^3 \times 5x^{-3}y^8.$$

Remember to press the tab key **(tab)** to move through the fields.



4. Notice that the calculator expresses answers using positive indices.

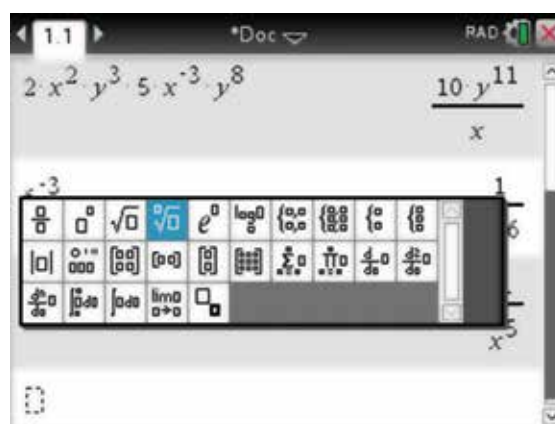


5. Surds can be evaluated by changing them into fractional powers or by using the Maths expression palette.

To evaluate $\sqrt[3]{27}$ using the Maths expression palette, press:

- **($\sqrt[n]{}$)**.

Move across to the surd template highlighted on the screen. Press the enter key **(enter)**.

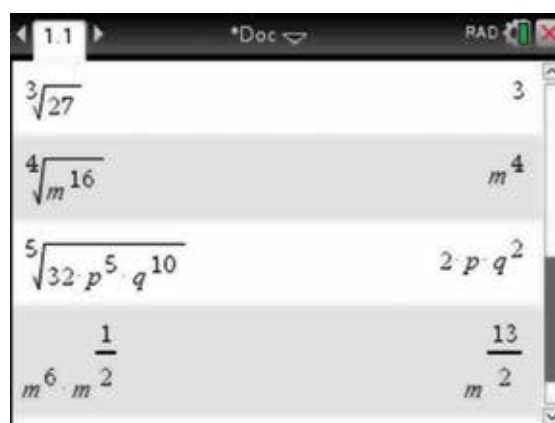


6. Then press:

- **3:** **(3)**
- **tab** **(tab)**
- **27** **(2)** **(7)**
- **tab** **(tab)**
- **ENTER** **(enter)**.

Try the following.

(a) $\sqrt[4]{m^{16}}$ (b) $\sqrt[5]{32p^5q^{10}}$ (c) $m^6 \times m^{\frac{1}{2}}$



6.2 How to plot an exponential graph

- Complete the following table and use the table to plot the graph of $y = 2^x$.

x	-4	-3	-2	-1	0	1	2	3	4
y									

Start with a new document and add a Lists & Spreadsheets page (see section 3.1). Type the x -values into column A. Label column A as x values (see section 3.1), then press the enter key **enter**.

A	xvalues	B	C	D
=				
6	1			
7	2			
8	3			
9	4			
10				

- Use the Touchpad to move the cursor to column B.

Label column B as y values, then press the enter key **enter**.

Then move the cursor into the header (grey) cell below B and complete the entry line as:

$$= 2^{xvalues}$$

Note: A shortcut to fill column B is to type $=2^{[a]}$. The brackets appear automatically.

A	xvalues	B	yvalues	C	D
=			xvalues		
6	1				
7	2				
8	3				
9	4				
10					

yvalues = 2^xvalues

- To fill column B with the y -values, press:

- ENTER** **enter**.

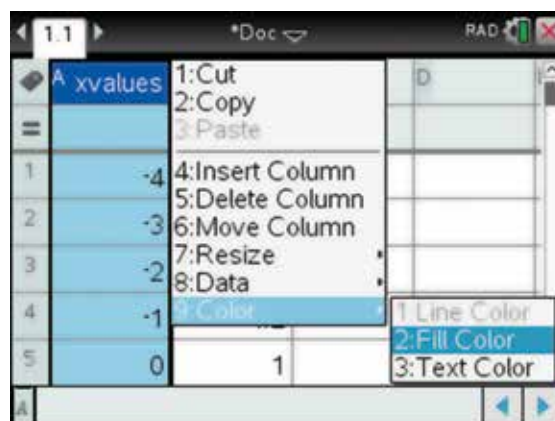
The results are shown as fractions if the Calculation Mode in the General Settings are on Auto or Exact.

A	xvalues	B	yvalues	C	D
=			=2^xvalue		
1	-4		1/16		
2	-3		1/8		
3	-2		1/4		
4	-1		1/2		
5					

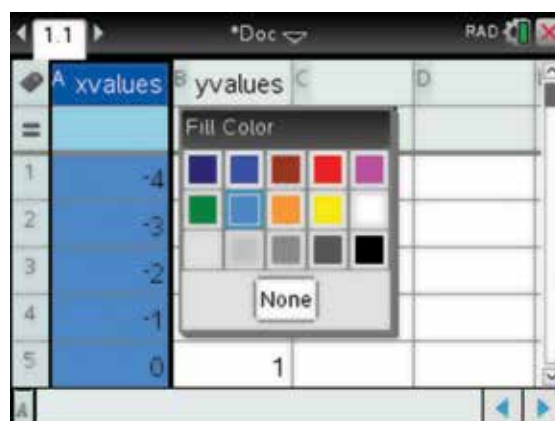
= 1/16

4. Now using the Touchpad, move the cursor to the top of column A and select this column. To colour the background or fill colour of the cells, press:

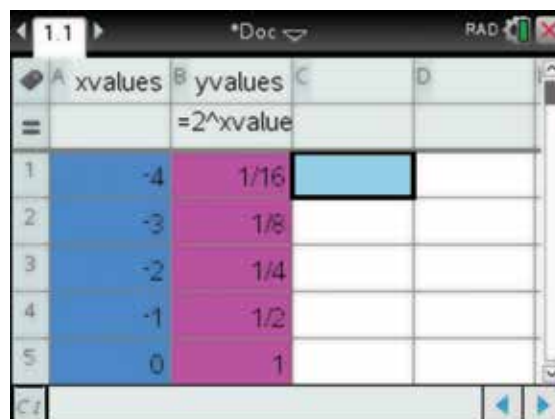
- **ctrl** (ctrl)
- **MENU** (menu)
- **9: Color** (9)
- **2: Fill Color** (2).



5. Use the Touchpad, to choose a colour for the background of column A.



6. Repeat and choose a different colour for the background of column B. Leave the Text Colour as black.




7. To draw the full-screen graph on a new page, complete the following steps. Press:

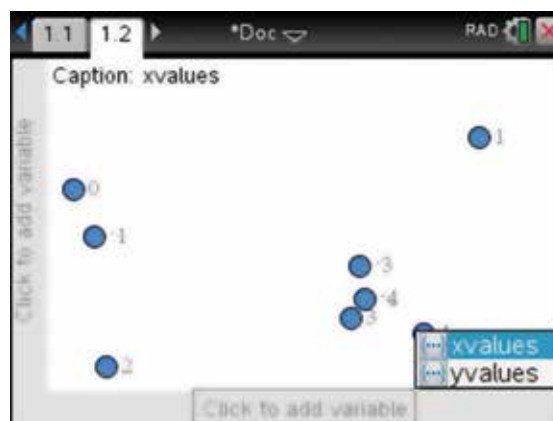
- **HOME** (home).

Select the Data & Statistics icon and press the enter key (enter).



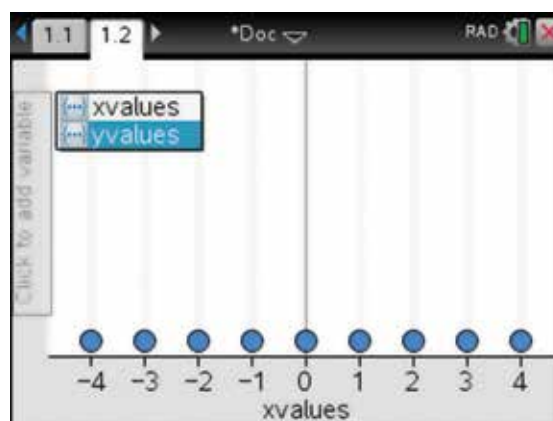
8. The Data & Statistics page appears as shown.

To move to the horizontal axis, press the tab key **(tab)**. Then select xvalues by pressing the click key .

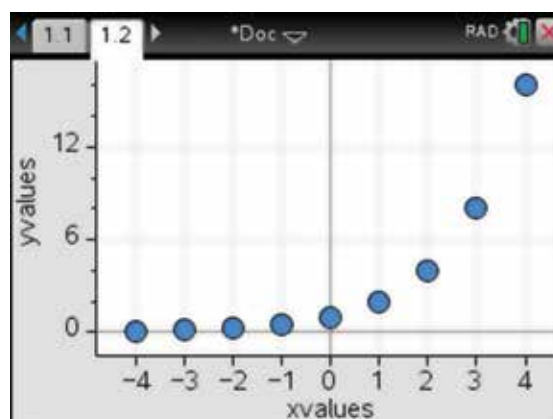


9. To move to the vertical axis, press the tab key **(tab)**.

Then select yvalues and press the click key .

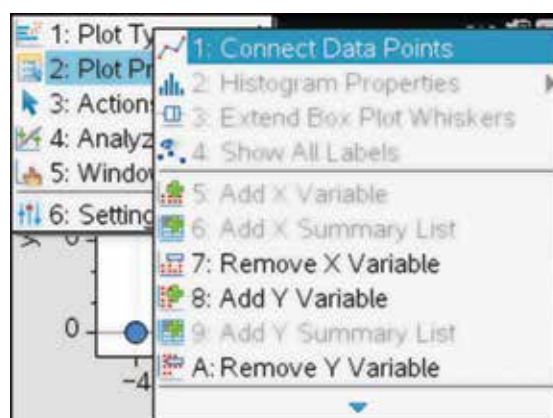


10. The points from the table of values are plotted as shown.

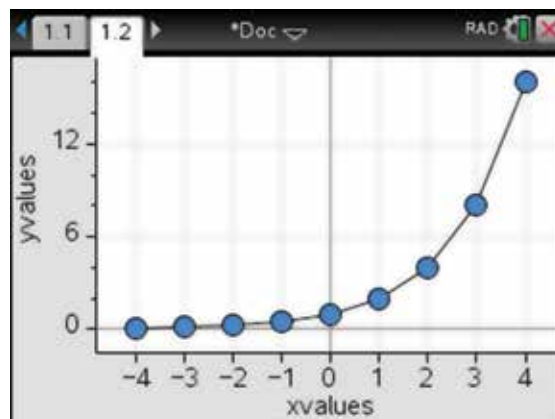


11. Join the points by pressing:

- **MENU** **(menu)**
- **2: Plot Properties** **(2)**
- **1: Connect Data Points** **(1)**.



12. The plot of the exponential relationship $y = 2^x$ is shown.



6.3 How to sketch exponential functions

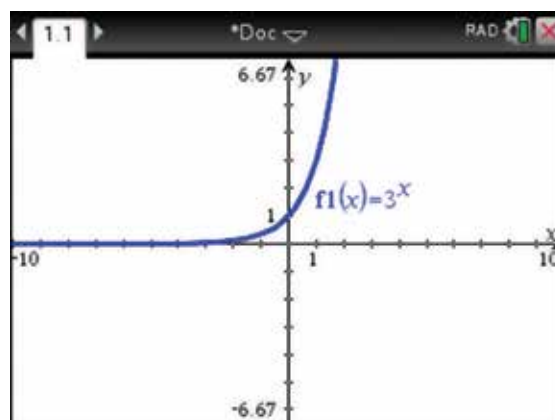
1. Open a new document and add a Graphs page (see section 3.4).

To sketch the graph of $f(x) = 3^x$, complete the function entry line as:

$$f1(x) = 3^x.$$

Then press:

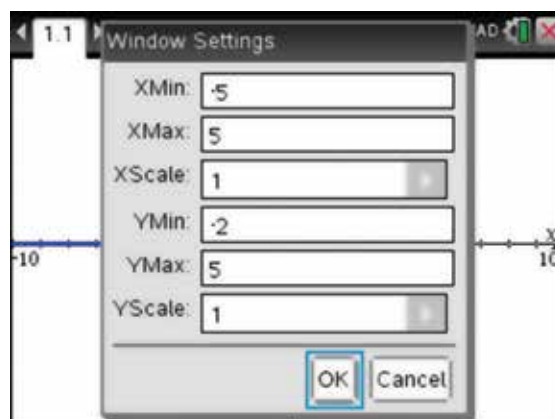
- **ENTER** (enter).



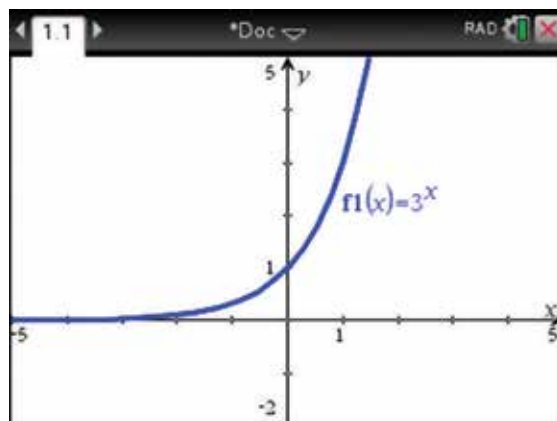
2. Change the scale of the axes by pressing:

- **MENU** (menu)
- **4: Window / Zoom** (4)
- **1: Window Settings** (1).

Enter the values shown. Use the tab key (tab) to select OK, then press the enter key (enter). Press the tab key (tab) to move between fields.



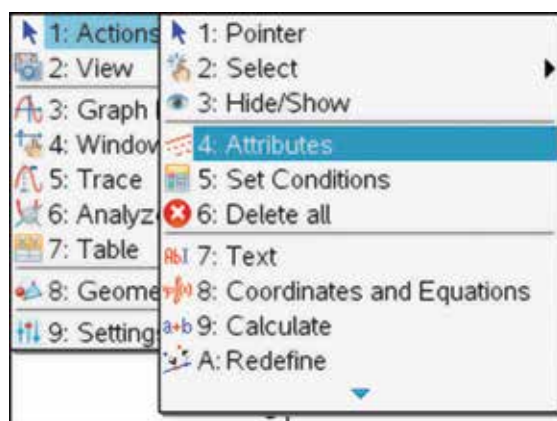
3. Note the change in the scale of the axes.



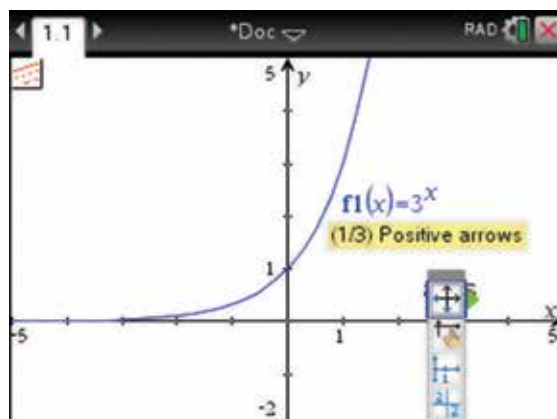
4. To change the axes attributes, press:

- **MENU**
- **1: Actions**
- **4: Attributes**

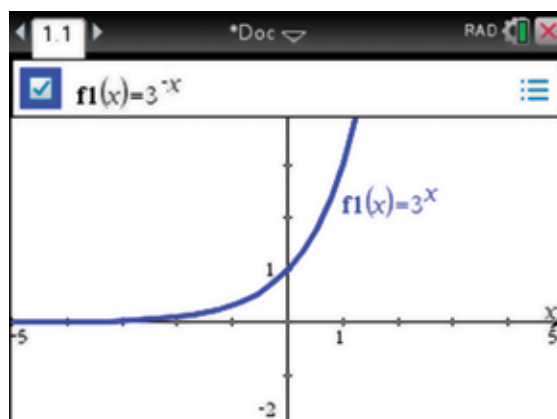
The Attributes icon appears in the top left corner of the graph work area. Move the pointer onto an axis so the word 'axes' appears; then press the enter key .



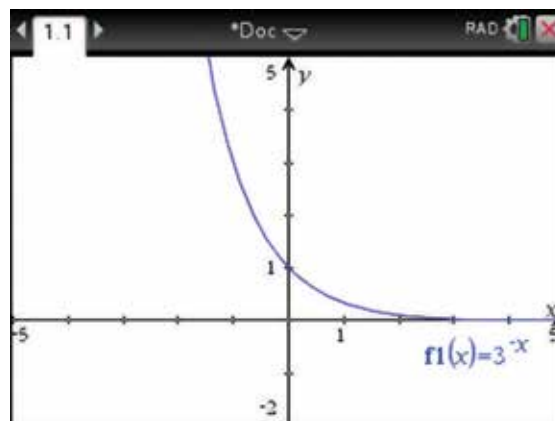
5. The Attributes icon appears in the top left-hand corner of the graph work area. Move the pointer onto an axis so the word 'axes' appears; then press the enter key . The axes attributes options bar will appear. Scroll through to see all of the changes that can be made to the axes settings. The option shown is to place arrows on the end of all axes. Press the esc key to get rid of the Attributes icon.



6. To view transformations, move the pointer until it hovers over the equation of the graph. Press the click key . The equation can be edited by double-clicking on the equation. The function entry line will appear. Edit the equation to $f1(x) = 3^{-x}$. Then press the enter key .



7. The graph should appear as shown, reflected in the y-axis.



6.4 How to calculate logarithms

1. The calculator can calculate logarithms to any base.

To evaluate $\log_{10}(10)$ in a Calculator page, press:

- **ctrl** (**ctrl**)
- (**10^x**).

The logarithm template appears as shown. Type 10 into the base position; then press the tab key (**tab**) to move to the next field.



2. Follow these steps to calculate the following logarithms.




- $\log_2(8)$
- $\log_5(1)$
- $\log_e(4)$

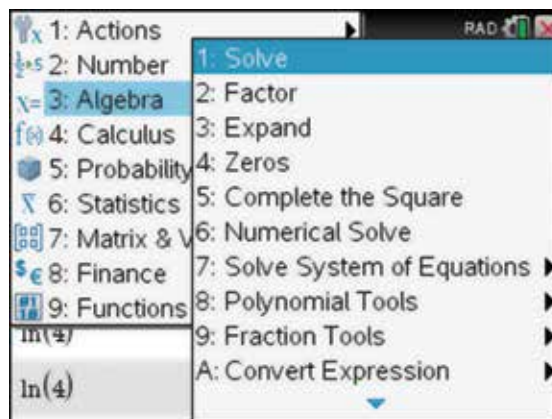
Notes: The final answer on the screen is given as a decimal approximation by pressing the ctrl key (**ctrl**) and the enter key (**enter**). To get the natural logarithm $\log_e(x) = \ln(x)$, press the ctrl key (**ctrl**) and (**e^x**).

$\log_{10}(10)$	1
$\log_2(8)$	3
$\log_5(1)$	0
$\ln(4)$	$2 \cdot \ln(2)$
$\ln(4)$	1.3863

6.5 How to solve exponential and logarithmic equations

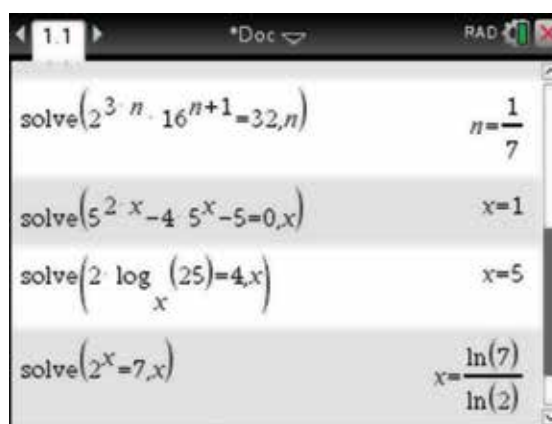
1. Exponential and logarithmic equations can be solved using the Solve application. To do this, press:

- MENU 
- 3: Algebra 
- 1: Solve .



2. Solve each of the following by completing each entry line as shown in the screen opposite.



- (a) $2^{3n} \times 16^{n+1} = 32$
- (b) $5^{2x} - 4 \times 5^x - 5 = 0$
- (c) $2 \log_x 25 = 4$
- (d) $2^x = 7$



6.6 How to use the TI-Nspire CAS in applications of exponential and logarithmic functions

1. A new document with a Graphs page is needed for the problem below.

To get this, press:

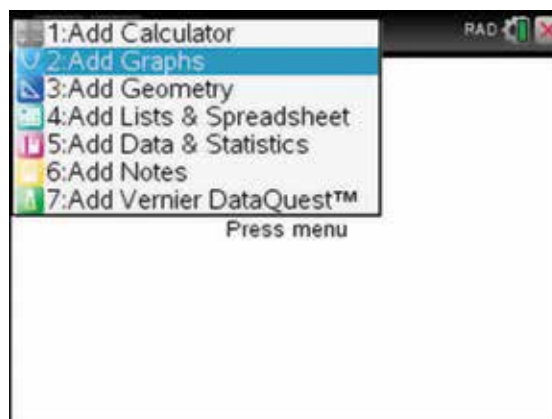
- ctrl 
- N .

Select:

- No.

Press:

- ENTER 
- 2: Add Graphs .



The calculator can be used to work through problems graphically such as the one shown here.

The temperature, T ($^{\circ}\text{C}$), of a cooling cup of coffee in a room at 20°C can be modelled by

$$T = 90 \left(3^{-\frac{t}{20}} \right) \text{ for } t \geq 0, \text{ where } t \text{ is the number of minutes after it is poured.}$$

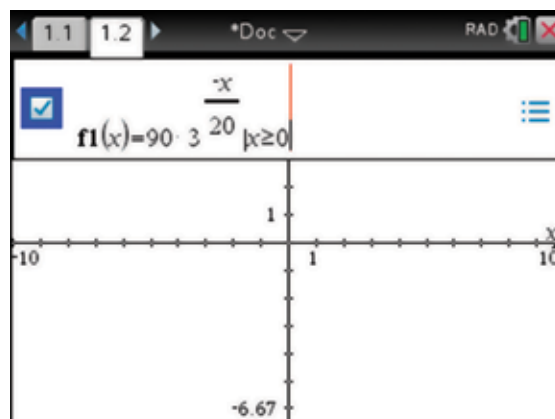
- Find the initial temperature.
- Find the temperature: (i) 5 minutes after pouring (ii) 10 minutes after pouring.
- How long is it until the temperature reaches half its initial value?

2. Complete the function entry line as:

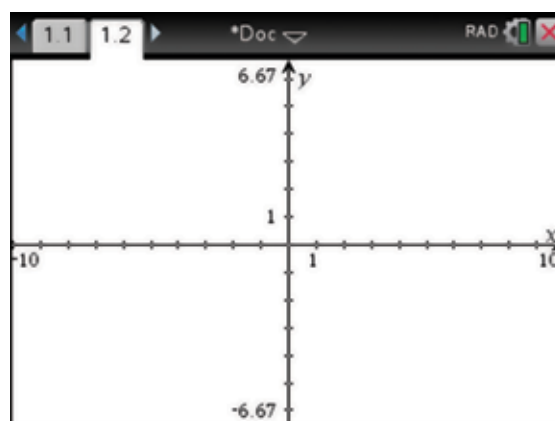
$$f1(x) = 90 \cdot 3^{-\frac{x}{20}} | x \geq 0$$

Then press the enter key **enter**.

Note: We have defined a restricted domain function, so the graph will only be drawn for $x \geq 0$.

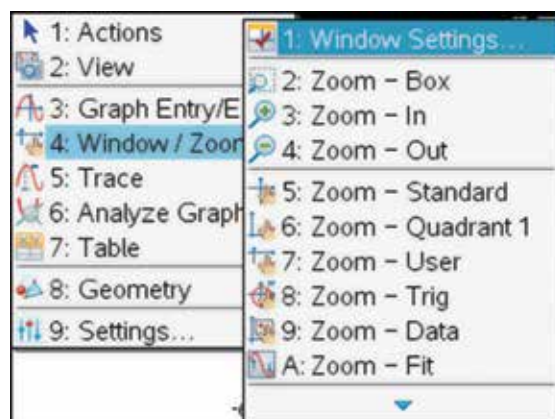


3. Notice that it appears as though no function has been entered. This is because the standard viewing window is not appropriate for this function.

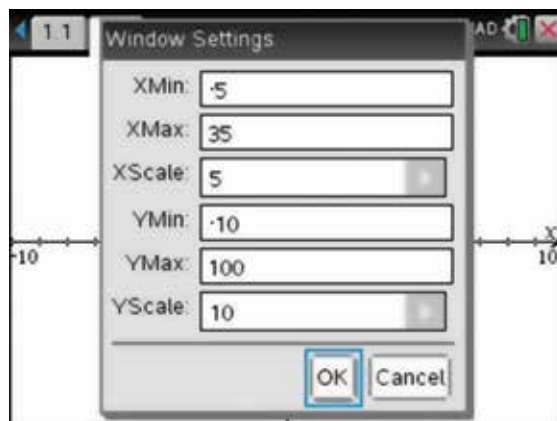


4. The variables for this function $f(x) = 90 \times 3^{-\frac{x}{20}}$ are time and temperature. Change the window settings to a suitable scale by pressing:

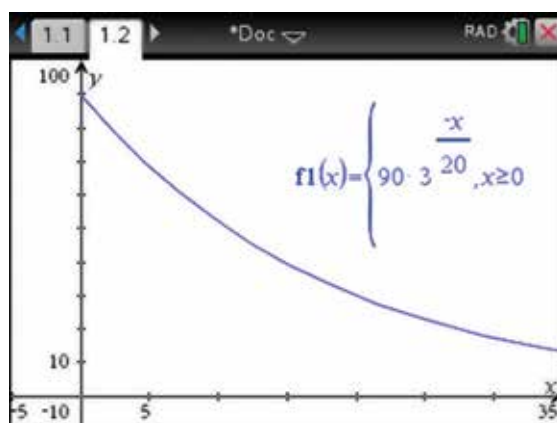
- MENU** **menu**
- 4: Window / Zoom** **4**
- 1: Window Settings** **1**.



5. Complete as shown, then select OK and press the enter key **(enter)**.



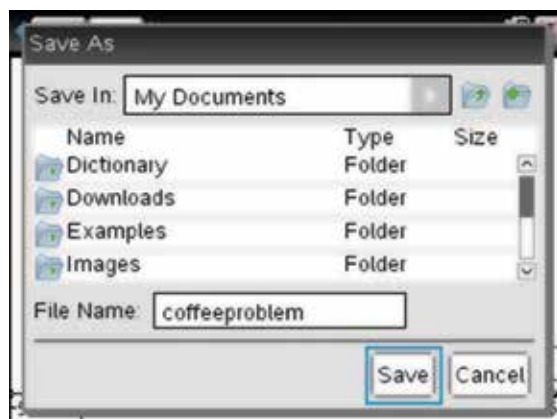
6. The graph will appear as shown, and only for $x \geq 0$.



7. Since we are finding several parts to this problem, we will save the document and give it a name. To save, press:

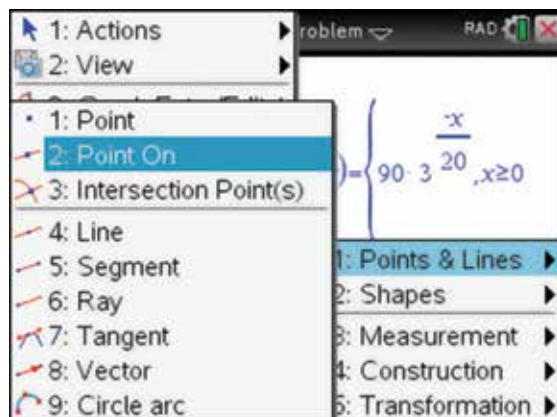
- **ctrl** **(ctrl)**
- **S** **(S)**


This will bring up a dialog box in which we can save the document and give it a name. Press the tab key **(tab)** to move through the fields and name the document 'coffeeproblem'. Use the tab key **(tab)** to select Save, then press the enter key **(enter)**. We will now see the document name at the top of the screen.



8. To answer the questions, all the required coordinates can be located and placed on the graph. To do this, press:

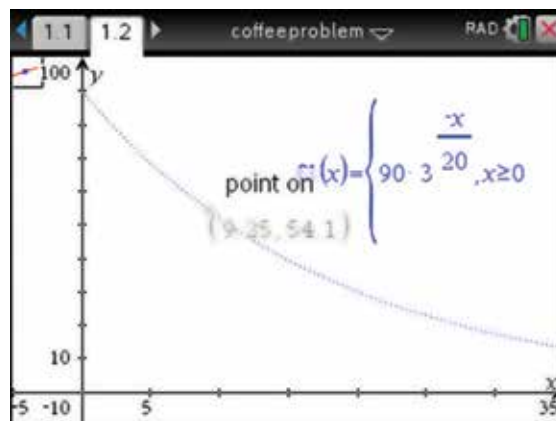
- **MENU** **(menu)**
- **8: Geometry** **(8)**
- **1: Points & Lines** **(1)**
- **2: Point On** **(2)**






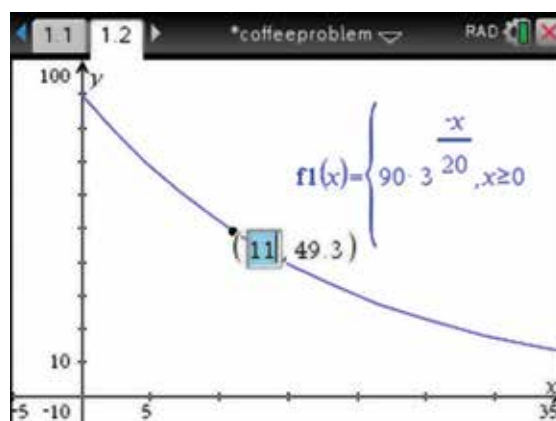
9. Move the pencil  over the graph; when 'point on' is revealed with the coordinates dimmed, press:


- **ENTER** 
- **esc** .

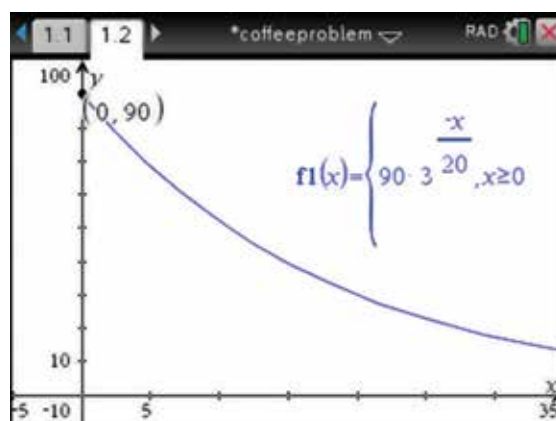
This allows you to edit the x -coordinate or the y -coordinate at will and displays the corresponding coordinate.



10. The question requires the initial temperature, which is at $t = 0$. On our graph, this is when $x = 0$. To locate this point, move the pointer  until it hovers over the x -value of the coordinates. When it changes into an open hand , press the click key  twice.



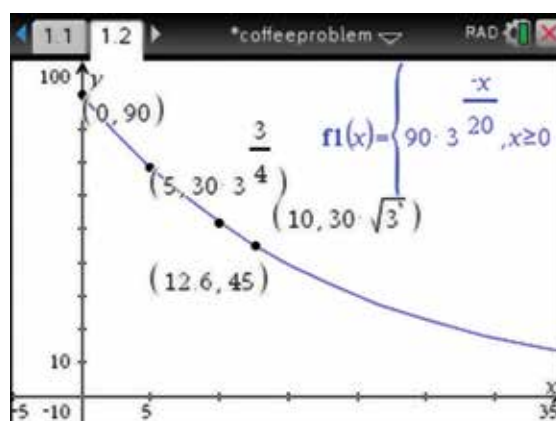
11. A text box appears. Delete the existing x -value and type in 0. Press the enter key . This places the point (0, 90) in the correct position on the graph.






12. To find the answers to parts (b) and (c) for this problem, repeat the above process using the following coordinates:

- (5, ?)
- (10, ?)
- (?, 45).

Note that for a given x -value, an exact y -value is now displayed. This will happen if the General Settings, for the Calculation Mode are set to Exact or Auto. Note that some of the labels of the coordinates have been moved to display the coordinates neatly.

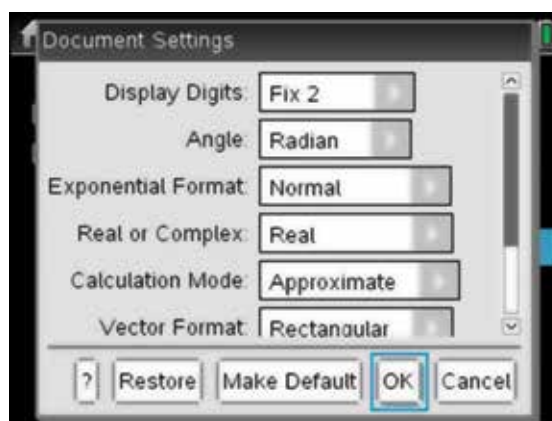


13. If answers are required to two decimal places, press:

- HOME  on
- 5: Settings 
- 2: Document Settings .



14. Then change the General settings for the Calculation Mode to Approximate and set the number of decimal places to Fix 2.



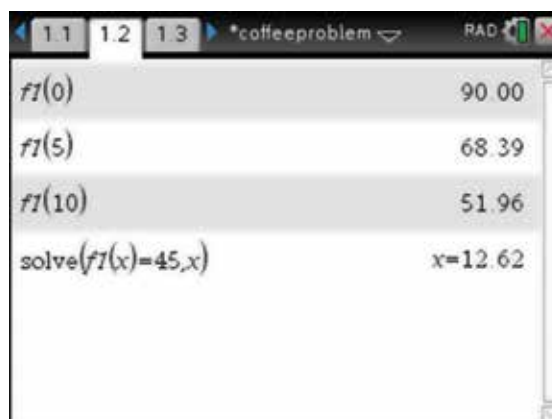
15. Click on OK, then press:

- 4: Current 

to return to the current document.



16. We can add a Calculator page to the document to check algebraically the answers to all of the above. Since $f1(x)$ is already defined on the Graphs page, we can use it in the Calculator page, complete as shown.



CHAPTER 7

Advanced functions

This chapter deals with absolute value functions, hybrid functions and inverse functions. Although not individually mentioned, other functions such as square roots, hyperbolas, truncuses and polynomials can be graphed and their important features examined using the processes outlined in this chapter.

7.1 How to graph the absolute value function

1. Open a new document and add a Graphs page. To do this, press:

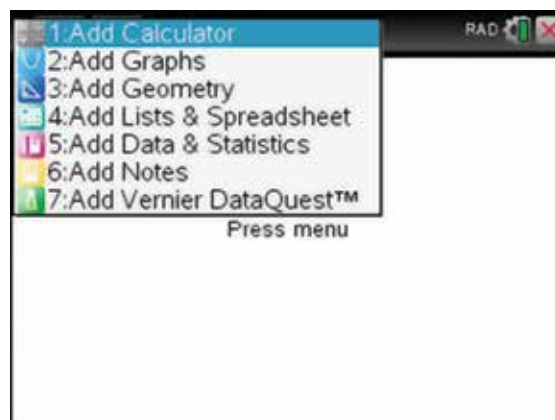
- **ctrl** **(ctrl)**
- **N** **(N)**.

Select:

- **No.**

Press:

- **ENTER** **(enter)**
- **2: Add Graphs** **(2)**.

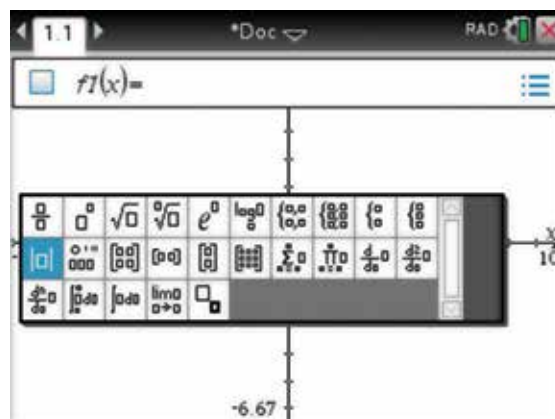


2. To graph $f(x)=|x|$, complete the following steps.

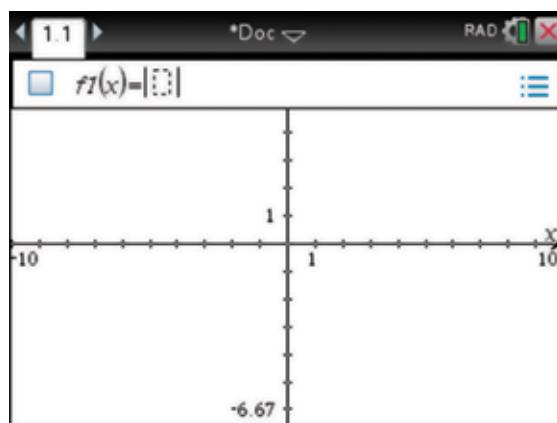
The Absolute value template is located in the Maths Expression palette. To access this, press:

- **(MATH)**.

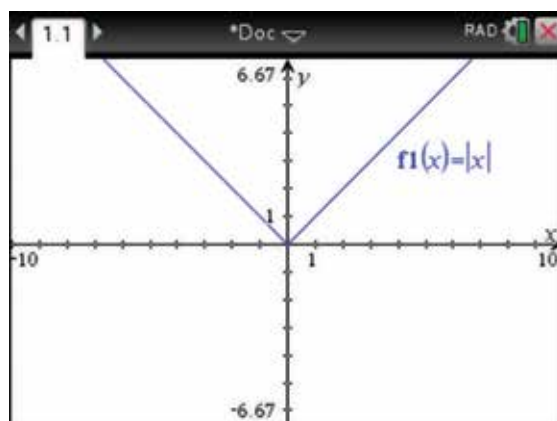
Use the Touchpad to locate the template highlighted in the screen opposite. Then press the enter key **(enter)**.



3. The page shown opposite will appear. Type x into the field and press the enter key **enter**.

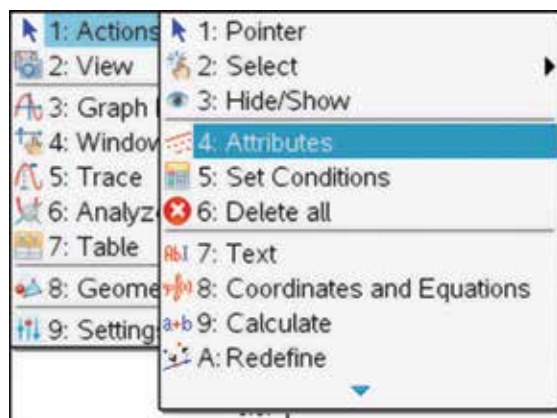


4. The graph of $f1(x)=|x|$ will appear as shown.



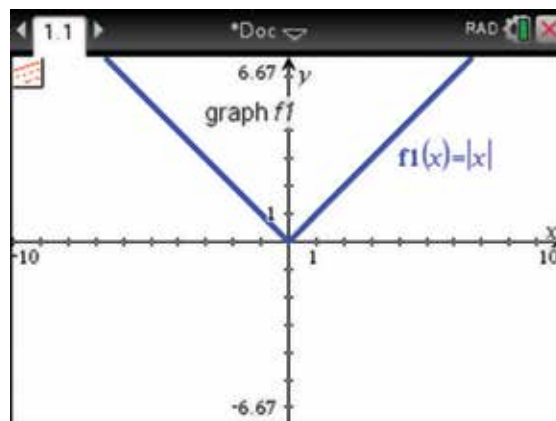
5. To make this initial function more visible, we can alter its appearance or attributes. To do this, press:



- **MENU** **menu**
- **1: Actions** **1**
- **4: Attributes** **4**.

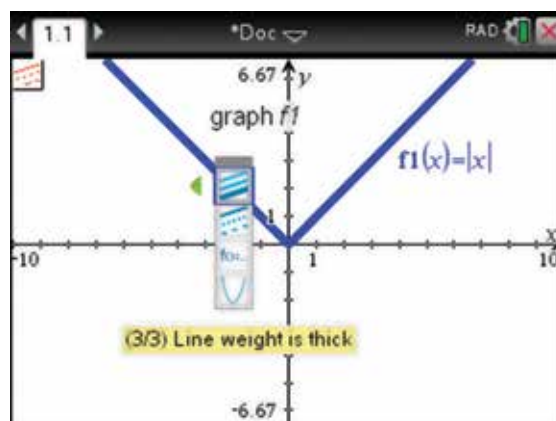


6. Notice the Attributes icon in the top left corner of the graph work area. Position the pointer near the line until its shape changes to a horizontal arrow and 'graph f1' appears. Press:


- ENTER .




7. An attributes options bar will appear. The first option is for the thickness of the line. Using the right arrow on the Touchpad, select: (3/3) Line weight is thick. Press the enter key . Press the escape key  to exit attributes.

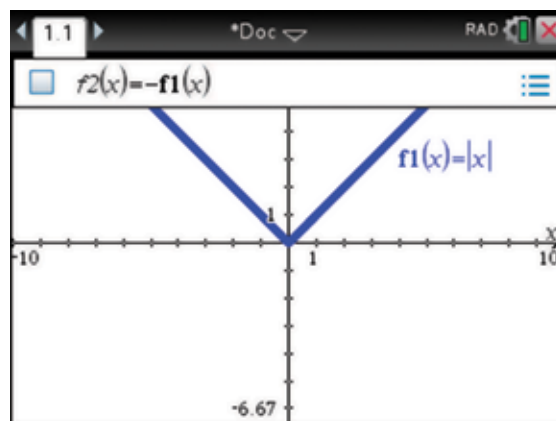


8. By using this as the basic graph, it is easy to illustrate the effect of any transformations on this function.

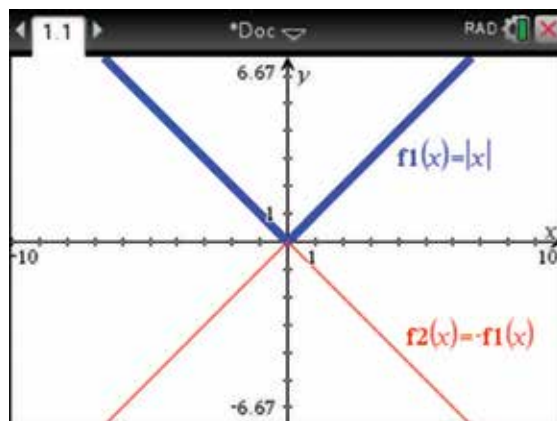
To return to the function entry line, press the tab key . To see the graph of $y = f(x)$ reflected in the x -axis, complete the function entry line as:

$$f2(x) = -f1(x).$$

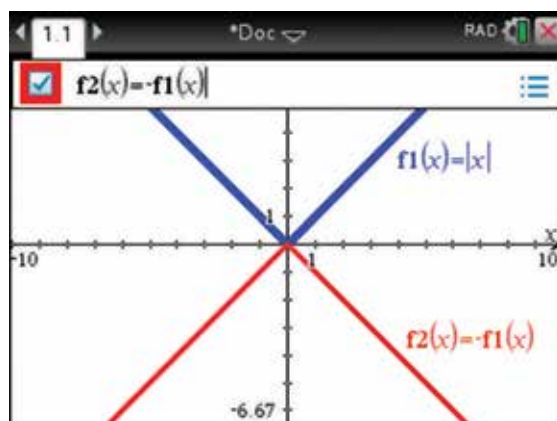
Press the enter key .



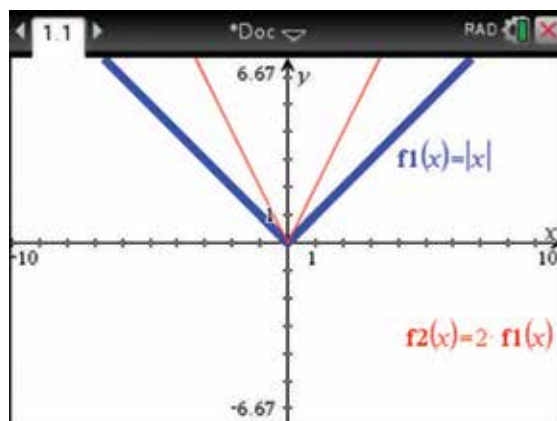
9. The equations of the graphs are shown.



10. To illustrate another transformation, a dilation, move the pointer until it hovers over the equation $f2(x) = -f1(x)$ and changes into an open hand. Then press the click key twice. The function entry line will appear.

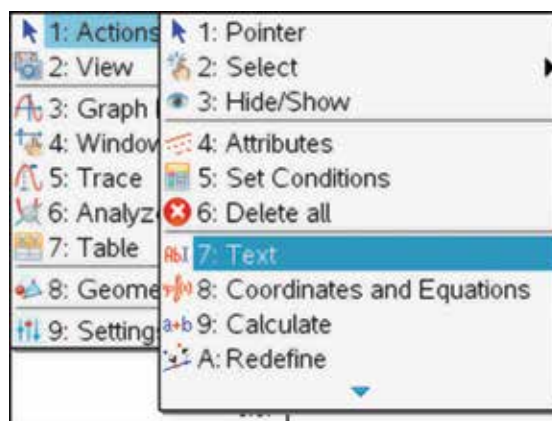



11. Try transforming the graph by a dilation factor of 2 from the x-axis or parallel to the y-axis. To do this, change the equation to $f2(x) = 2f1(x)$ as shown. Press the enter key **enter** to see the new graph.

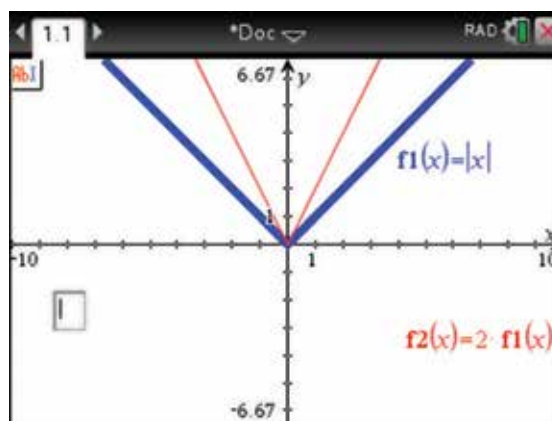


12. It is possible to write a description of this dilation in the graph work area. To do this, press:

- MENU **menu**
- 1: Actions **1**
- 7: Text **7**.

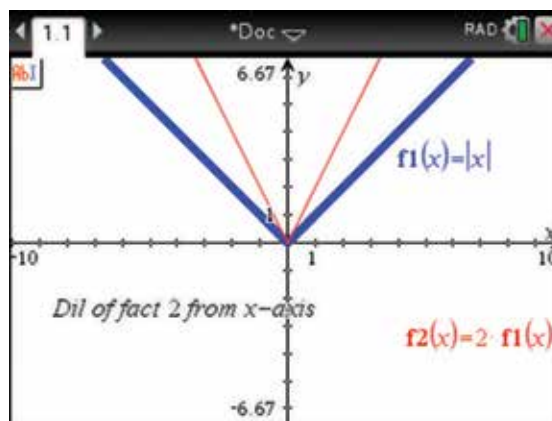


13. Use the Touchpad to move the pointer I to a suitable position and press the click key  to open a text box.



14. Write the relevant annotation, such as 'Dil of fact 2 from x -axis' as shown on the screen. Then press the enter key **enter**.

Note: Abbreviations are used to save space.



7.2 How to graph hybrid functions

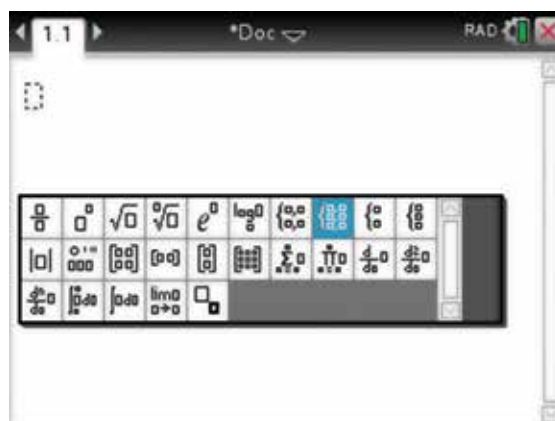
1. Open a new document and add a Calculator page (see section 2.1). To graph the hybrid function




$$f(x) = \begin{cases} x & \text{for } x < 0 \\ x+1 & \text{for } 0 \leq x < 2 \\ 5-x & \text{for } x \geq 2 \end{cases}$$

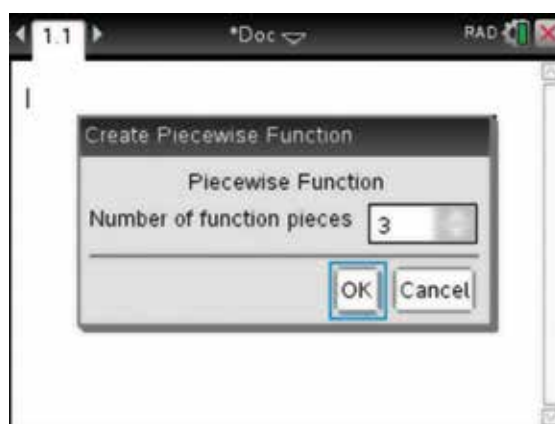
select the hybrid function template from the Maths Expression palette by pressing:


- .




Use the Touchpad to move across to the selection highlighted in the screen opposite.

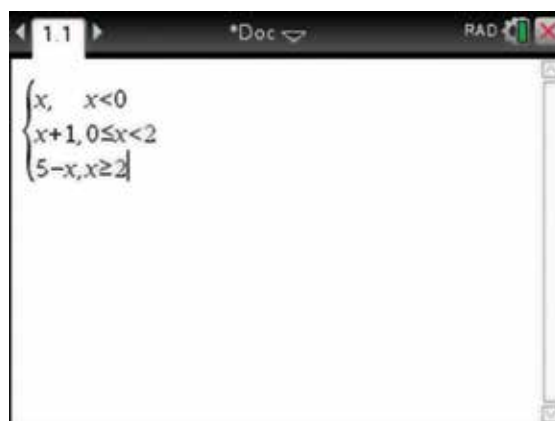


2. Press the enter key . The screen opposite appears. Use the Touchpad to select 3 function pieces. Use the tab key  to select OK, then press the enter key .





3. Use the tab key  to move through the six boxes and type the expressions as shown.

Note: You must press the ctrl key  and  each time to get the template to insert the inequality signs, such as \leq . Do not press the enter key  as we have not yet defined the function.

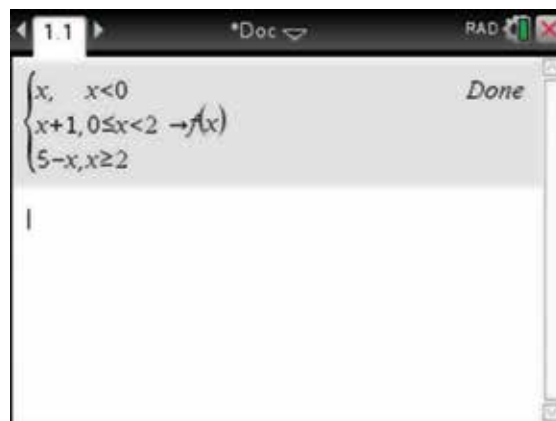


4. To define this hybrid function as $f(x)$, complete the following steps. Press:

- **tab** 
- **ctrl** 
- **var** 

Type $f(x)$, then press:

- **ENTER** 



5. To draw the graph of the hybrid function $f(x)$, add a Graphs page to the document. Press:

- **HOME** 

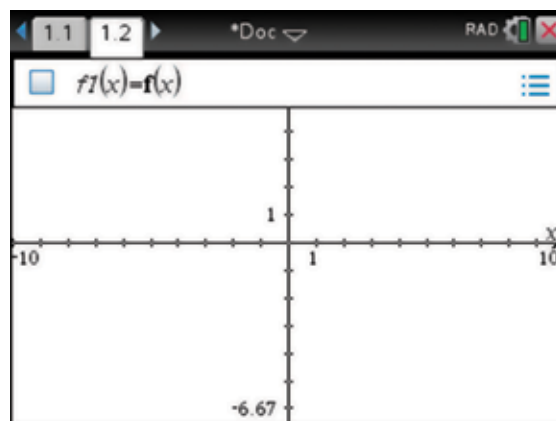
Select the Graph icon and press:

- **ENTER** 

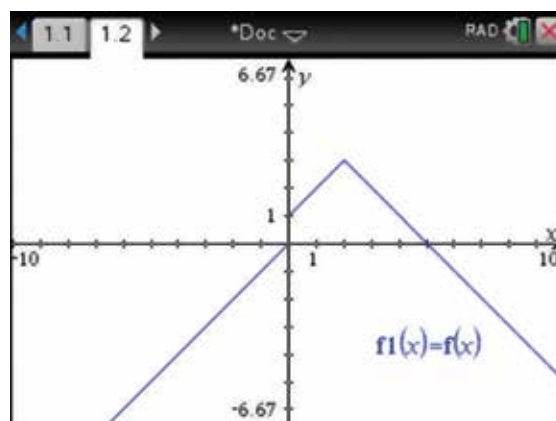


6. Complete the function entry line as:

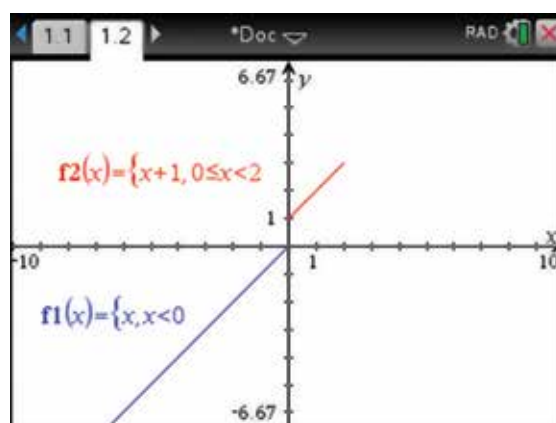
$$f1(x) = f(x).$$



7. Press the enter key **enter** to display the graph.



8. An alternative way to graph this hybrid function is to draw the graph of three restricted domain functions. However, each graph will be drawn in a different colour, as shown.



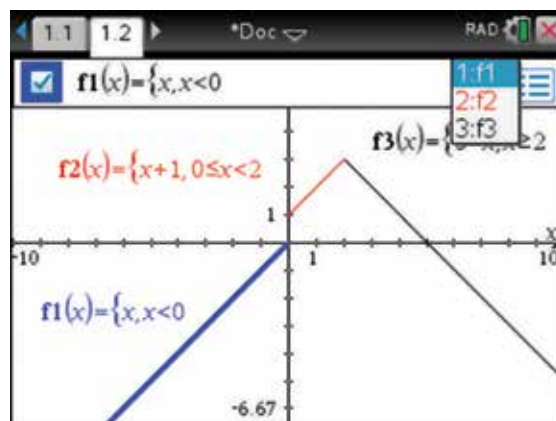
9. Note that the three functions were typed into the function entry line as:

$$f1(x) = x \mid x < 0$$

$$f2(x) = x + 1 \mid 0 \leq x < 2$$

$$f3(x) = 5 - x \mid x \geq 2.$$

However, when we press the enter key, the functions are displayed as shown.



7.3 How to find an inverse algebraically

1. Often it is necessary to find the inverse of a function.

This can be done easily on the CAS calculator. An example is shown below.

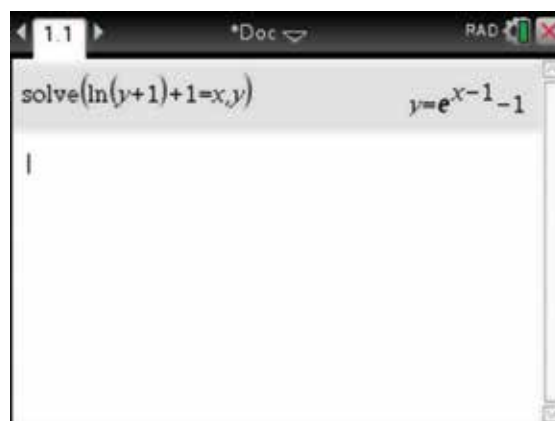
If $f(x) = \log_e(x+1) + 1$, find f^{-1} .

Let $y = f(x)$, so $y = \log_e(x+1) + 1$.

Interchange x and y and use the Solve application to solve for y as shown on the screen opposite.

Thus $f^{-1}(x) = e^{x-1} - 1$.

Note: An alternative method is to define $f(x)$ and then solve $(f(y) = x, y)$.



7.4 How to find an inverse graphically

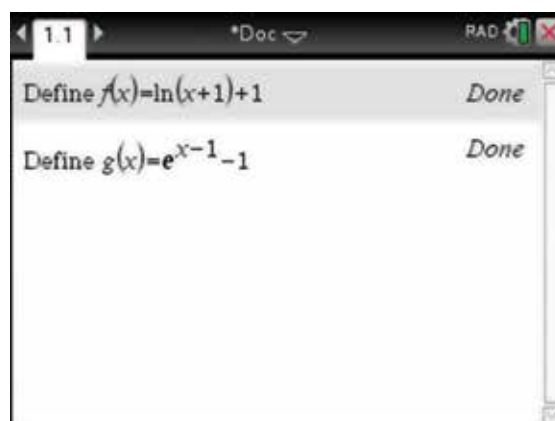
1. To draw the graph of $f(x) = \log_e(x+1) + 1$ and its inverse $f^{-1}(x) = e^{x-1} - 1$, open a new document and complete the following steps. Define these functions on a Calculator page. Press:

- **MENU** (menu)
- **1: Actions** (1)
- **1: Define** (1).

Complete the entry line as shown:

Define $f(x) = \ln(x+1) + 1$.

Press the enter key (enter). Repeat the process to define the inverse function $f^{-1}(x) = e^{x-1} - 1$ as $g(x)$.



2. To draw these functions, add a Graphs page to the document. Press:

- **HOME** (on).

Select the graph icon and press:

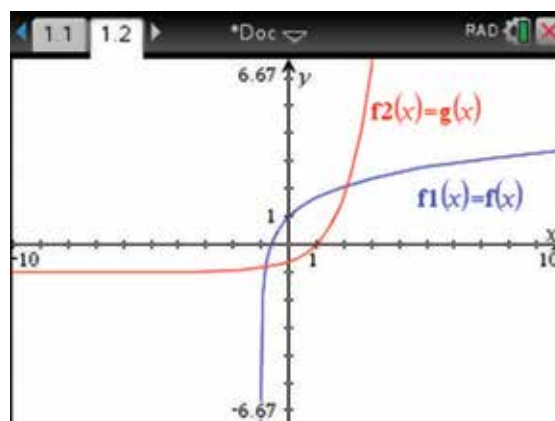
- **ENTER** (enter).

To draw $f(x)$, complete the function entry line as:

$f1(x) = f(x)$. Press the enter key (enter).

To draw $g(x)$, use the tab key (tab) to get back to the function entry line as: $f2(x) = g(x)$.

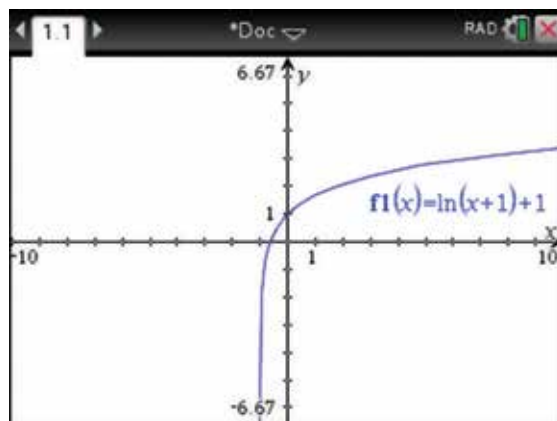
Press the enter key (enter).



3. An alternative method of drawing inverse functions, worthy of exploration, is to use the dynamic geometry functionality of the calculator. Open a new document and add a Graphs page to draw the graph of $f(x) = \log_e(x+1) + 1$. To draw $f(x)$, complete the function entry line as:

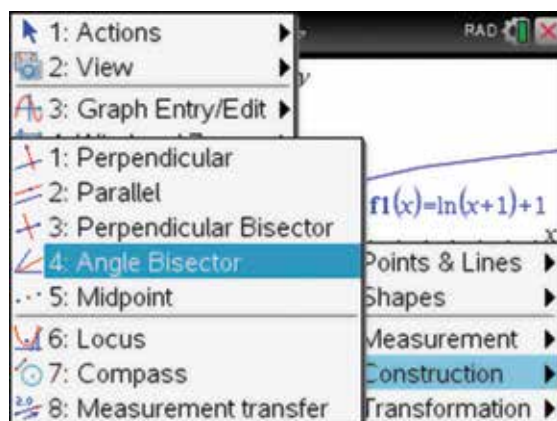
$$f1(x) = \ln(x+1) + 1.$$





Then press the enter key **enter**.



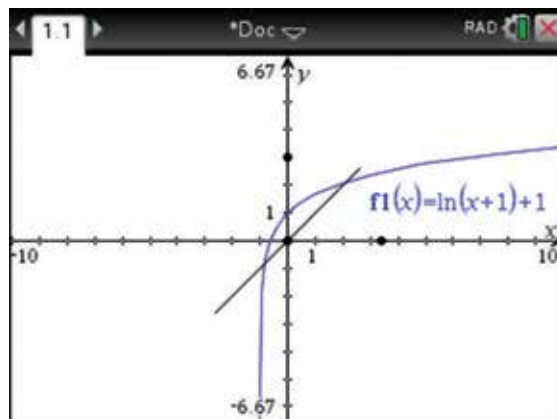
4. To draw the inverse function, we need to construct the line $y = x$ geometrically. To do this we need to bisect the angle between the axes. Press:

- **MENU** **menu**
- **8: Geometry** **8**
- **4: Construction** **4**
- **4: Angle Bisector** **4**.



5. You are returned to the graph work area, where the pencil  has appeared. Using the Touchpad, move the pencil, press the click key  once on the positive y-axis, press the click key  once on the origin and press the click key  once on the positive x-axis. The line $y = x$ appears. Press the escape key **esc**.

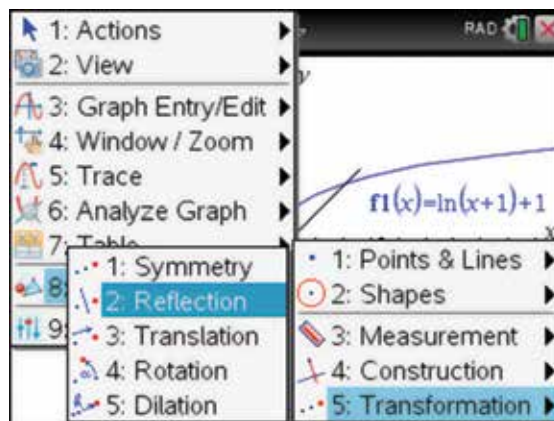
Note: To bisect an angle, the vertex of the angle must be clicked second.





6. Now you can reflect a point on $f_1(x)$ in the line $y = x$.

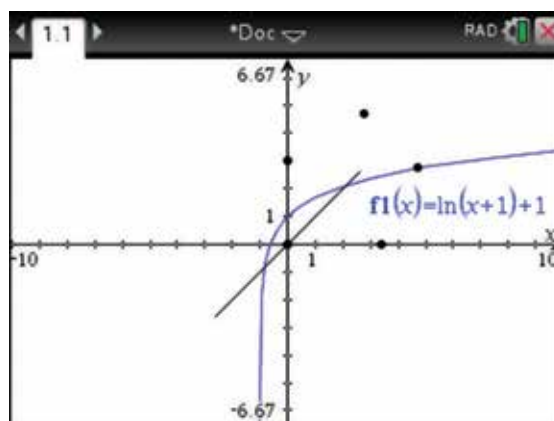
To do this, press:

- **MENU** **menu**
- **8: Geometry** **8**
- **5: Transformation** **5**
- **2: Reflection** **2**.



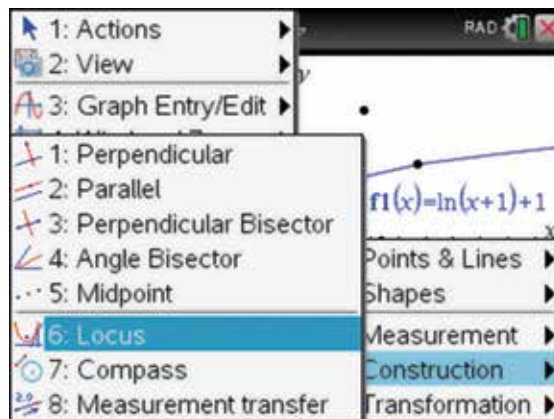
7. Move the pencil onto the $f_1(x)$ graph and press the click key  to create a point. Then move the pointer onto the graph of $y = x$ and press the click key .



The reflected point will appear. Press the escape key **esc**.



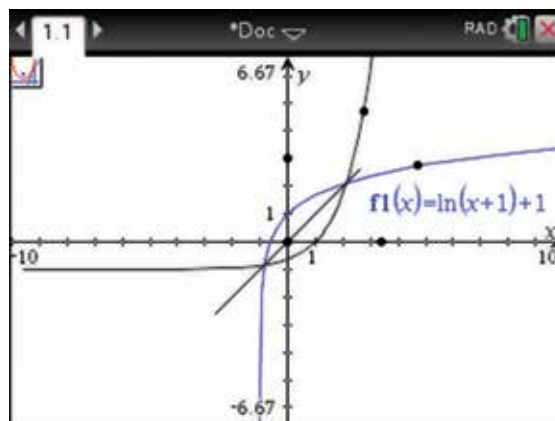
8. Then press:

- **MENU** **menu**
- **8: Geometry** **8**
- **4: Construction** **4**
- **6: Locus** **6**.



- Move the pointer onto the reflected point, then press the click key . Move the pointer back to the original point on the function $f1(x)$ and then press the click key . The graph of the inverse will appear.

Note: That this graph does not have an equation; it is really just an image of reflected points.

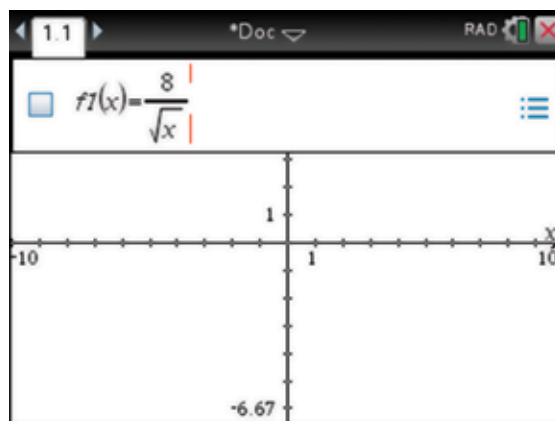


7.5 How to find exact values on a graph

- Open a new document and add a Graphs page (see section 7.1). Complete the entry line as shown:

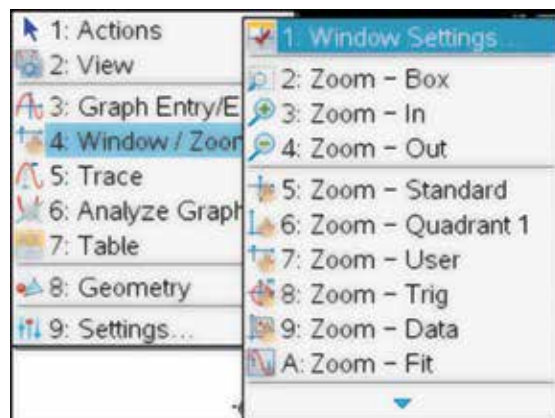
$$f1(x) = \frac{8}{\sqrt{x}}$$

Press the enter key .

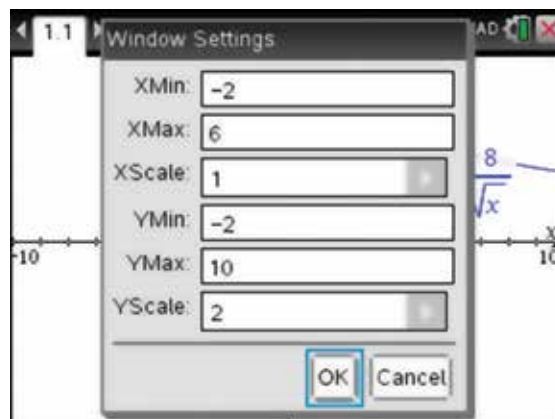


- The graph is shown. Choose a suitable viewing window, press:

- MENU** 
- 4: Window/Zoom** 
- 1: Window Settings** .

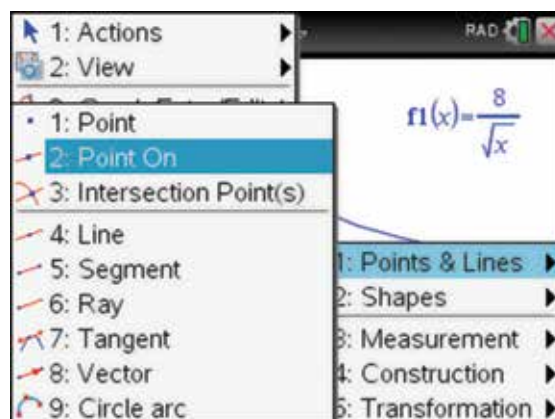


3. Complete as shown.

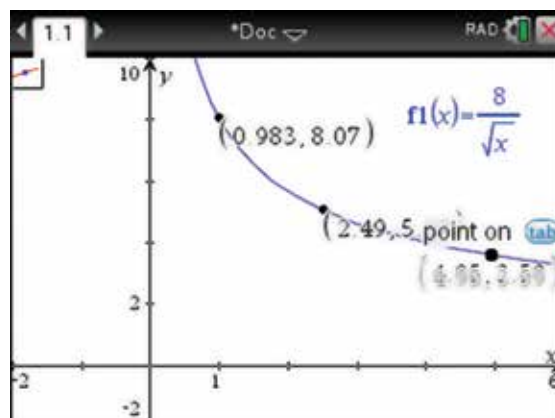





4. To place four points on the graph, press:

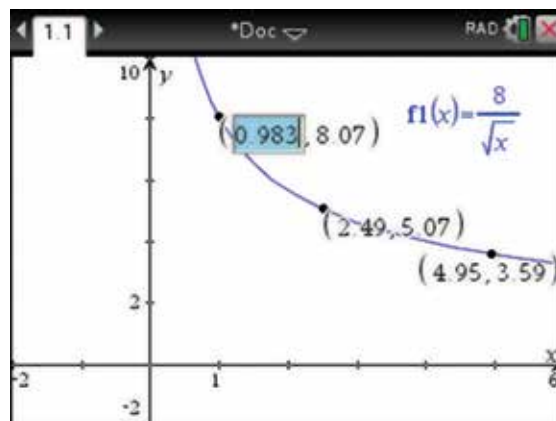
- MENU (menu)
- 8: Geometry (8)
- 1: Points & Lines (1)
- 2: Point On (2).



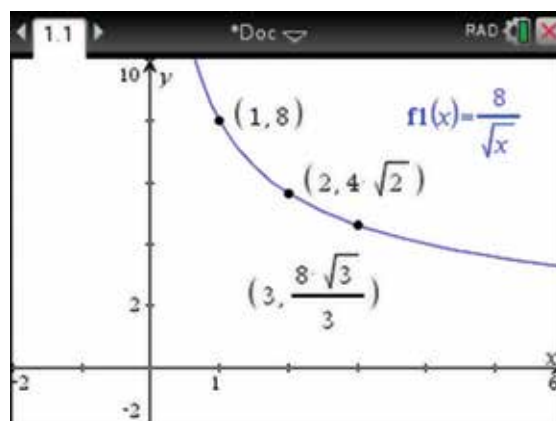
5. Position the pencil over the graph and press the enter key (enter). Repeat to add the other points on the graph. Note that their coordinates are displayed.



6. Press the escape key **(esc)** to stop adding points, and move the cursor  over the graph until it is over one of the points. When the open hand appears  with the word 'text', press  twice. This will open up a text box that allows you to edit the x -coordinate or the y -coordinates.



7. Repeat the above to change the x -coordinates of each point to 1, 2 and 3. Move the coordinates to a suitable position. Note that the exact coordinates are shown on the graph; this will happen provided the General Settings in the Calculation Mode are in either Exact or Auto mode.



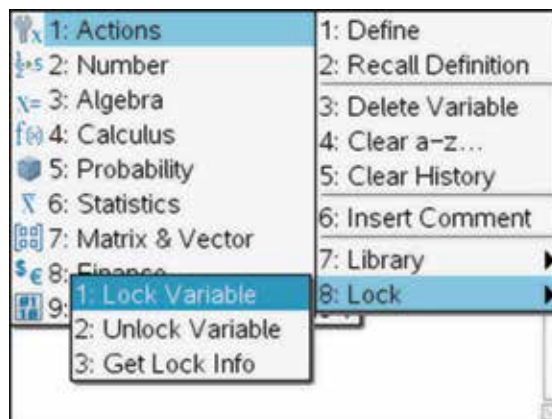
7.6 How to lock variables

1. Open a new document and add a Calculator page. Assign x the value of 5. Press the enter key **(enter)**.



2. To lock the variable x , press:

- MENU **menu**
- 1: Actions **1**
- 8: Lock **8**
- 1: Lock Variable **1**.



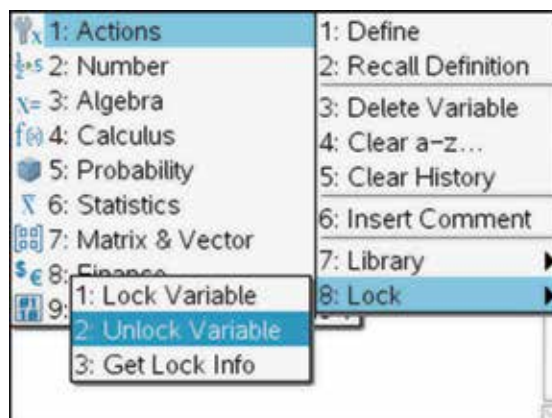
3. If we try to modify, change or delete the variable x , we get a message as shown.



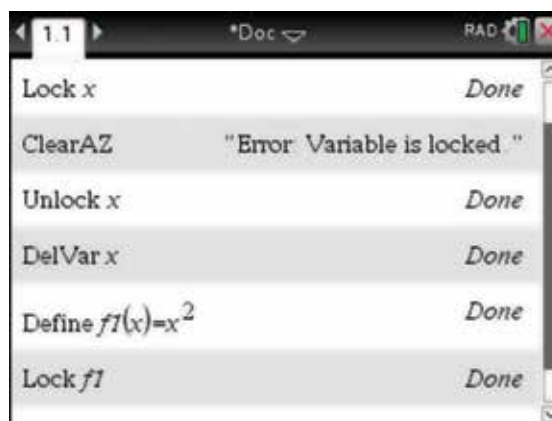
4. To delete or modify the variable, we must unlock the variable. To unlock, press:

- MENU **menu**
- 1: Actions **1**
- 8: Lock **8**
- 2: Unlock Variable **2**.

Locking a variable can be useful to prevent unwanted accidental changes.



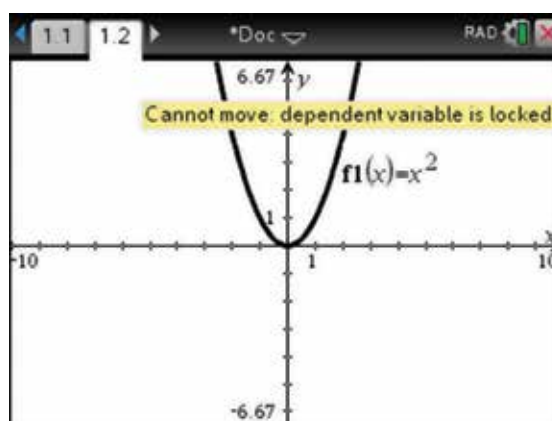
5. Now define a function $f1(x) = x^2$ and lock the function, as shown.



6. Open a Graphs page. Since we have already defined the function, press the up arrow \blacktriangle , then press the enter key **enter** and the graph will appear.

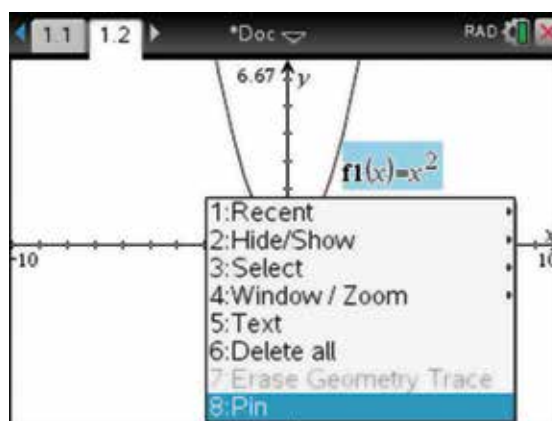
Now grab the graph and try to move it. A message appears, but as the graph is locked it cannot be moved or modified. Also we cannot modify or edit the equation. This technique is useful, as when you have defined and graphed a function, you may want to avoid accidentally moving it and changing its equation.

Note that we can still move the function label around the screen.



7. We can also pin down a label. Move the pointer \blacktriangle until it is over the label. When the open hand appears hand icon with the word 'label', press:

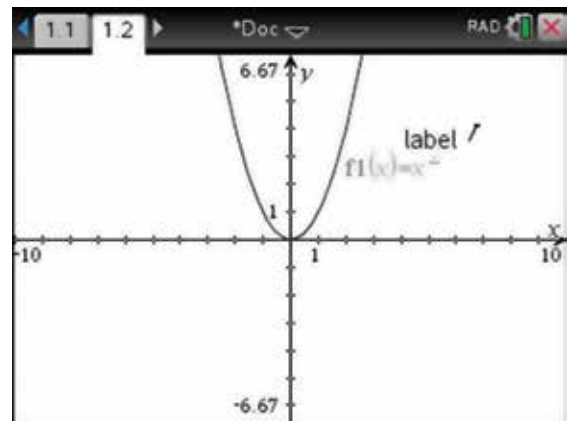
- **ctrl** **(ctrl)**
- **MENU** **(menu)**
- **8: Pin** **(8)**.



8. Now we cannot move the label, as it is pinned down.

If we decided that we really wanted to move the label to a different position on the screen, we could just undo the last command by pressing:

- **ctrl** (ctrl)
- **esc** (esc).




CHAPTER 8


Geometry

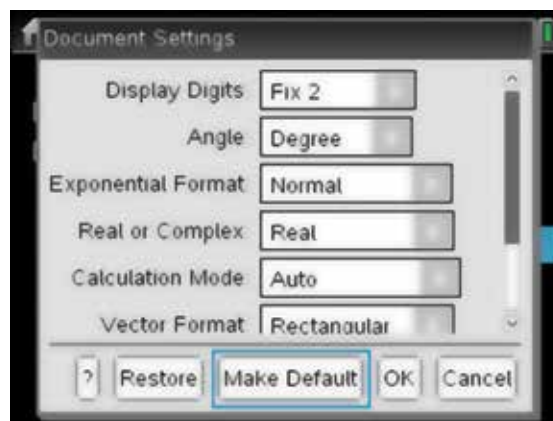
This chapter introduces the geometry applications on the TI-Nspire. This application allows you to view the size, shape, position and other important features of various shapes. A number of theorems are explored here.

8.1 How to calculate the interior angle sum of a triangle

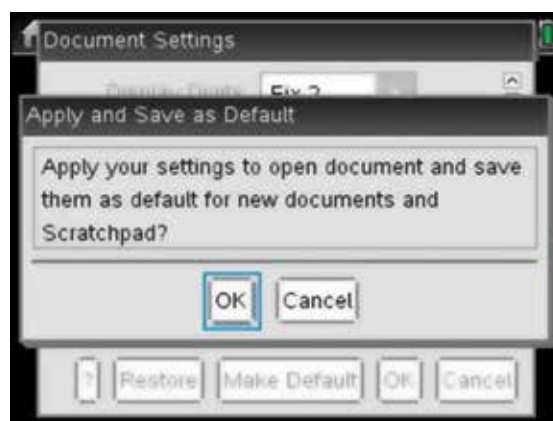
1. When working with Geometry applications, the angle should be set in degrees. To set this, press:

- **HOME**  **on**
- **5: Settings** **5**
- **2: Document settings** **2**.

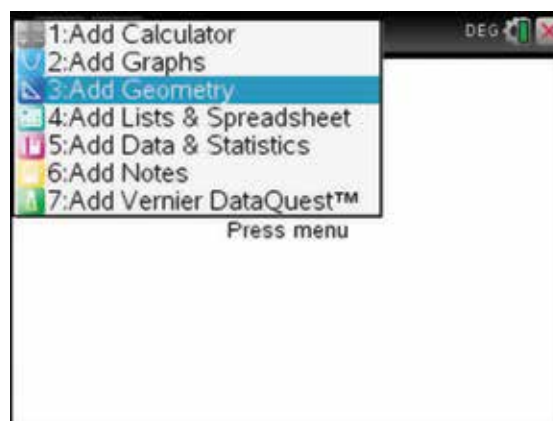
Press the tab key **tab** to select Angle, then press the click key  and select Degree. Press the tab key **tab** to select Make Default, then press the enter key **enter**.



2. The following screen will appear. Press enter **enter**.



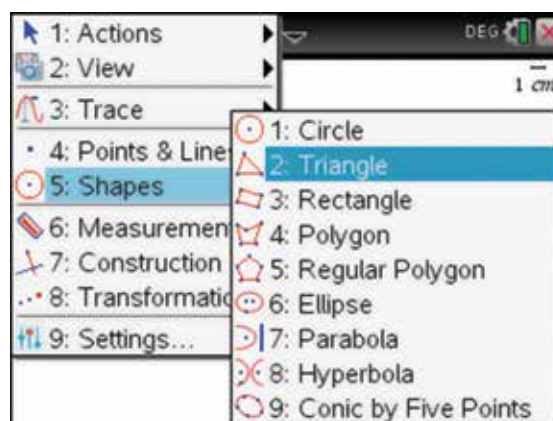
3. Start with a new document and add a Geometry page.









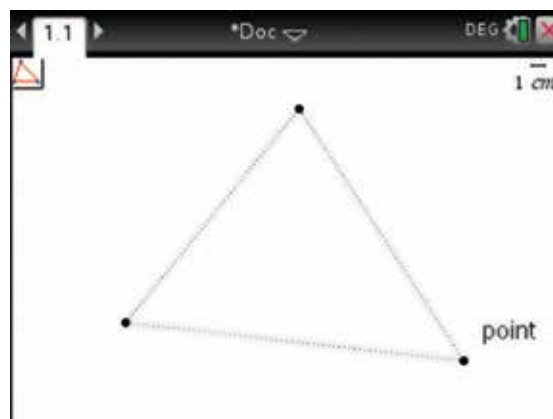
4. To draw a triangle, complete the following steps.

Press:

- **MENU** **(menu)**
- **5: Shapes** **(5)**
- **2: Triangle** **(2)**.

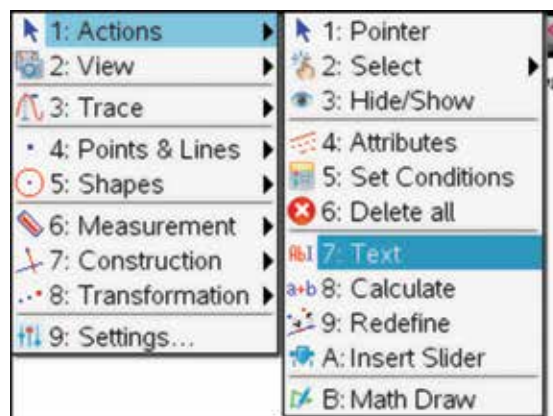





5. Move the pencil  to the desired position for the first vertex, then press the click key . Move the pencil  to the desired position for the second vertex, then press the click key . Move the pencil  to the desired position for the third vertex, then press the click key . Press the escape key **(esc)**.



6. To label the vertices of the triangle, press:

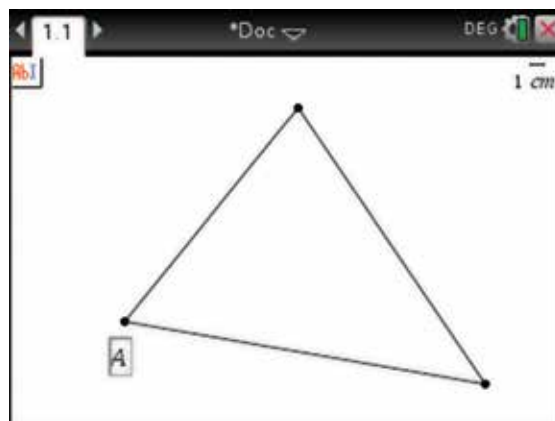
- **MENU** **(menu)**
- **1: Actions** **(1)**
- **7: Text** **(7)**.






7. Move the pointer  to a vertex, then press the click key . A text box will open. Label the vertex A , press the shift key  to capitalise the letter A .



Press the enter key .

Repeat this process for the other two vertices, labelling them B and C .





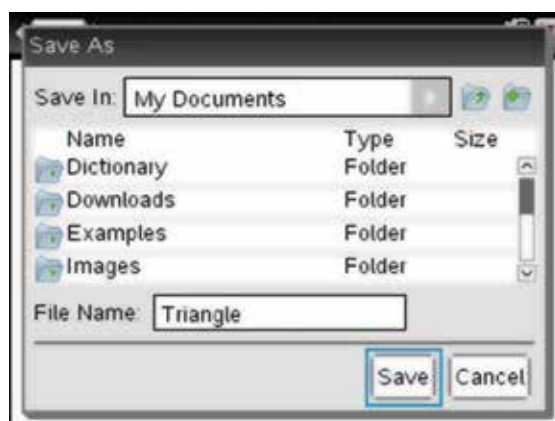
8. When you have finished with the text, press the escape key . We will save this document. Press:

- **ctrl** 
- **save** .




Type 'Triangle' in the File Name box. Use the tab key  to select Save and press the enter key .

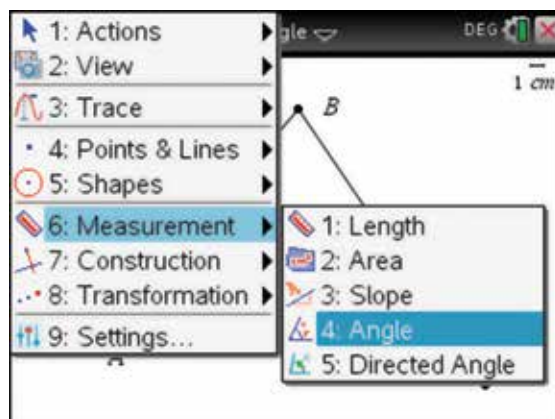
Note: To resave at any time, simply press:









- **ctrl** 
- **S** .

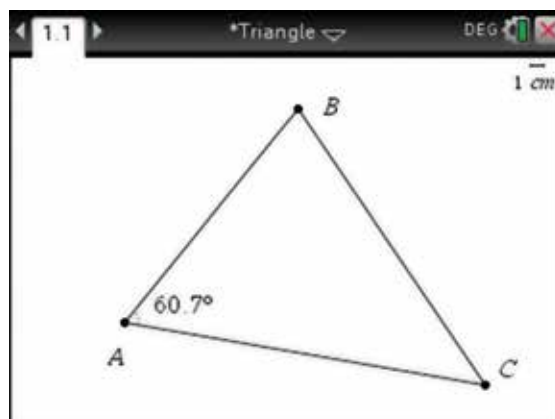


9. To measure the magnitudes of the angles of the triangle, press:

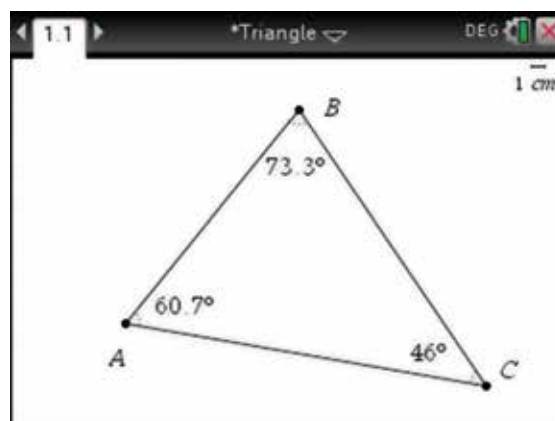
- **MENU** 
- **6: Measurement** 
- **4: Angle** .



10. To measure $\angle BAC$, move the pencil  to vertex B . When the hand with a pointing finger  appears, press the enter key . Then move the pencil  to vertex A and press the enter key . Then move the pencil  to vertex C and press the enter key . The angle symbol is displayed to indicate which angle measurement is being calculated. Press the enter key  again on vertex A to display the measured angle.



11. Repeat this process to obtain the measurements of the other angles.



8.2 How to illustrate a circle theorem (#1)

One circle theorem states: The angle subtended at the centre of a circle is always twice the size of the angle subtended at the circumference of the circle and standing on the same arc.

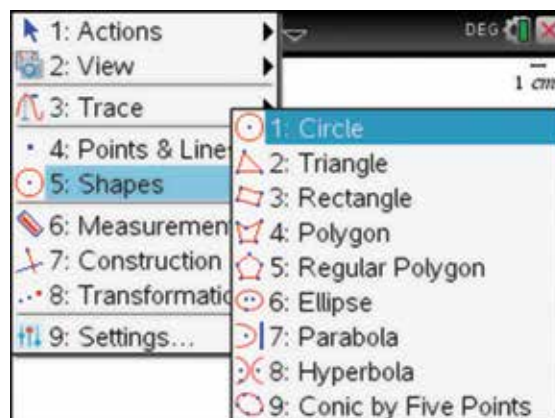
1. To verify this theorem, start with a new document.


Press:

- **ctrl** (ctrl)
- **N** (N)
- **3: Add Geometry** (3).

To draw a circle, press:

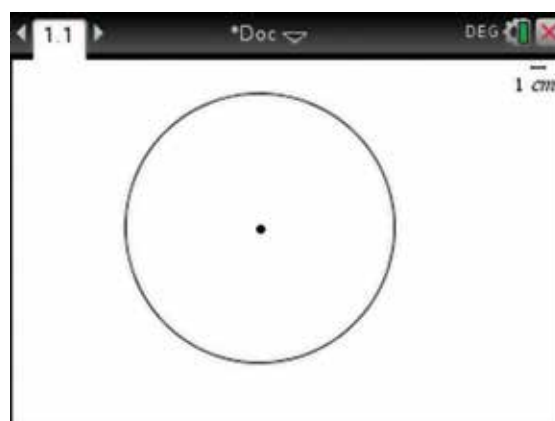
- **MENU** (menu)
- **5: Shapes** (5)
- **1: Circle** (1).



2. Position the pencil  at the chosen location for the centre of the circle. Press the enter key (enter).

Use the Touchpad to enlarge the circle. Then press:

- **ENTER** (enter)
- **esc** (esc).

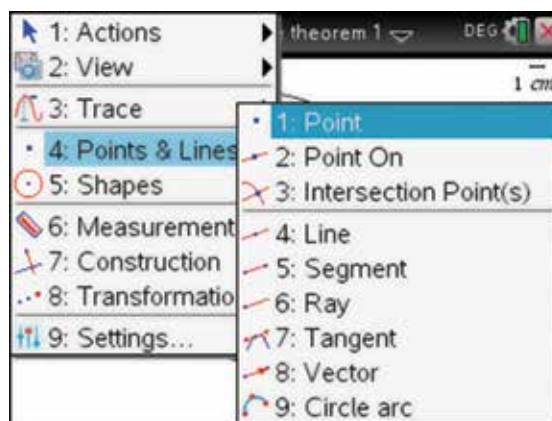


3. We will save this document. Press:

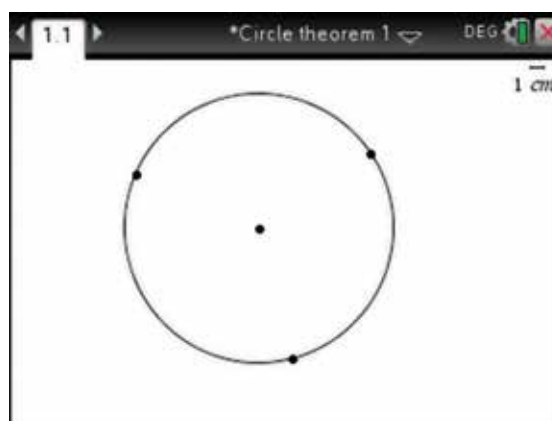
- **ctrl** (ctrl).
- **save** (save icon).

In the File Name: box, type 'Circle Theorem 1'. Use the tab key (tab) to select Save and press the enter key (enter). Now Press:

- **MENU** (menu).
- **4: Points & Lines** (4).
- **1: Point** (1).

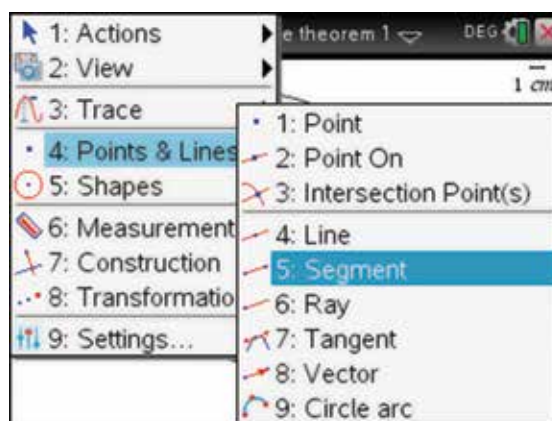


4. Move the pencil (pencil icon) to a position on the perimeter of the circle, then press the enter key (enter). Move the pointer to another point on the perimeter of the circle, then press the enter key (enter). Move the pointer to a third point on the perimeter of the circle, then press the esc key (esc), to stop adding more points.



5. To draw a line segment between points on the circle, press:

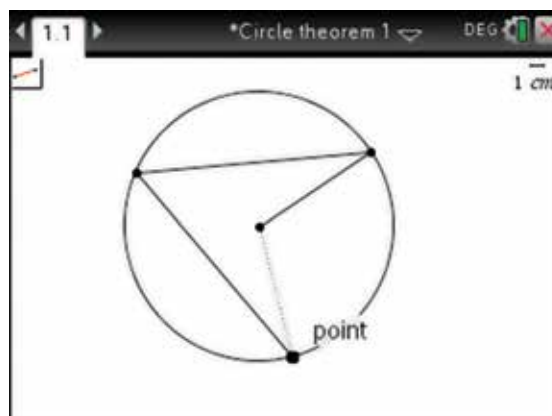
- **MENU** (menu).
- **4: Points & Lines** (4).
- **5: Segment** (5).



6. Move the pencil (pencil icon) to one of the points until it turns into the hand with the finger (hand icon); then press the enter key (enter). Move the pointer to a point you wish to join with a line segment, then press the enter key (enter).

Repeat for each pair of endpoints as shown.

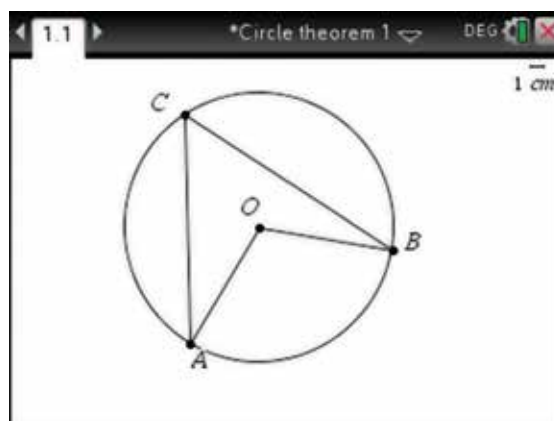
Press the esc key (esc).



7. Label the vertices for ease of reference. To do this, press:

- **MENU** **menu**
- **1: Actions** **1**
- **7: Text** **7**.

Then press the esc key **esc**. Move the points around the circumference of the circle and position the labels for a better view. Resave the document.

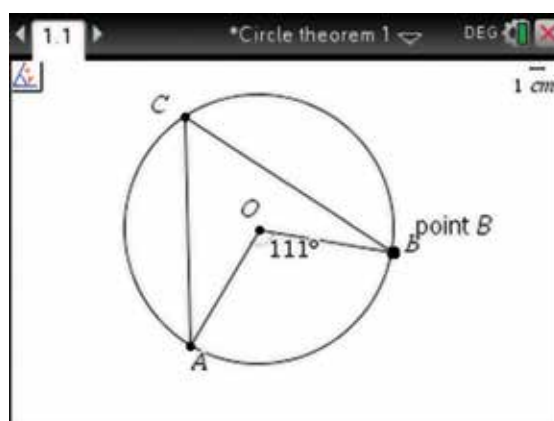


8. To measure $\angle AOB$ and $\angle ACB$, complete the following steps, as in section 8.1.

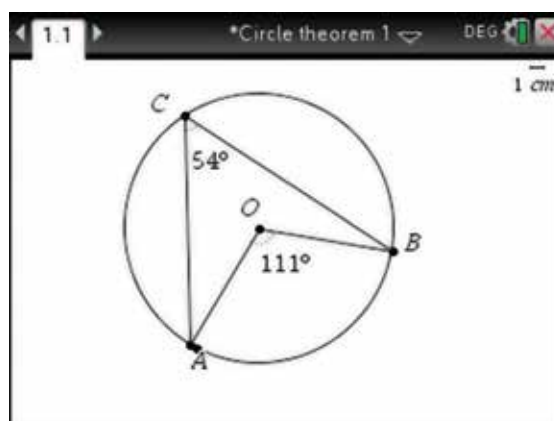
- **MENU** **menu**
- **6: Measurement** **6**
- **4: Angle** **4**.

Move the pointer over vertex A and press the enter key **enter**. Repeat this for vertex O and vertex B.

Note: The vertices must be selected in the order given.



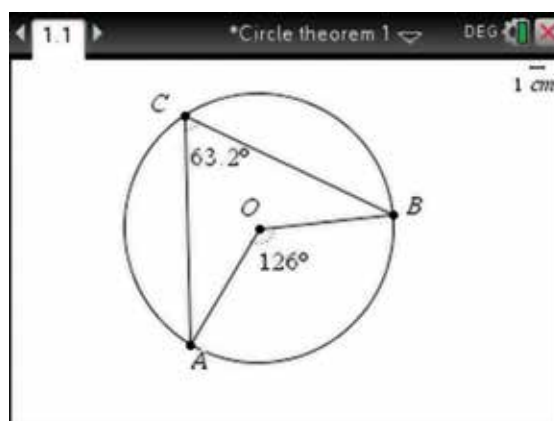
9. Repeat this process for $\angle ACB$.



10. Notice that the angle at the centre of a circle is always twice the size of the angle subtended at the circumference of the circle and standing on the same arc. To check if this theory holds true, drag a vertex and change the values. To do this, move the pointer until it hovers over a point. When the shape changes to an open hand, press:

- **ctrl** **ctrl**
- **CLICK**

The hand will close. Drag the vertex and watch the measurements change accordingly. Resave the document. The angle sizes always stay in the ratio of $\angle ACB : \angle AOB = 1 : 2$.

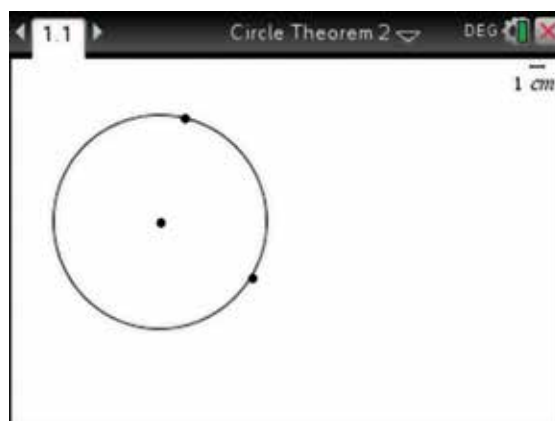


8.3 How to illustrate a circle theorem (#2)

1. To show that two tangents that meet outside a circle are equidistant from the point of contact on the circle, complete the following steps.

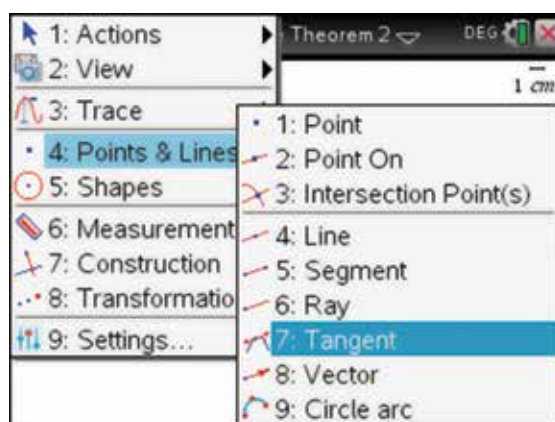
Start with a new document and add a Geometry page.

Draw a circle as shown, and place two points on the circumference of the circle. Save the document under the name 'Circle Theorem 2'. The steps to do this are described in section 8.2.

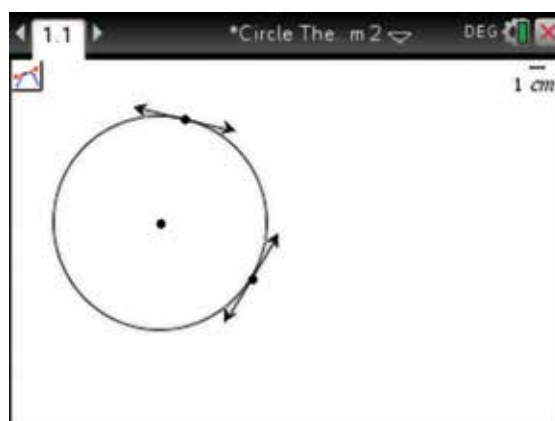


2. To draw a tangent to the circle at each point, complete the following:

- MENU **menu**
- 4: Points & Lines **4**
- 7: Tangent **7**.

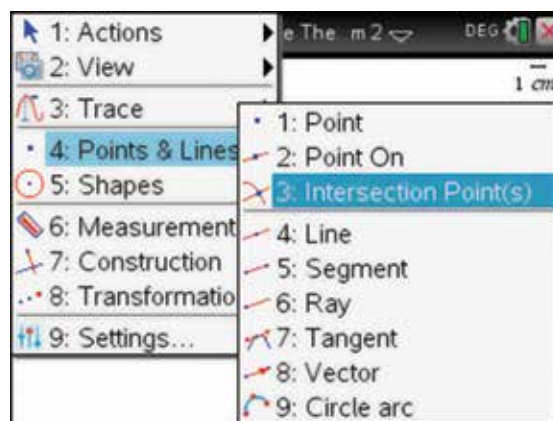









3. Click on each point on the circumference, then press the esc key **esc**.

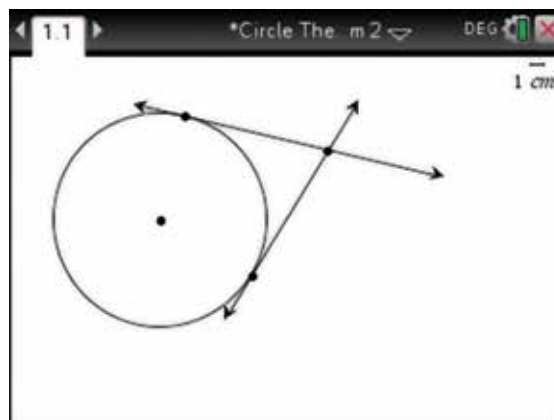


4. To find the point of intersection of the tangents, press:




- **MENU** 
- **4: Points & Lines** 
- **3: Intersection Point(s)** 

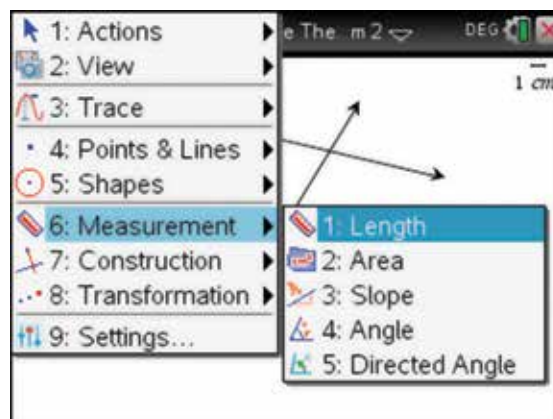






5. Move the pointer  over one of the tangents. When the word 'line' appears with the finger , press the click key  or the enter key . Then move the finger  over the other tangent and press the click key . The tangents will be extended and the point of intersection will be displayed, then press the esc key .




6. To measure the length of each line segment, press:

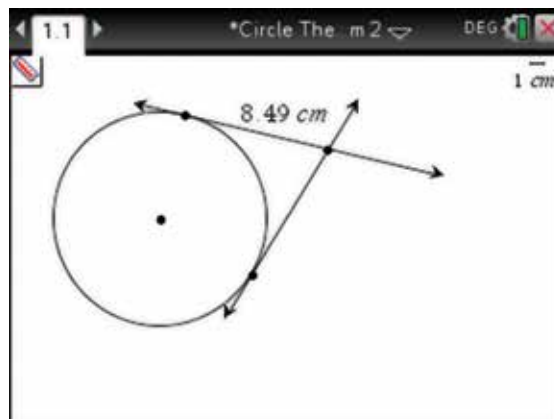
- **MENU** 
- **6: Measurement** 
- **1: Length** 



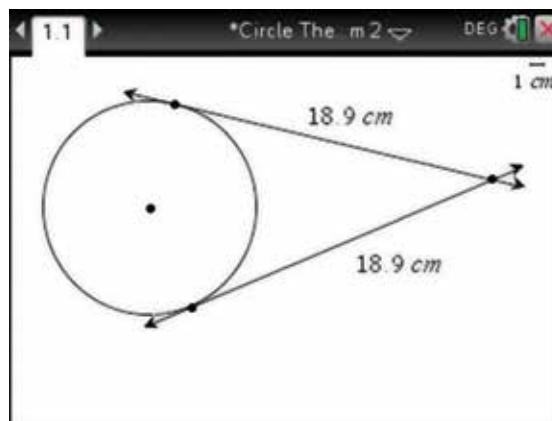
7. Move the pointer  to the point of intersection and press the enter key . Then move the pointer  to the other point on the line and press the enter key .

The length of the line will be displayed.

Press the enter key  to display the measured length.



8. Repeat this process for the second line. Then press the **esc** key. This shows that two tangents that meet outside a circle are equidistant from the point of contact on the circle. You can reposition the labels to get a better view and resave the document. To check that this is always true, grab and move the points on the circumference or change the radius of the circle. The lengths of the line segments remain equal.

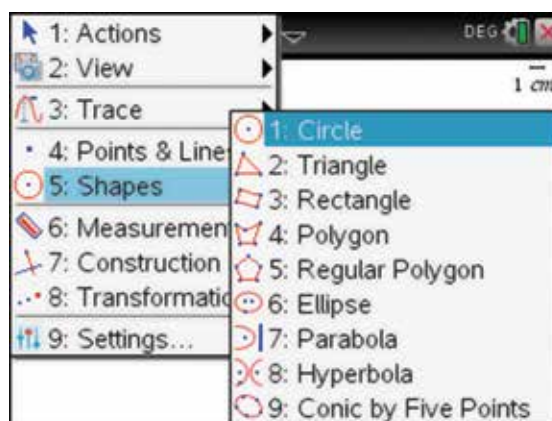


8.4 How to optimise a rectangle inscribed in a circle

Consider the following optimisation problem. 'Given a circle with a radius of r , inscribe a rectangle with base length l and width w , such that all vertices of the rectangle lie on the circle. Find the ratio of the largest rectangle, to that of the circle.'

1. To solve this problem geometrically, open a new document and add a Geometry page. To draw a circle, press:

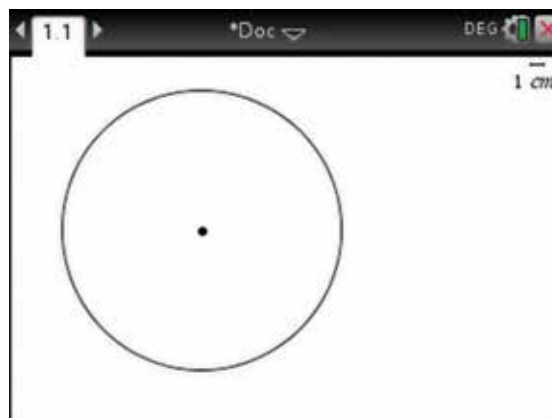
- **MENU** **menu**
- **5: Shapes** **5**
- **1: Circle** **1**.






2. Position the pencil at the chosen location for the centre of the circle. Press the **enter** key.

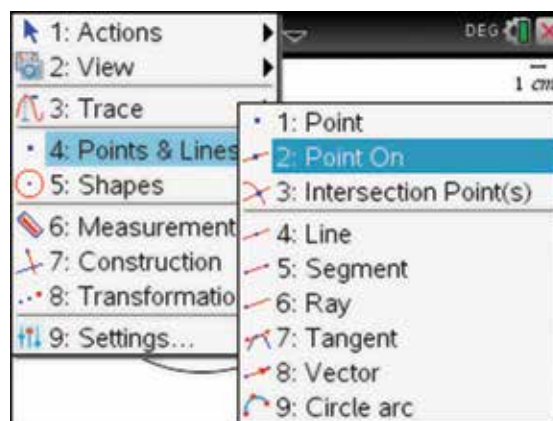
Use the Touchpad to enlarge the circle. Then press:






- **ENTER** **enter**
- **esc** **esc**.

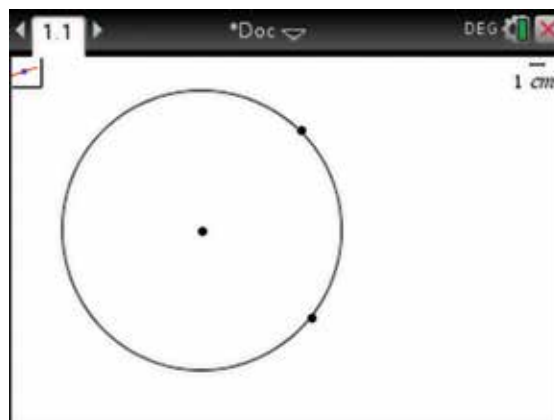


3. Now place a point on the circumference of the circle. To do this, press:




- **MENU** 
- **4: Points & Lines** 
- **2: Point On** .

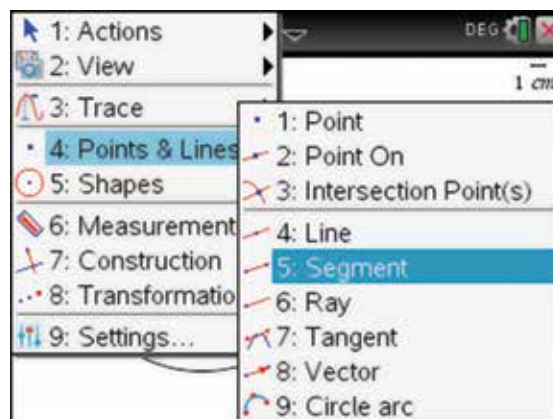









4. Position the pointer  over the circle. When the pointer changes into the finger  and the word 'circle' appears, press the enter key  or the click key . Move to another point on the circumference and place another point on the circle, then press the escape key .

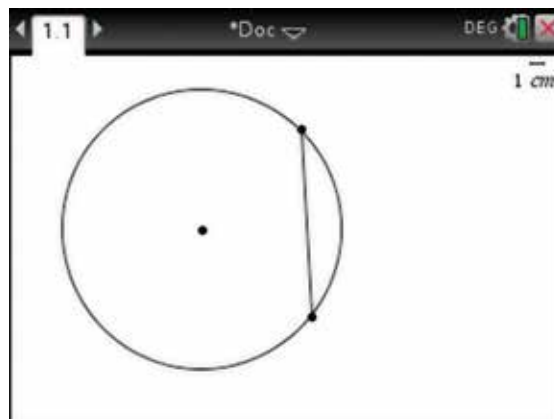


5. Now join these two points with a line segment. To do this, press:

- **MENU** 
- **4: Points & Lines** 
- **5: Segment** .

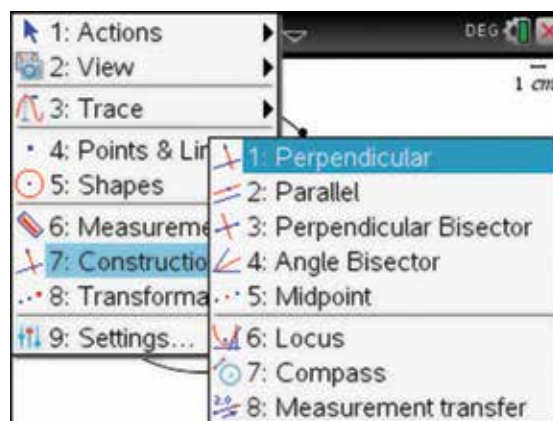






6. Position the pointer  over the first point. When the pointer changes into the finger  and the word 'point' appears, press the enter key  or the click key . Move to the other point and press the enter key  or the click key , then press escape .

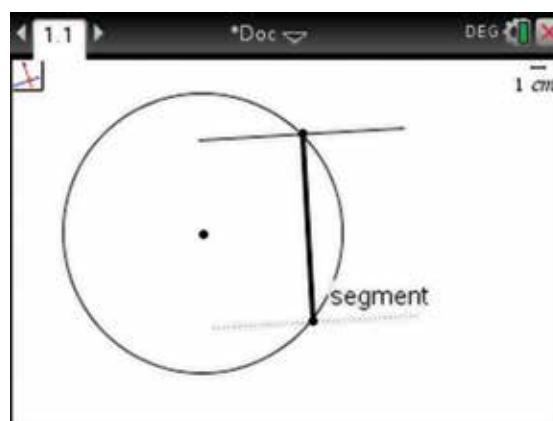


7. Now construct lines perpendicular to this line segment. To do this, press:

- **MENU** **(menu)**
- **7: Construction** **(7)**
- **1: Perpendicular** **(1)**.

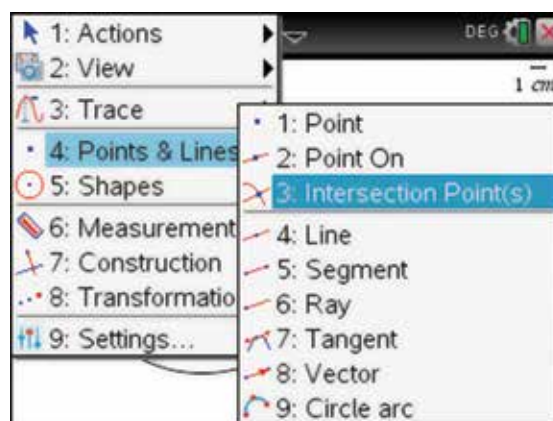


8. Position the pointer  over the first point. When the pointer changes into the finger  and the word 'point' appears, press the enter key **(enter)** or the click key . Move to the segment, then press the enter key **(enter)** or the click key . Repeat for the other point as shown, then press escape **(esc)**.

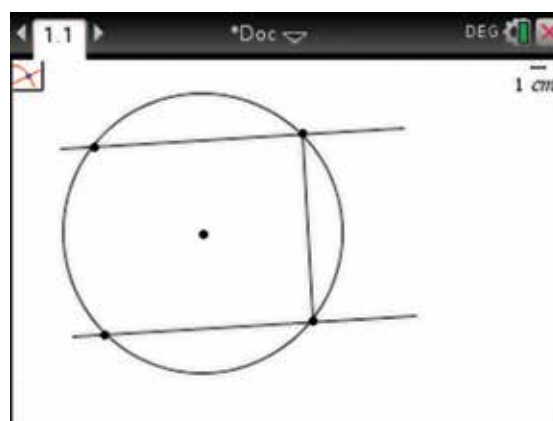


9. Now find the point of intersection between these perpendicular lines and the circle. To do this, press:

- **MENU** **(menu)**
- **4: Points & Lines** **(4)**
- **3: Intersection Point(s)** **(3)**.

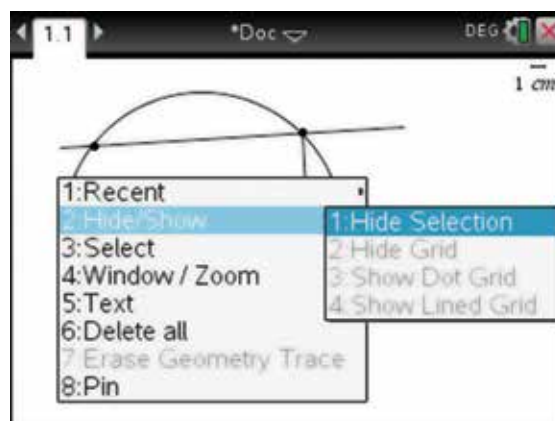


10. Click on the line, then click on the circle. Note the lines will automatically be extended and the points of intersection will be shown.

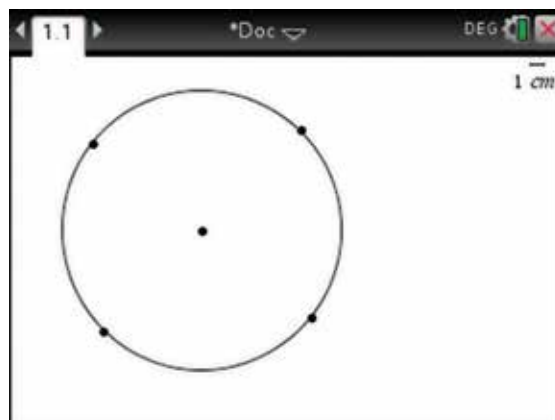


11. These lines are only for construction, so we want to hide them. To do this, press:

- **ctrl** (ctrl)
- **MENU** (menu)
- **2: Hide/Show** (2)
- **1: Hide Selection** (1).

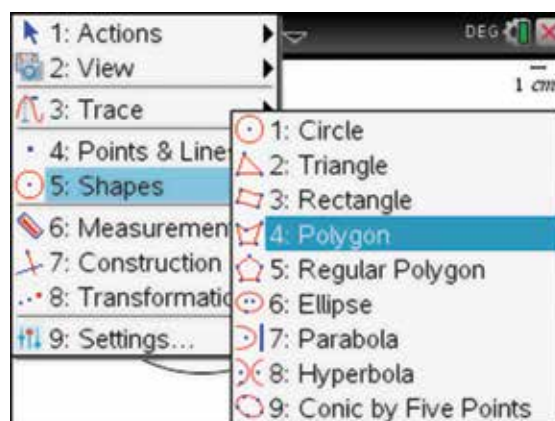


12. Click on the line and it will disappear. Repeat for the other line and the line segment.

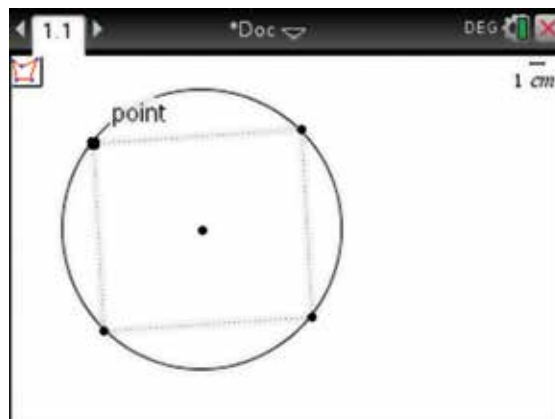




13. Now we will draw a polygon over these four points. (A rectangle would not work, as it would not enforce the fact that the four points lie on the circle.) To draw the polygon, press:




- **MENU** (menu)
- **5: Shapes** (5)
- **4: Polygon** (4).

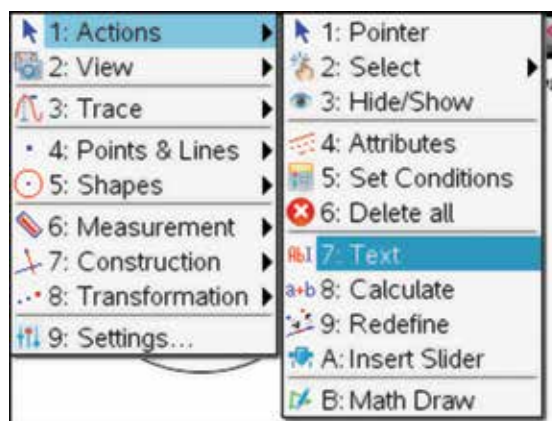



14. Now click on the four points, in either clockwise or anti-clockwise order, but we must click on the first point again to tell the tool we have enough vertices. Then press escape (esc).

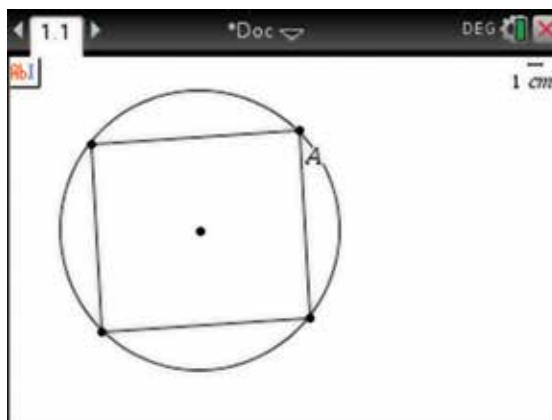


15. Now label the five points. To do this, move the pointer  over a point. When the pointer changes into the finger  and the word 'point' appears, press:

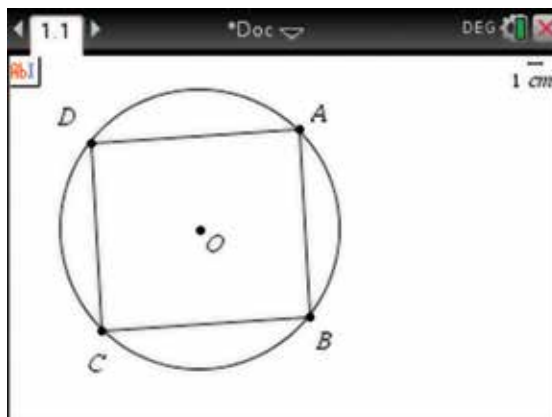
- MENU 
- 1: Actions 
- 7: Text .





16. A text box will open. To type in the label for the point, type *A*, then press the enter key .





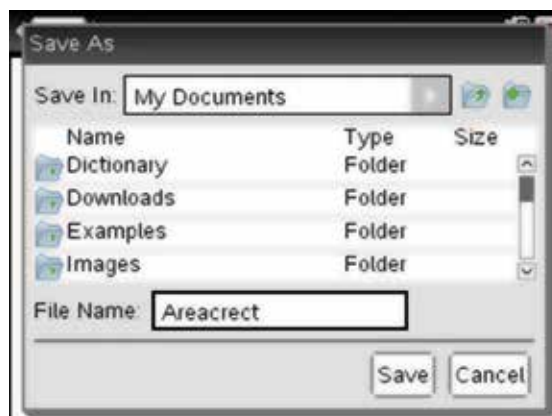
17. Repeat and label the other vertices of the rectangle *B*, *C* and *D* and the centre of the circle *O*.



18. Now we should save the document. To do this, press:

- ctrl 
- save 

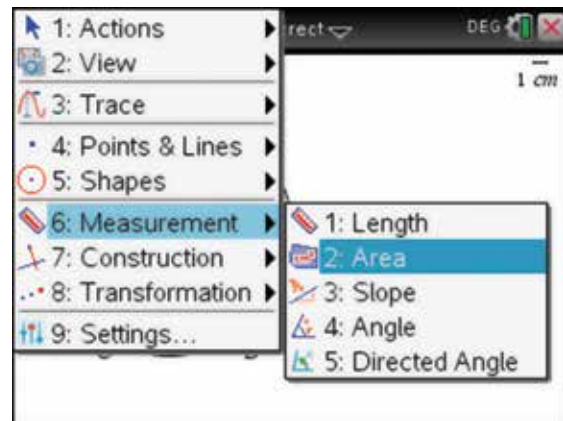
and give it the name 'arearect'. Tab  to Save and press the enter key .




19. Now we want to measure various quantities. To measure the area of the circle, press:

- MENU **menu**
- 6: Measurement **6**
- 2: Area **2**.

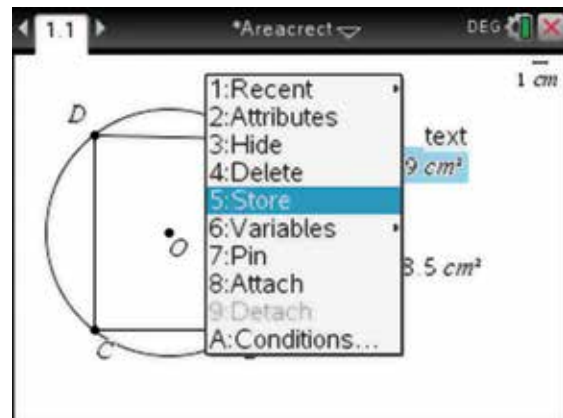
Click on the circle and its area appears. Press the enter key **enter**, then click on the polygon and its area appears; press the enter key **enter**, then press escape **esc**.



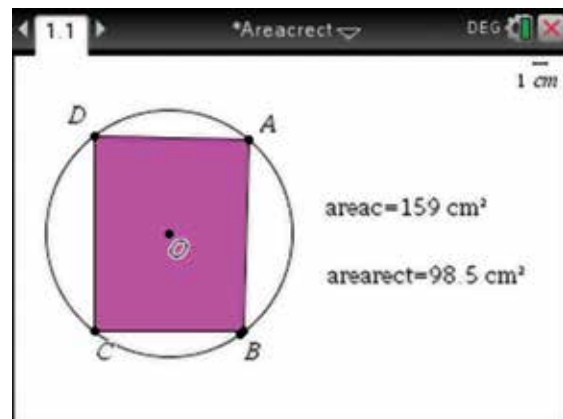
20. Move the labels to a more suitable position on the screen and move the pointer  over the label for the area of the circle, then press:

- ctrl **ctrl**
- MENU **menu**
- 5: Store **5**.

In the text box that appears, type in 'areac' as the variable to store the area of the circle.

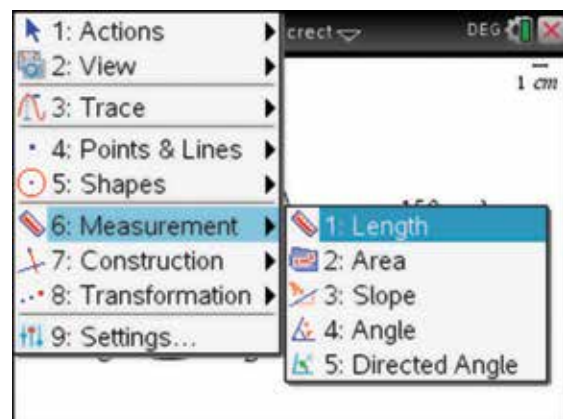






21. Repeat for the area of the rectangle. Use 'arearect' as the variable to store the area of the rectangle. The polygons can be coloured by selecting them, then pressing ctrl, menu, selecting Color and then selecting Fill Color.

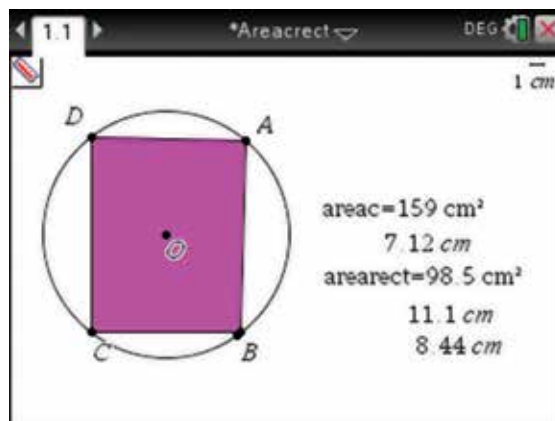


22. Now measure the lengths of the sides, AB CD and the radius OA of the circle. To do this, press:

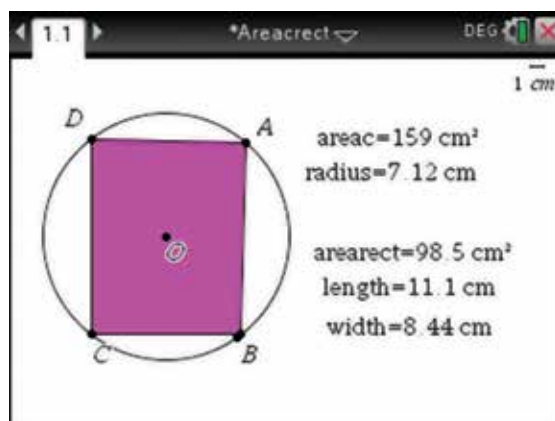
- MENU **menu**
- 6: Measurement **6**
- 1: Length **1**.



23. Move the pointer  over the point A , press enter , move over the point B , and press enter . The length of AB will appear. Repeat for points C to D to measure CD and points O to A to measure OA , then press escape  to stop measuring lengths. Move these labels to appropriate positions.



24. Repeat the process in step 21 to store these length labels with variable names as shown. Use 'length' and 'width' as the variable names to store the length and width of the rectangle, and 'radius' to store the radius of the circle.






CHAPTER 9

Trigonometry


This chapter deals with finding trigonometric ratios, converting between degrees and radians, solving trigonometric equations and drawing trigonometric functions.

9.1 How to set the mode to radians or degrees

1. Put the calculator in degree mode. To do this, press:


- HOME  on
- 5: Settings  5
- 2: Document Settings ...  2.



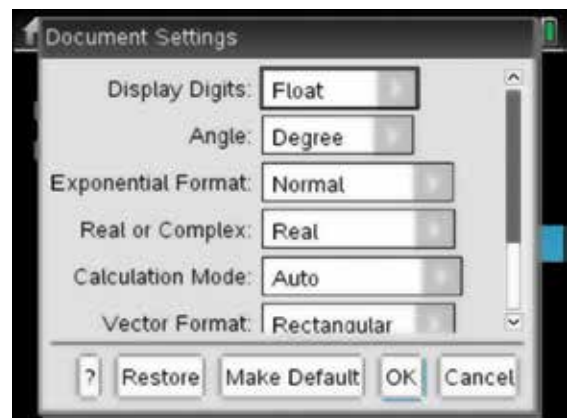
2. Use the tab key  to scroll down to Angle, then press:

- CLICK .

Select Degree.

Press the tab key  a number of times to select Make Default, then press:

- ENTER .



3. Now open a new document and add a Calculator page by pressing:

- **1: New Doc** **1**.

Select No (if you do not want to save the current document). Press:

- **ENTER** **enter**.

Open a new Calculator page by pressing:

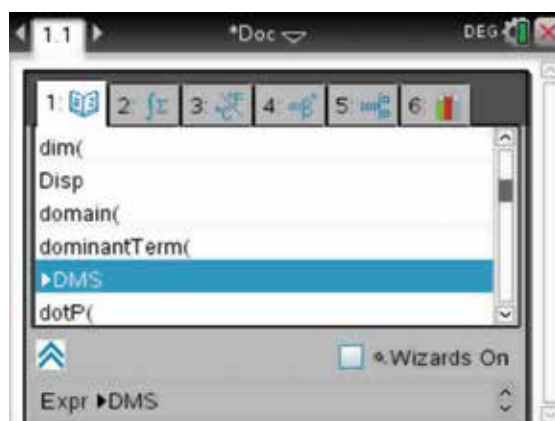
- **1: Add Calculator** **1**.



4. To convert an angle expressed in decimal degrees to degrees, minutes and seconds, complete the following steps. For example, express 25.34° in degrees, minutes and seconds. Type 25.34 onto the Calculator page, then press:

- **catalog**
- **1** **1**
- **D** **D**.

Scroll down and select **DMS**, then press the enter key **enter**.



5. When you return to the Calculator page, press the enter key **enter**.



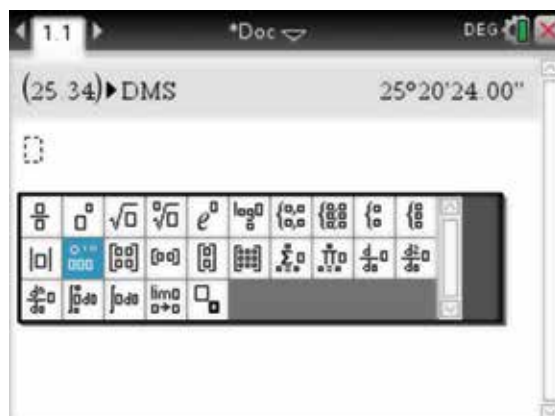
6. To convert an angle expressed in degrees, minutes and seconds into decimal degrees, complete the following steps. For example, convert $67^\circ 28'$ into degrees only.

Use the Maths Expression palette to enter the angle.

To do this, press:

-

Select the template option highlighted on the screen opposite. Then press the enter key **enter**.

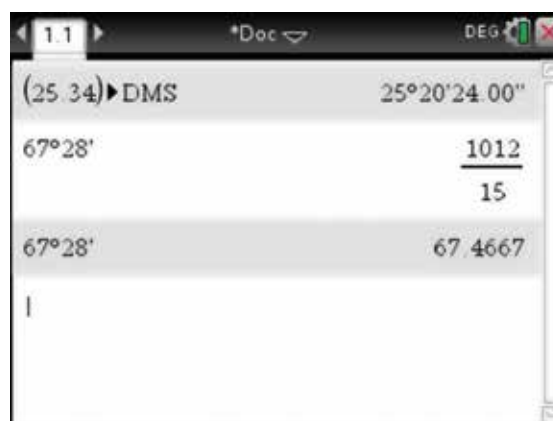



7. Type the angle, using the tab key **tab** to move between fields. Then press the enter key **enter**.



8. The answer is given as an improper fraction because the Calculation Mode is set to Auto. To get a decimal approximation, press:


- **ctrl** **ctrl**
- **ENTER** **enter**.

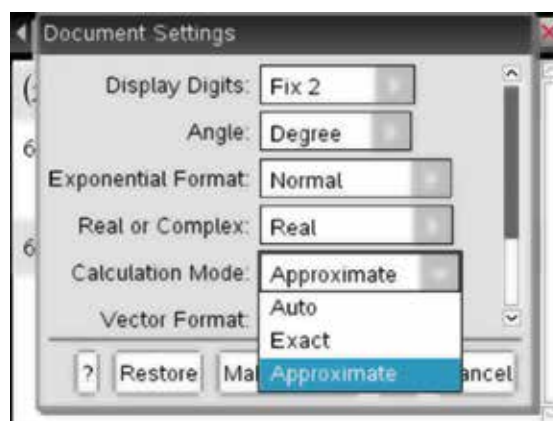


9. It may be more useful to change the Calculation Mode in the Document Settings to Approximate. To do this, move the pointer  over the Settings icon (at the top of the screen, next to the close box) and press:

- **CLICK** 
- **2: Document Settings** **2**.

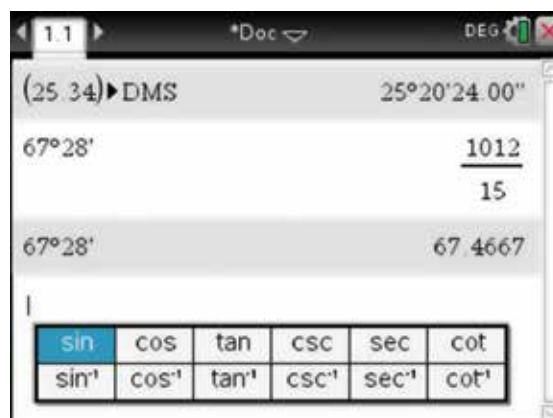
Use the tab key **tab** to select Calculation Mode. Press:

- **CLICK** . Select Approximate, then press the enter key **enter** twice.



10. To calculate trigonometric ratios such as those shown, press the trig key **trig** and select the appropriate trigonometric function from the palette.

- $\sin(84^{\circ})$
- $\cos(45^{\circ}16')$
- $\tan(156^{\circ}18'50'')$



11. Enter the information as shown on the screen opposite, using the Mathematical Expressions palette as required.


67°28'	1012 15
67°28'	67.4667
$\sin(84)$	0.994522
$\cos(45^\circ 16' 0'')$	0.703808
$\tan(156^\circ 18' 30'')$	-0.438796


12. To find the value of an angle to the nearest second, such as those shown, follow the instructions below.

(a) $\sin(\theta) = 0.6471$

(b) $\cos(\theta) = 0.3182$

(c) $\tan(\theta) = 1.25$

To access the inverse trigonometric functions, press the trig key  and select the appropriate function.

The outer brackets appear automatically when you press the enter key . Round answers approximately.

$\cos(45^\circ 16' 0'')$	0.703808
$\tan(156^\circ 18' 30'')$	-0.438796
$(\sin^{-1}(0.6471)) \blacktriangleright \text{DMS}$	40°19'23.9141"
$(\cos^{-1}(0.3182)) \blacktriangleright \text{DMS}$	71°26'45.2278"
$(\tan^{-1}(1.25)) \blacktriangleright \text{DMS}$	51°20'24.6903"

13. The calculator can also be used to solve problems such as the one below.

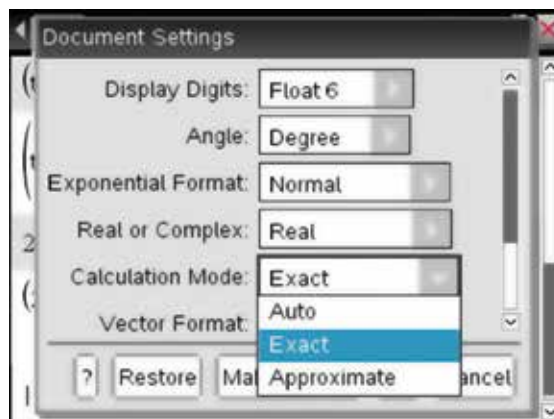
A woman walks 6 km south from her campsite and then turns and walks 2.5 km west. What is the true bearing from her campsite to the nearest minute?

The answer to the nearest minute is $202^\circ 37'$.

$(\tan^{-1}(1.25)) \blacktriangleright \text{DMS}$	51°20'24.6903"
$(\tan^{-1}(\frac{2.5}{6})) \blacktriangleright \text{DMS}$	22°37'11.5138"
$22^\circ 37' 11.5138129476'' + 180$	202.62
$(202.61986494804) \blacktriangleright \text{DMS}$	202°37'11.5138"

9.2 How to work with exact values

1. To work in exact values, return to the General Settings screen and select Exact from the Calculation Mode (see section 9.1).



2. Try answering the following.

- (a) $\sin(60^\circ)$
- (b) $\cos(45^\circ)$
- (c) $\tan(330^\circ)$

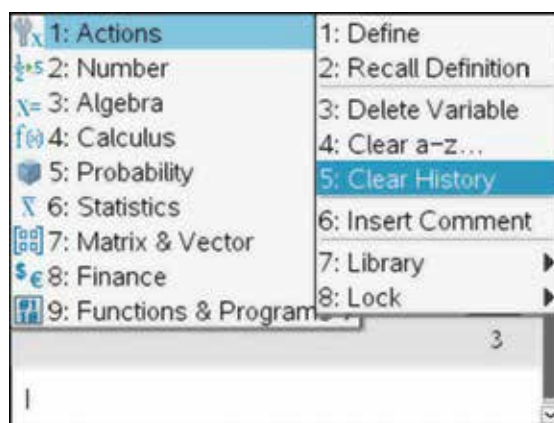
Note that we can always check what mode the Angle setting is in, by moving the cursor over the settings icon in the top right hand corner of the screen, next to the close box.



9.3 How to solve trigonometric equations

1. Before beginning, it is a good idea to clear the history. To do this, press:




- **MENU** (menu icon)
- **1: Actions** (1)
- **5: Clear History** (5).

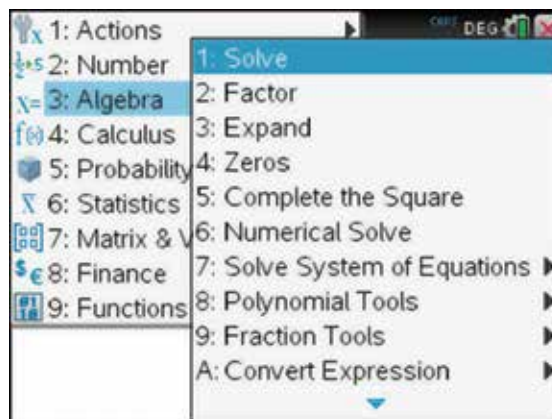


To solve the following equations for $x \in [-180, 180]$, complete the steps shown.

- (a) $\sin(x) = \frac{\sqrt{3}}{2}$
- (b) $\cos(x) = \frac{1}{2}$
- (c) $\tan(x) = -1$

2. To use the Solve application for part (a), press:

- **MENU** 
- **3: Algebra** 
- **1: Solve** .







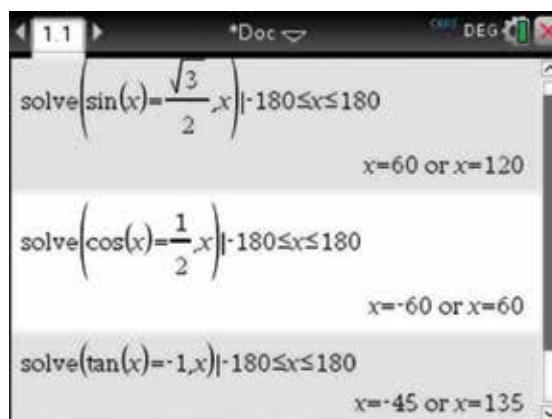
3. Complete the entry line as:


$$\text{solve}\left(\sin(x)=\frac{\sqrt{3}}{2},x\right)|-180\leq x\leq 180$$

Press the enter key .


Repeat this process for parts (b) and (c).

Note: The given key can be accessed from the palette, by pressing the ctrl key  and , or from the template, by pressing the ctrl key  and .




4. Change the Document Settings to Radian when you wish to work in this mode. To do this, move the pointer  over the Settings icon at the top of the screen next to the close box and press:

- **CLICK** 
- **2: Document Settings** .

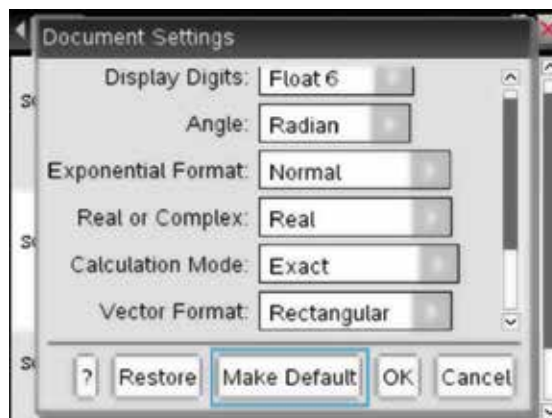
Use the tab key  to scroll down to Angle, press:

- **CLICK** .

Select Radian. Use the tab key  to select Make Default, then press:

- **ENTER**  twice.

You will be returned to the last screen, the Calculator page on which you were working.



5. For example, solve $\cos(2x) = \frac{1}{\sqrt{2}}$ for $x \in [-2\pi, 2\pi]$.

1.1 *Doc RAD

$\text{solve}(\tan(x)=-1,x)|-180 \leq x \leq 180$

$x=-45 \text{ or } x=135$

$\text{solve}(\cos(2 \cdot x)=\frac{1}{\sqrt{2}},x)|-2 \cdot \pi \leq x \leq 2 \cdot \pi$

$x=\frac{-15 \cdot \pi}{8} \text{ or } x=\frac{-9 \cdot \pi}{8} \text{ or } x=\frac{-7 \cdot \pi}{8} \text{ or } x=\frac{-\pi}{8} \text{ or } x=\frac{7 \cdot \pi}{8} \text{ or } x=\frac{9 \cdot \pi}{8} \text{ or } x=\frac{11 \cdot \pi}{8} \text{ or } x=\frac{15 \cdot \pi}{8}$

6. Sometimes general solutions are required. They appear when no domain is specified for the equation.

For example, solve $\sin(2x) = \frac{1}{\sqrt{2}}$.

For general solutions, the bold figures are elements of the set of integers and represent rotations around the unit circle. Each time you solve a general solution, the number next to n increases by 1. Note that exact radian values are given as the calculator is set to Exact mode with the Angle mode in Radians.

1.1 *Doc RAD

$x=\frac{-15 \cdot \pi}{8} \text{ or } x=\frac{-9 \cdot \pi}{8} \text{ or } x=\frac{-7 \cdot \pi}{8} \text{ or } x=\frac{-\pi}{8} \text{ or } x=\frac{7 \cdot \pi}{8} \text{ or } x=\frac{9 \cdot \pi}{8} \text{ or } x=\frac{11 \cdot \pi}{8} \text{ or } x=\frac{15 \cdot \pi}{8}$

$\text{solve}(\sin(2 \cdot x)=\frac{1}{\sqrt{2}},x)$

$x=\frac{(8 \cdot \mathbf{n}+1) \cdot \pi}{8} \text{ or } x=\frac{(8 \cdot \mathbf{n}+3) \cdot \pi}{8}$

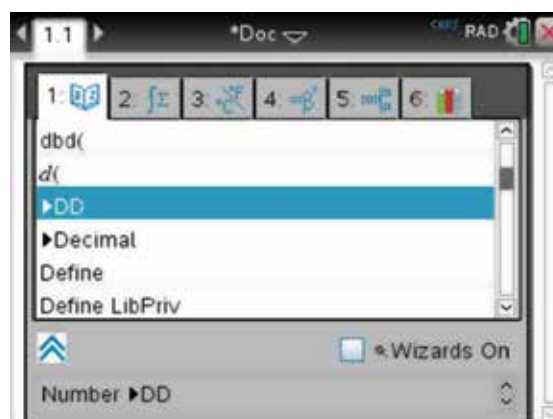
9.4 How to convert between radians and degrees

1. Begin by clearing the history.

To convert $\frac{5\pi}{4}$ into degrees, type the angle using the fraction template, then press:

- catalog
- 1
- D

Select **DD** and press the enter key twice.



2. There was no need to type the radian symbol 'r' as the General Settings are in radians. If required, this symbol can be found in the symbol palette or from the π key.



3. To convert 60° into radians, type 60 on the Calculator page. To get the degree symbol, press:

- **ctrl** π
- **catalog** π .

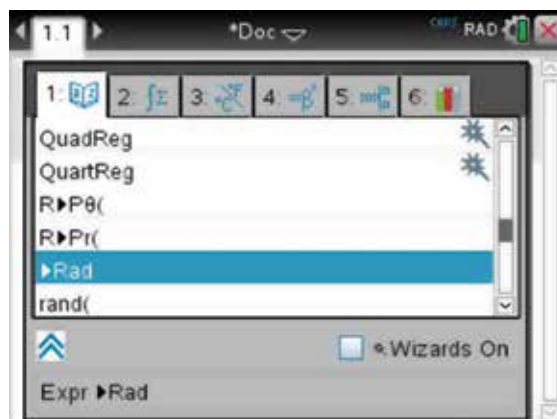
Select the degree symbol as highlighted in the screen opposite. Then press:

- **ENTER** π .



4. Now press:

- **catalog** π
- **R** π .



5. Select \blacktriangleright Rad and press the enter key π twice.

Note: The degree symbol had to be used because the current Document Settings are in radians.



9.5 How to graph trigonometric functions

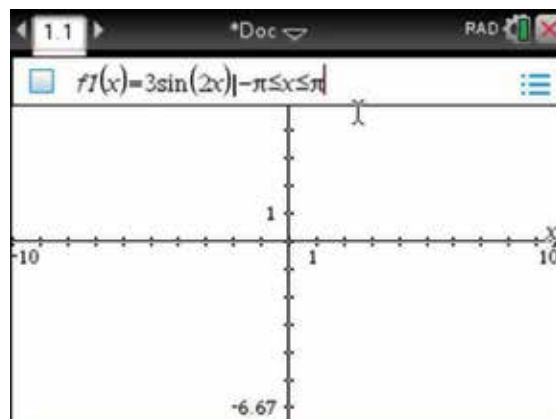
1. To graph the function
 $f: [-\pi, \pi] \rightarrow \mathbb{R}$ where $f(x) = 3\sin(2x)$, complete the following steps.

Open a new document and add a Graphs page (see section 7.1).

Complete the function entry line as:

$$f1(x) = 3\sin(2x) \mid -\pi \leq x \leq \pi$$

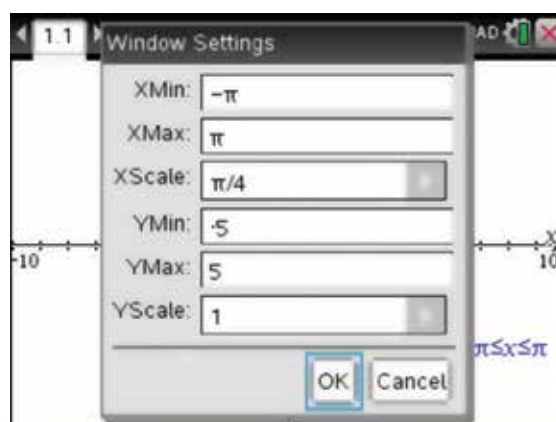
Press the enter key **enter**.



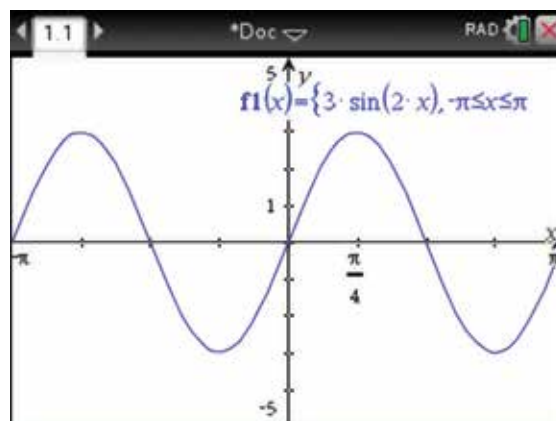
2. To change the viewing window for this graph, press:

- **MENU** **menu**
- **4: Window / Zoom** **4**
- **1: Window Settings** **1**.




Change to the settings as shown and press the enter key **enter**. These values were chosen as the amplitude is 3.



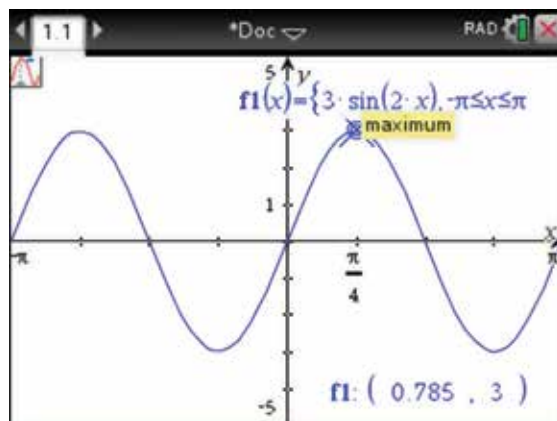
3. The graph appears.






4. To find important features of the graph, press:

- **MENU** 
- **5: Trace** 
- **1: Graph Trace** .

As you trace the graph, the coordinates of the maximum point will be displayed when the word 'maximum' appears in a box as shown. Note that the coordinates are shown as decimal values; however, we can check on a calculator page for these exact values. You can also use this method to find the coordinates of maximum points and the x -intercepts.



5. Open a Calculator page and press:

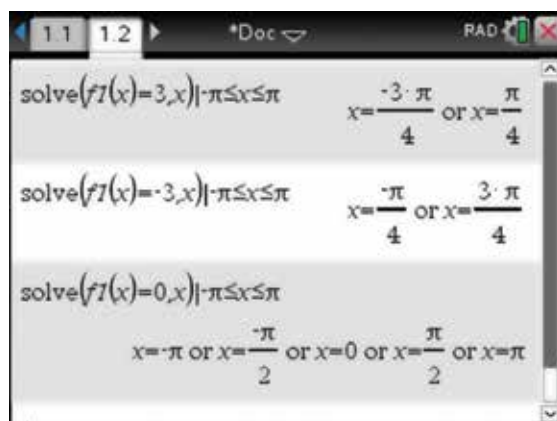
- **MENU** 
- **3: Algebra** 
- **1: Solve** .

Complete the function entry line as:

$$\text{solve}(f1(x) = 3, x) | -\pi \leq x \leq \pi$$

Press the enter key .

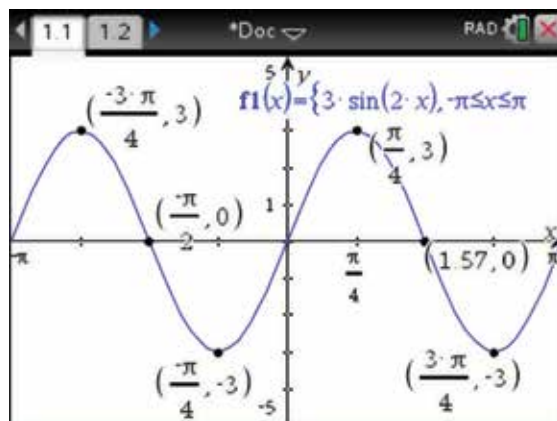
Repeat for the minimum and x -intercepts, as shown on the Calculator page. This is a very useful method when exact values are required.



6. To return to the graph on page 1.1, press:

- **ctrl** 
- **left arrow** .

As shown above, edit the coordinates of the maximum and minimum points and the two x -intercepts to show on the graph the exact values of these critical points.






The Graph and Calculator pages of the calculator are useful for solving problems such as the one below.

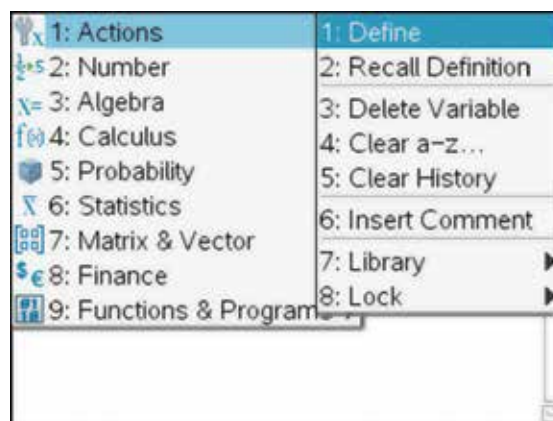
The height (cm) that a clock's pendulum swings above its base can be approximated by the function

$$H(t) = 14 + 6\cos\left(\frac{3\pi t}{2}\right) \text{ for } t \geq 0 \text{ at any time } t \text{ seconds after being released.}$$

- Find the exact maximum and minimum heights that the pendulum reaches, and the first times these occur.
- Sketch the graph of the function over the first 8 seconds.
- On the same set of axes, sketch the median position.


7. Open a new document and add a Calculator page (see section 2.1). Then press:

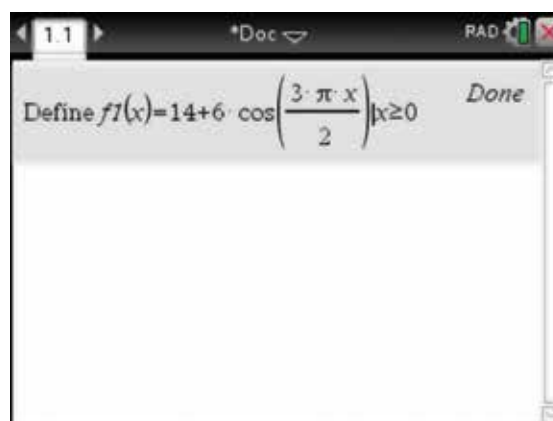
- **MENU** 
- **1: Actions** 
- **1: Define** 



8. Complete the entry line as:


Define $f1(x) = 14 + 6 \cos\left(\frac{3\pi x}{2}\right) | x \geq 0$

Then press the enter key .





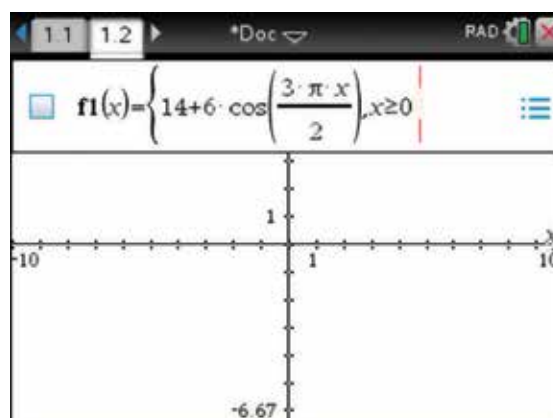
9. To draw a graph of H , now defined as $f1(x)$, you will need to add a new Graph page to the document. Press:

- **HOME** 

Select the Graphs icon, then press the enter key .



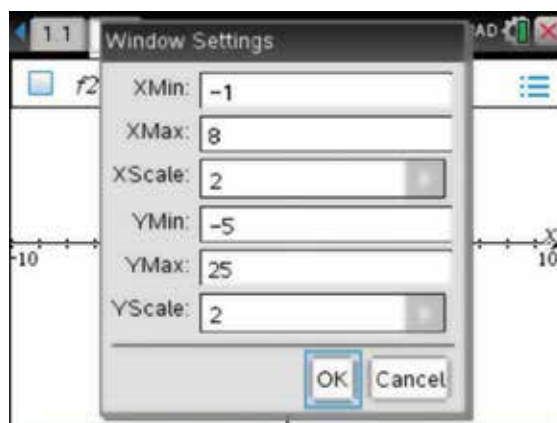
10. Use the Touchpad and the up arrow  to display $f1(x)$, which has already been defined in this document. Press the enter key .



11. A more appropriate window needs to be set. To do this, press:

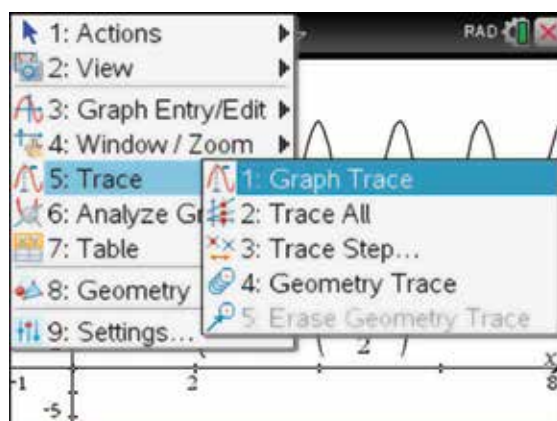
- **MENU** **(menu)**
- **4: Window / Zoom** **(4)**
- **1: Window Settings** **(1)**.

Select the values as shown, tab **(tab)** to OK and press the enter key **(enter)**. These settings shown were chosen because the minimum height equals $14 - 6 = 8$ and the maximum height equals $14 + 6 = 20$, and we also see the axes along with the scale.

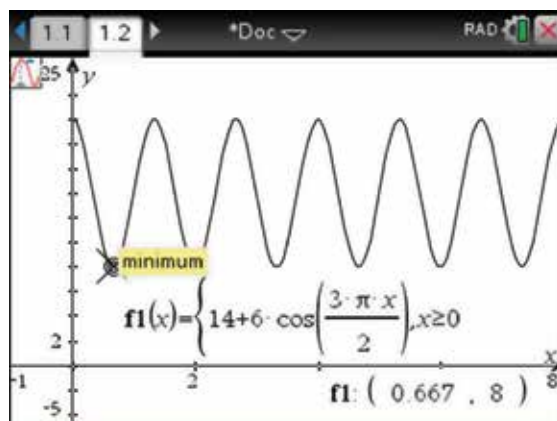


12. To find the first maximum and minimum values, you will need to trace along the graph. To do this, press:

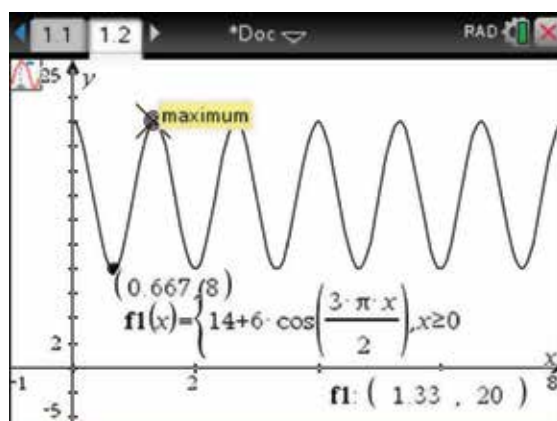
- **MENU** **(menu)**
- **5: Trace** **(5)**
- **1: Graph Trace** **(1)**.



13. Use the Touchpad to move along the graph. When the first minimum is displayed in a box press the enter key **(enter)**.



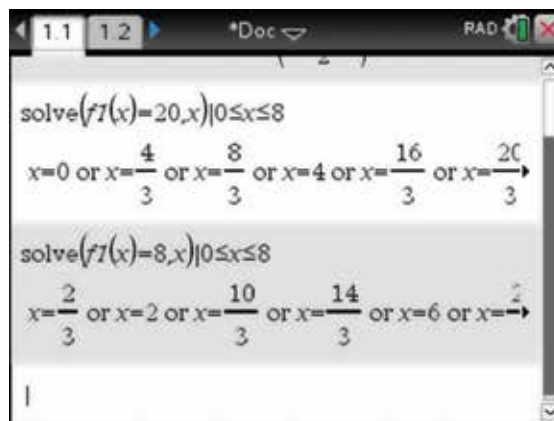
14. When the first maximum is displayed in a box, press the enter key **(enter)**, then press escape **(esc)**.



15. To find the exact times of these maximum and minimum values, return to page 1 of the document. To do this, press:

- **ctrl** **(ctrl)**
- **left arrow** **◀**.

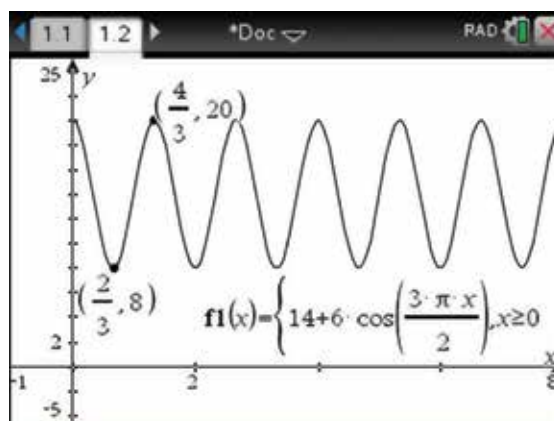
Complete the entry lines as shown.



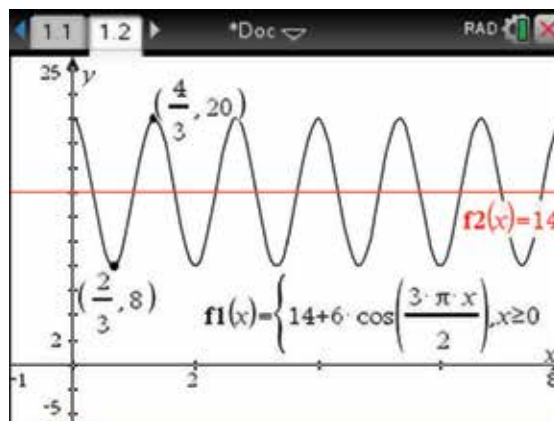
16. Return to the graph page. To do this, press:

- **ctrl** **(ctrl)**
- **right arrow** **▶**.

Edit the coordinates on the graph to display the first maximum and minimum values with their exact values as shown.



17. The median position is the centre of the curve, which falls halfway between the maximum and the minimum. This is at $H = 14$ cm. To draw this median line, complete the function entry line as: $f2(x) = 14$. Press the enter key **(enter)**. Note that the period of the function is $\frac{2\pi}{\frac{3\pi}{2}} = \frac{4}{3}$, so the maximum and minimum values repeat every $\frac{4}{3}$ seconds.



CHAPTER 10

Financial mathematics

The general principles of growth and decay can be modelled by equations or graphs. Many examples of growth and decay can be found in the world of business and finance.

10.1 How to calculate simple interest

1. Start with a new document. To do this, press:

- HOME  on
- 1: New Doc .

Select No.

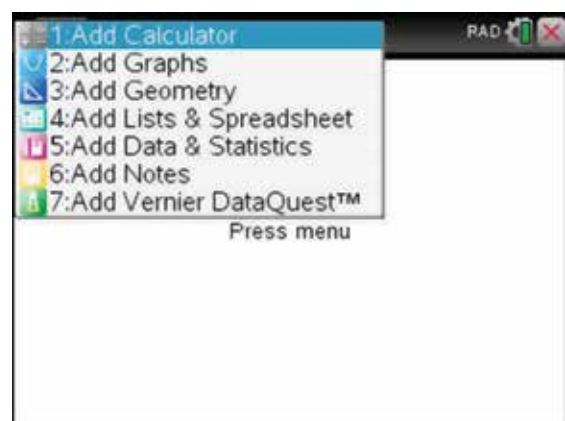
Press:

- ENTER .



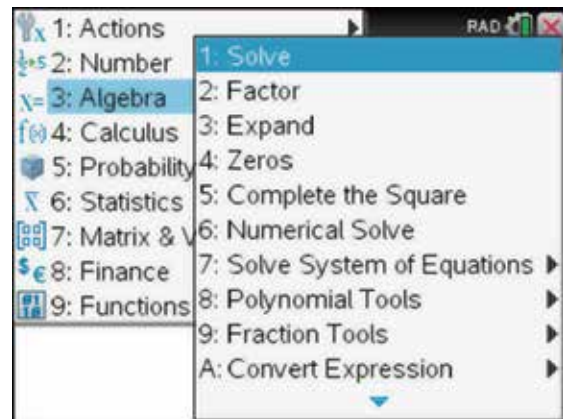
2. To calculate the simple interest if \$8320 is invested for 3 years at 6.45% p.a., select:

- 1: Add Calculator .



3. To calculate the simple interest earned, press:

- MENU **menu**
- 3: Algebra **3**
- 1: Solve **1**.

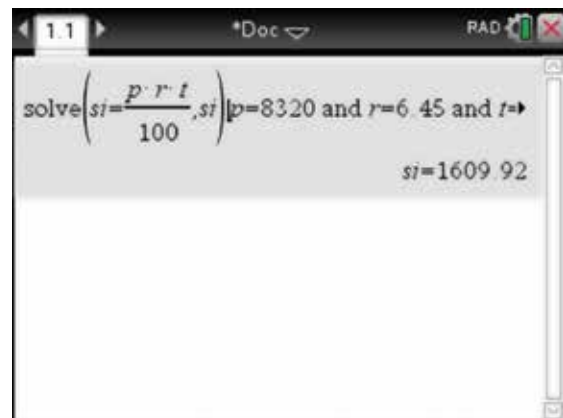


4. Complete the entry line as:

$$\text{solve}\left(\text{si} = \frac{p \times r \times t}{100}, \text{si}\right) | p = 8320 \text{ and } r = 6.45 \text{ and } t = 3$$

Press the enter key **enter**.

The interest earned is \$1609.92.



5. To calculate, to the nearest month, how long it would take to earn \$1950 simple interest if \$16325 is invested at 9.75% p.a., press:

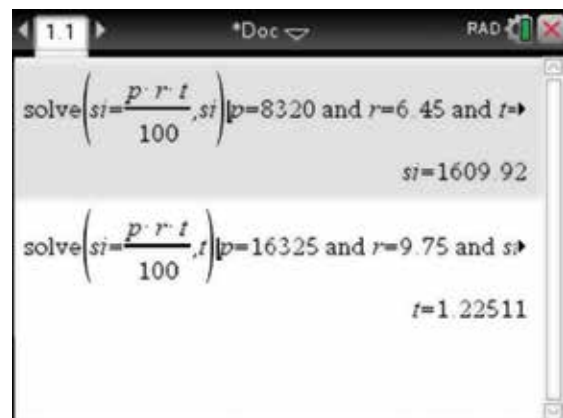
- MENU **menu**
- 3: Algebra **3**
- 1: Solve **1**.

Complete the entry line as:

$$\text{solve}\left(\text{si} = \frac{p \times r \times t}{100}, t\right) | p = 16325 \text{ and } r = 9.75 \text{ and } \text{si} = 1950$$

Press the enter key **enter**. The time in years is 1.225.

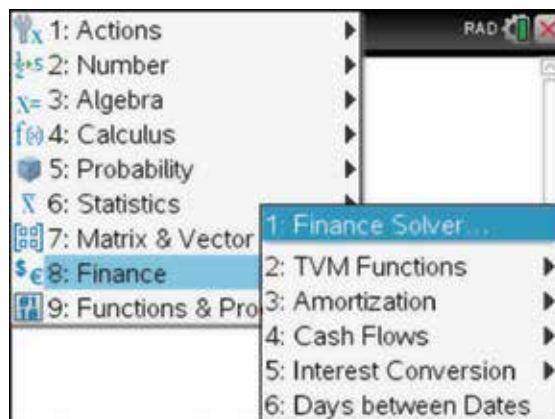
The time to the nearest month is 1 year and 3 months.



10.2 How to calculate compound interest

1. To calculate compound interest, the compound interest formula $A = P\left(1 + \frac{r}{100}\right)^n$ can be used in a manner similar to the simple interest formula, as shown in section 10.1. Alternatively, the Finance Solver table can be used. To find the amount of interest earned if \$3200 is invested for 5 years at 6% p.a. compounded quarterly using the Finance Solver table, begin with a new document and add a calculator page. Press:

- **MENU** **menu**
- **8: Finance** **8**
- **1: Finance Solver** **1**.



2. Complete the fields as shown.

N is the number of payments (20).

I(%) is the interest rate p.a. (6).

PV is the amount to be invested (3200).

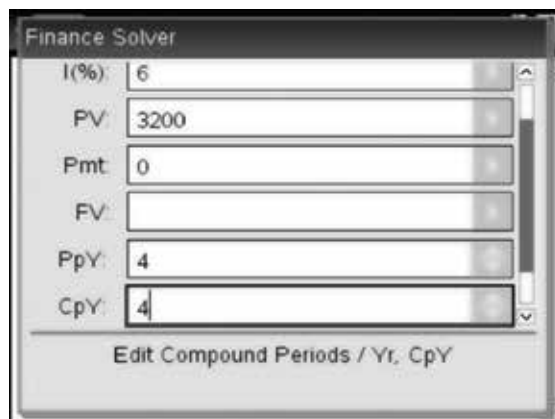
Pmt is the regular payment (\$0).

FV is the future value of the investment (to be determined).

PpY is the number of payments per year (4).

CpY is the number of compounding period per year (4).

Press the tab key **tab** to move between fields.

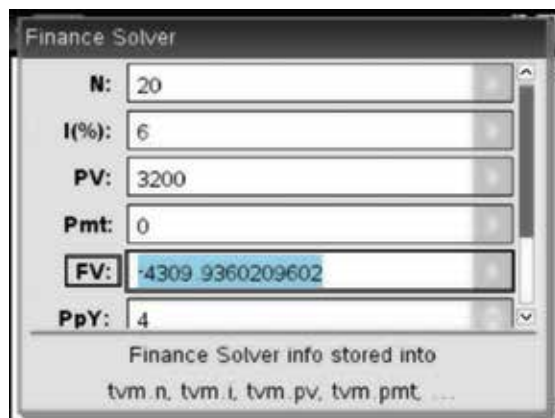


3. Press the tab key **tab** to return to the FV field and press the enter key **enter**.

The investment is worth \$4309.94 after 5 years.




The interest earned is $\$4309.94 - \$3200 = \$1109.94$.

Note: The future value (FV) is given as a negative value.



10.3 How to calculate the interest rate

1. To calculate the interest rate per annum (to 2 decimal places) that would enable an investment of \$3000 to grow to \$4000 over 2 years if interest is compounded quarterly, press:

- **MENU** 
- **8: Finance** 
- **1: Finance Solver** .

Complete the fields as shown.

N is the number of payments (8).

I(%) is the interest rate p.a. (to be determined).


PV is the amount to be invested (3000).


Pmt is the regular payment (\$0).

FV is the future value of the investment (−4000).

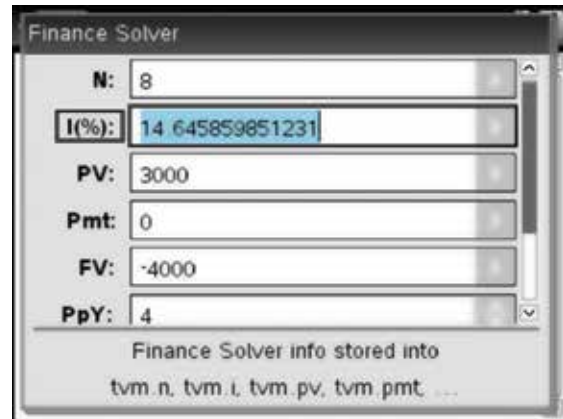
PpY is the number of payments per year (4).

CpY is the number of compounding period per year (4).

Press the tab key  to move between fields.

Press the tab key  to return to the interest I(%)

field. Press the enter key . The interest rate would be 14.65%.



The screenshot shows the 'Finance Solver' window with the following values entered:

Field	Value
N:	8
I(%):	14.645859851231
PV:	3000
Pmt:	0
FV:	-4000
PpY:	4

At the bottom, it says: 'Finance Solver info stored into tvm.n, tvm.i, tvm.pv, tvm.pmt, ...'

10.4 How to calculate the monthly repayment on a reducing balance loan

1. To find the monthly repayment on a reducing balance loan of \$94 000 to be repaid over 20 years at an interest rate of 7.5% adjusted monthly, press:

- **MENU** **menu**
- **8: Finance** **8**
- **1: Finance Solver** **1**.

Complete the fields as shown.

N is the number of payments ($20 \times 12 = 240$).

I(%) is the interest rate p.a. (7.5).

PV is the amount to be borrowed (94000).

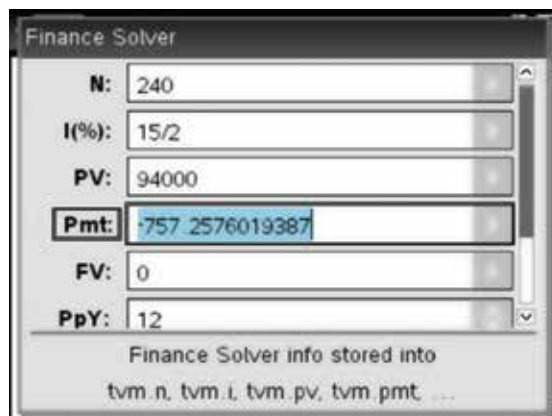
Pmt is the regular payment (to be determined).

FV is the future value of the investment (0).

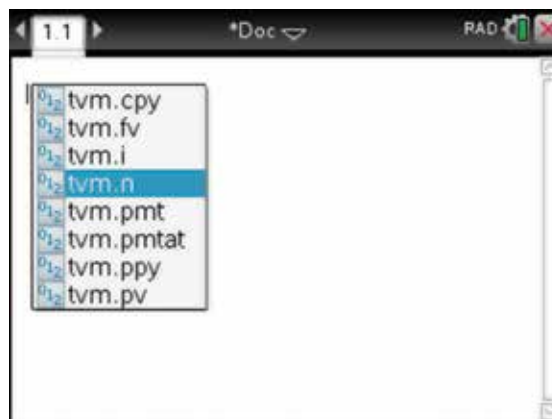
PpY is the number of payments per year (12).

CpY is the number of compounding period per year (12).

Press the tab key **tab** to move between fields. Press the tab key **tab** to return to the Pmt field. Press the enter key **enter**. The monthly repayment is \$757.26.







2. Press the escape key **esc** to return to the Calculator page. Note that the Finance Solver has automatically stored these variables for future use. To use any of these values in further calculations, press the var key **var** and select the appropriate variable. Doing this ensures that exact values are used in subsequent calculations instead of rounded values and will also ensure correct typing of the variables.




10.5 How to calculate the balance of the principal after any instalment

- Continuing with the example in section 10.4, now calculate the balance after one payment. To do this, press:

- **MENU** 
- **8: Finance** 
- **3: Amortization** 
- **2: Balance** .






Complete the entry line as:

`bal(1,tvm.n,tvm.i,tvm.pv,tvm.pmt,tvm.fv,
tvm.cpy,tvm.ppy)`

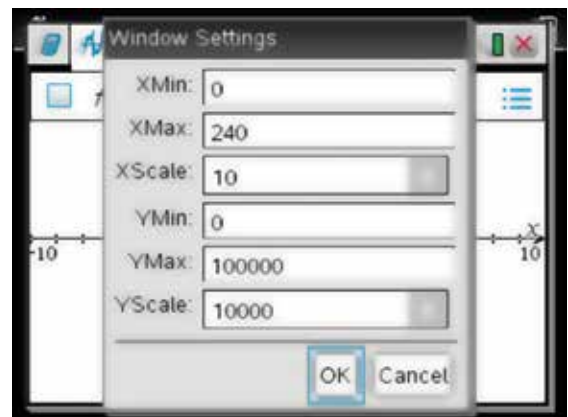
Press the var key  and use the Touchpad to select the appropriate variable in the correct order as shown. This shows that after one payment the balance of the loan is \$93 830.24.




- To show the progress of the loan on a graph, in a Scratchpad, first set the window settings. Press:

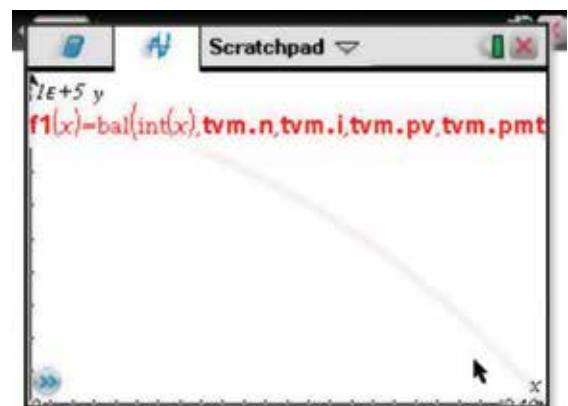
- **HOME** 
- **B: Graph** 
- **MENU** 
- **4: Window / Zoom** 
- **1: Window Settings** .

Enter the values as shown and select OK.



- To define the graph, press tab  and complete as shown, typing on the one line.
 $f1(x) = \text{bal}(\text{int}(x),$
`tvm.n,tvm.i,tvm.pv,tvm.pmt,tvm.fv,tvm.cpy,tvm.ppy)`

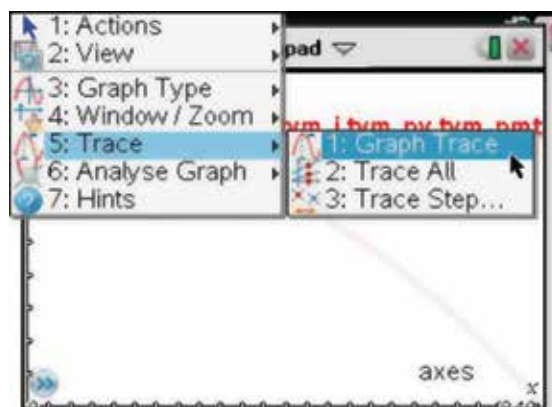
Note: `int(x)` has been used because the days and months are limited to integer values.



4. The graph should be displayed.

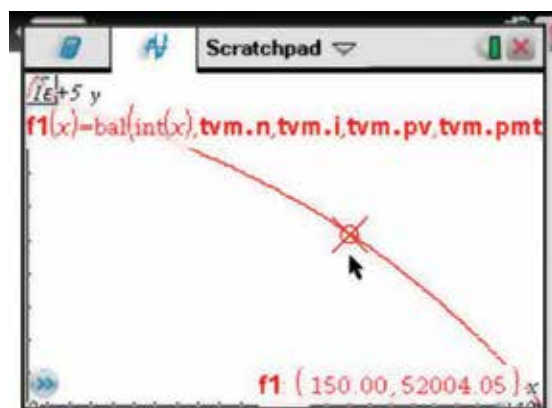
To trace the balance of the loan, press:

- MENU **menu**
- 5: Trace **5**
- 1: Graph Trace **1**.



5. Use the Touchpad to trace along the curve to see the balance over time.

Note: The values shown have been set to Fix 2, from the Display Digits from the Graphs and Geometry Settings; this will also apply to the Scratchpad.



10.6 How to calculate the number of days between dates

1. To find the number of days between:

(a) 21 April and 12 October

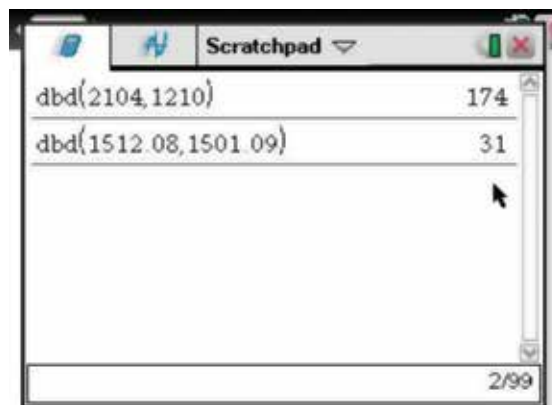
(b) 15 December 2008 and 15 January 2009

open a new Scratchpad and, on a Calculator page, complete the entry lines as shown.

dbd(2104,1210)

dbd(1512.08,1501.09)

Press the enter key **enter** after each line.



CHAPTER 11

Sequences and series

Recognition of the patterns formed by arithmetic and geometric sequences is important in analysing situations that occur normally in the real world.




11.1 How to find the n th term of an arithmetic sequence

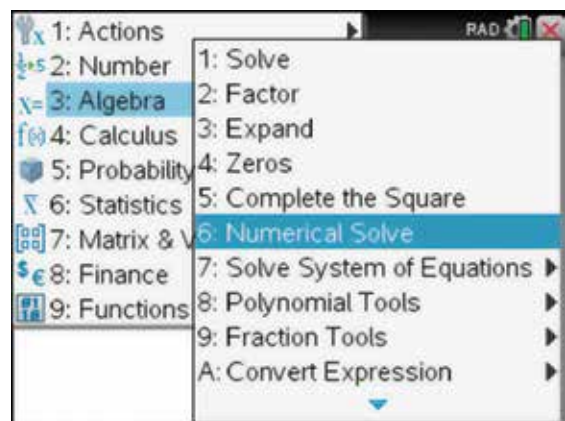
1. The calculator can be used very efficiently to answer questions such as those given below by using the instructions that follow.

Find the 20th term of the following arithmetic sequence:

5, 40, 75, 110, 145, ...

Begin with a new document and add a Calculator page (see section 2.1), then press:

- **MENU** 
- **3: Algebra** 
- **6: Numerical Solve** .



2. Complete the entry line as:

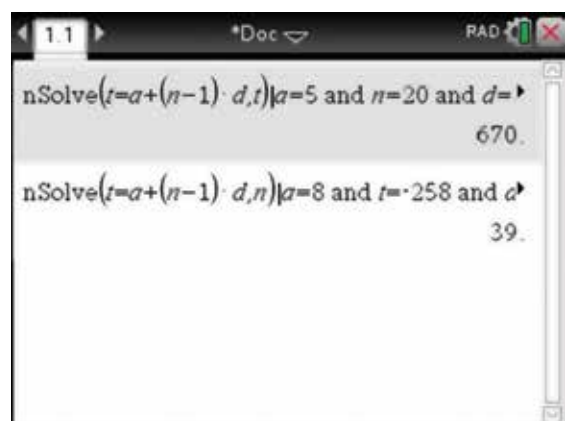
$\text{nSolve}(t = a + (n - 1) \times d, t) | a = 5 \text{ and } n = 20 \text{ and } d = 35$
Press:

- **ENTER** .

The numbers 8, 1 and -6 form the first three terms of arithmetic sequence. Which term is equal to -258 ?

Complete the entry line as:

$\text{nSolve}(t = a + (n - 1) \times d, n) | a = 8 \text{ and } t = -258 \text{ and } d = -7$



11.2 How to find the sum of an arithmetic sequence

1. If the third term of an arithmetic sequence is 4 and the 8th term is -11, what is the sum of the first 30 terms of the sequence?

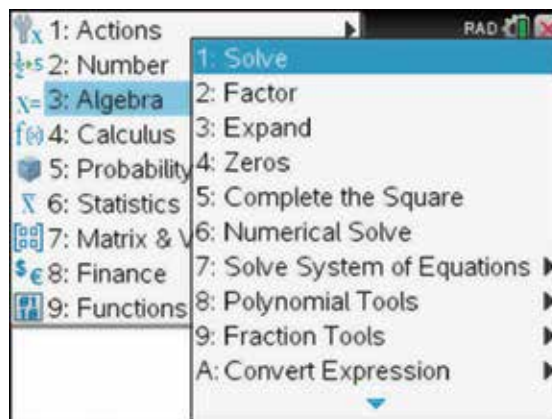
For an arithmetic sequence, $t = a + (n-1)d$:

(1) $a + 2d = 4$

(2) $a + 7d = -11$

Clear the history on the Calculator page, then press:

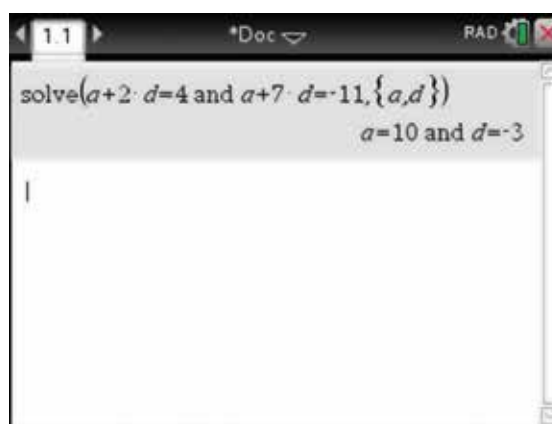
- **MENU** **menu**
- **3: Algebra** **3**
- **1: Solve** **1**.



2. Complete the entry line as:

$\text{solve}(a + 2d = 4 \text{ and } a + 7d = -11, \{a, d\})$

Then press the enter key **enter**.



3. To calculate the sum of the first 30 terms, press:

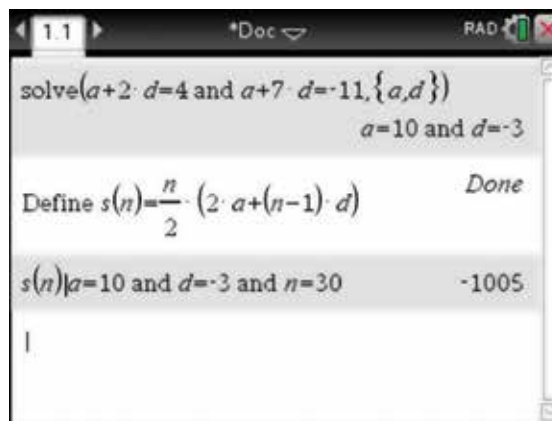
- **MENU** **menu**
- **1: Actions** **1**
- **1: Define** **1**.

Complete the entry line as:

Define $s(n) = \frac{n}{2} \times (2a + (n-1)d)$

Then press the enter key **enter**. Complete the entry line as: $s(n) | a = 10 \text{ and } d = -3 \text{ and } n = 30$

Then press the enter key **enter**.



11.3 How to find the n th term of a geometric sequence

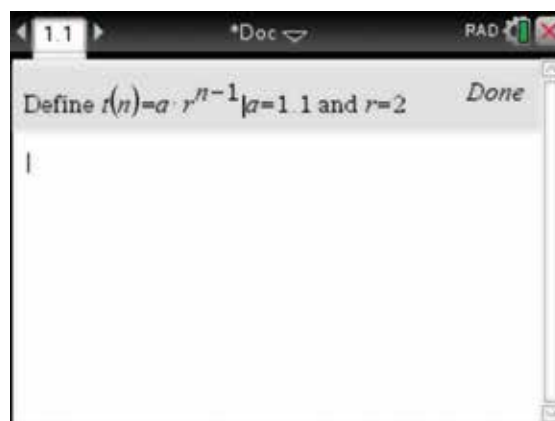
1. To find the 20th term of the geometric sequence 1.1, 2.2, 3.3, 4.4, 8.8, 17.6, ..., press:

- MENU **menu**
- 1: Actions **1**
- 1: Define **1**.

Complete the entry line as:

Define $t(n) = a \times r^{n-1} \mid a = 1.1 \text{ and } r = 2$

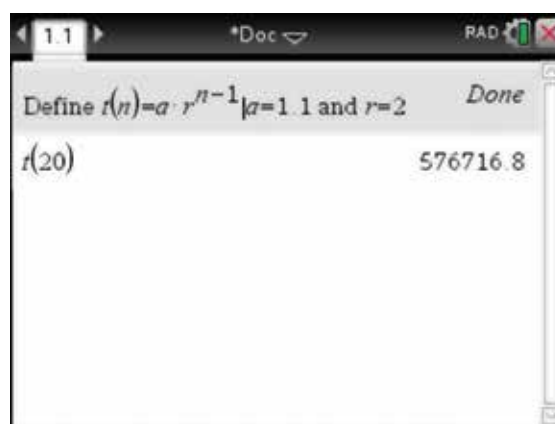
Then press the enter key **enter**.



2. To find the 20th term, complete the entry line as:

$t(20)$

Then press the enter key **enter**.



11.4 How to generate a list of terms from a difference equation

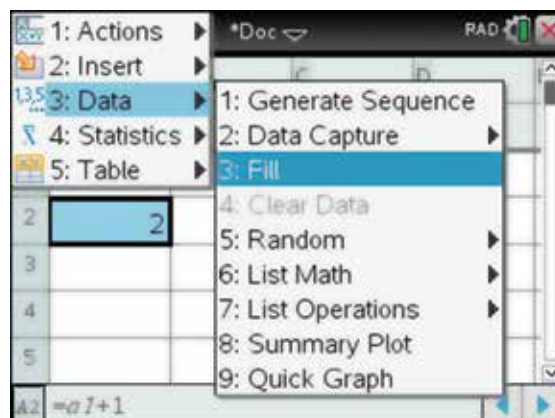
1. A sequence is defined by the first order difference equation: $t_{n+1} = 3t_n + 4$, $t_1 = 3$.

Open a new document and add a Lists & Spreadsheet page (see section 3.1). Label columns A and B as n and t_n respectively. Enter 1 into cell A1 and -3 into cell B1.

Complete cell A2 as: $=A1+1$

Press the up arrow on the Touchpad \blacktriangle to return into the cell A2. To fill down, press:

- MENU **menu**
- 3: Data **3**
- 3: Fill **3**.



- Press the down arrow on the Touchpad ▼ to highlight all cells to $n = 10$, and then press the enter key **enter**.

	A	n	B	tn	C	D
6		6				
7		7				
8		8				
9		9				
10		10				

- Complete cell B2 as:

$$= 3 \times b1 + 4$$

To fill down, press:

- MENU** **menu**
- 3: Data** **3**
- 3: Fill** **3**.

Use the Touchpad to highlight all cells to $n = 10$, and then press the enter key **enter**.

	A	n	B	tn	C	D
7		7	-731			
8		8	-2189			
9		9	-6563			
10		10	-19685			

11.5 How to generate a list of terms from a difference equation using an alternative method

- The population of a small town increases by 5% each year. If the initial population is 2500, calculate the population after 16 years.

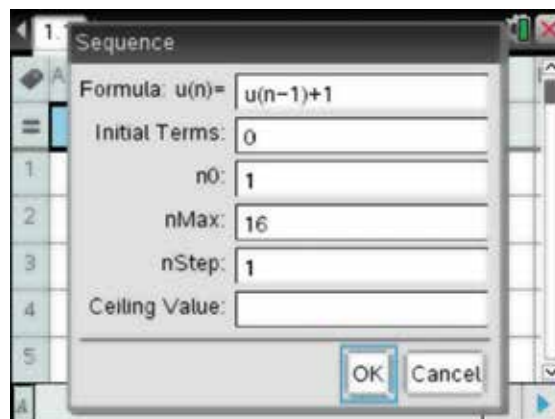
Open a new document and add Lists & Spreadsheet page (see section 3.1). Label column A as term. Move the cursor to the grey header cell of column A. Press:

- MENU** **menu**
- 3: Data** **3**
- 1: Generate Sequence** **1**.

	1: Actions	2: Insert	3: Data	4: Statistics	5: Table
2					
3					
4					
5					

- Complete the fields as shown, and then select OK.

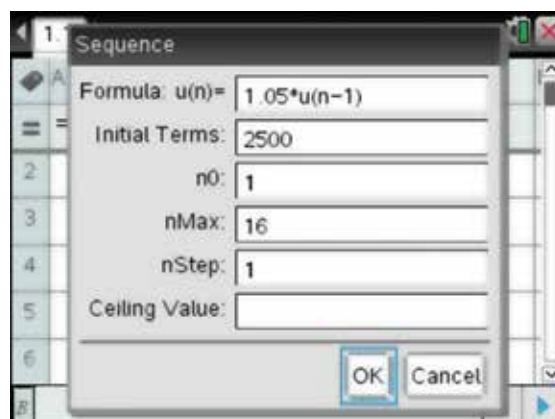
Note: $u(n)$ is used in place of t_n and the initial term is n_0 .



- Label column B as termvalue. Move the cursor to the grey header cell of column B. Press:

- MENU** **menu**
- 3: Data** **3**
- 1: Generate Sequence** **1**.

Complete the table as shown, and then select OK.




- Scroll down to find the population after 16 years.

	term	termv...	C	D
=	=seqgen(t	=seqgen('		
14	13.	4714.12		
15	14.	4949.83		
16	15.	5197.32		
17				
18				
B16	=5197.3204485283			

11.6 How to find the sum of a geometric sequence

1. The sum of a geometric sequence can be found by using the formula. It can also be found by finding the sum of column of values on a Lists & Spreadsheet page. Using the previous example, to find the sum of column B, termvalue, add a new Calculator page and complete the entry line as:

`sum(termvalue)`

Then press the enter key .



CHAPTER 12

Probability




Probability is an area of mathematics that has numerous practical applications. This chapter deals with a number of its key concepts.

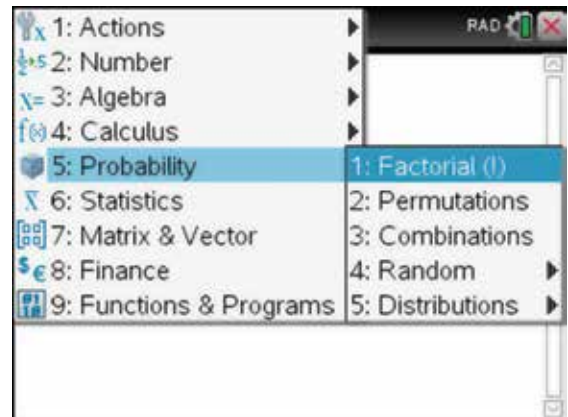
12.1 How to perform calculations using factorials, permutations and combinations

1. Open a new document and add a Calculator page (see section 2.1). Complete the steps outlined to calculate the following factorials.

- (a) $5!$
- (b) $30!$

Type 5 onto the calculator screen, then press:


- **MENU** 
- **5: Probability** 
- **1: Factorial** .



2. For part (b), this time we will get the factorial command from the symbol palette. To get this, press:

- **ctrl** 
- **catalog** .

Use the Touchpad to scroll to find the symbol highlighted on the screen opposite.

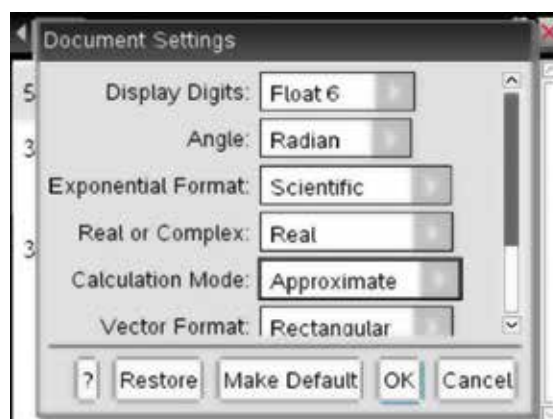
Press the enter key  twice.



3. $30! = 265252859812191058636308480000000$



4. Alternatively, the Exponential Format can be set to Scientific and the Calculation Mode set to Approximate in the Document Settings.



5. Then the answer may be given as

$30! = 2.65253\text{E}32$, which means 2.65253×10^{32} .

Answers may be given in this form when the numbers are particularly large.



6. To do calculations using permutations (nPr) and combinations (nCr), complete the instructions below.

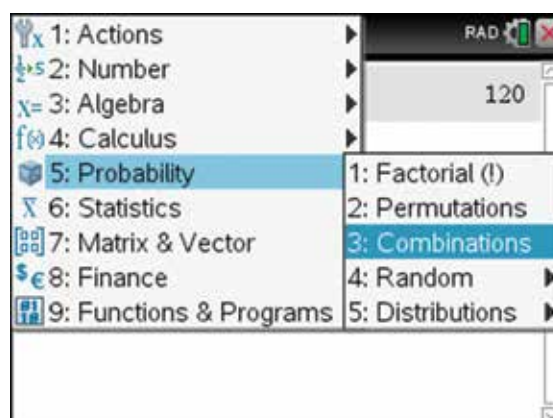
For example, calculate:

(a) 6C_3

(b) ${}^{10}P_2$.

Press:

- **MENU** (menu)
- **5: Probability** (5)
- **3: Combinations** (3).

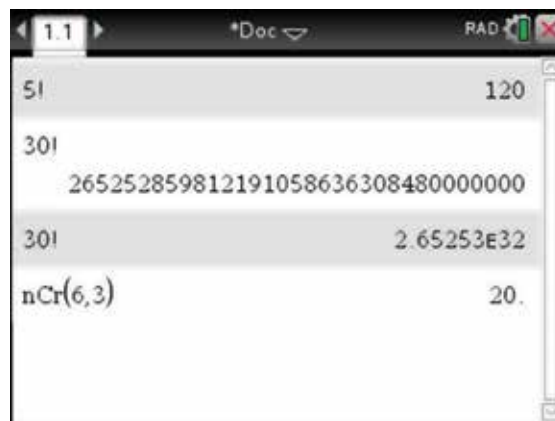


7. Complete the entry line as:

$nCr(6,3)$


Press the enter key .

Note that the Exponential Format has been changed back to Normal and the Calculation Mode set to Auto in the Document Settings.




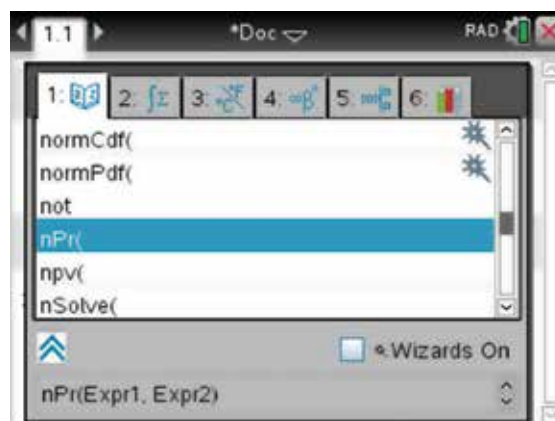
8. For part (b), this time locate nPr in the catalog. Press:

• **catalog** 

• **1:** 

• **N** .

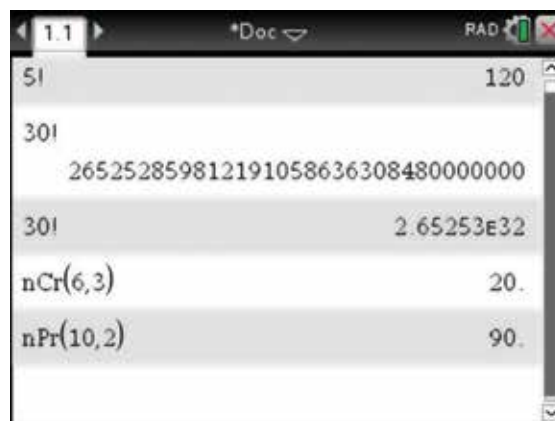
Using the Touchpad, locate nPr (, then press the enter key .



9. Complete the entry line as:

$nPr(10,2)$

You can also use the Touchpad to scroll back and highlight the previous operation. Copy it into your entry line so you can edit the previous entry and do another calculation quickly. We also observe that there are many ways to achieve these results.

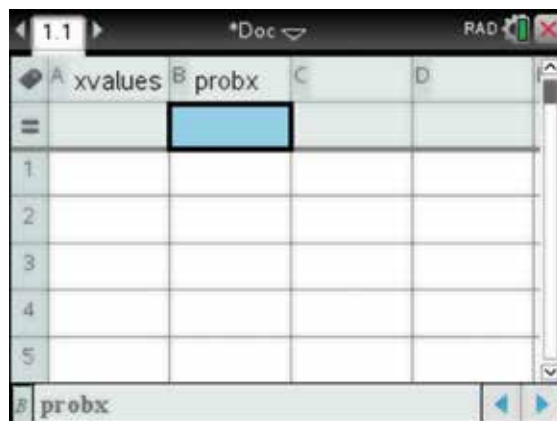


12.2 How to graph the binomial probability distribution and how to calculate binomial probabilities

Lauren's netball team has a known probability of winning 4 out of every 5 games they play. They play 12 games in a season. Construct the probability distribution for the number of games won in the season, graph the distribution and then find:

- the probability that they won all 12 games
- the probability that they won no more than 4 games
- the probability that they won more than 6 games.

1. Open a new document and add a Lists & Spreadsheet page (see section 3.1). We can use this to find the probabilities in the above problem and to draw a scatter plot of the distribution. To do this, place the cursor in the cell at the top of the column next to A. Label this column as xvalues. Press the enter key **enter**. Place the cursor in the cell at the top of the column next to B. Label this column as probx. Press the enter key **enter**.

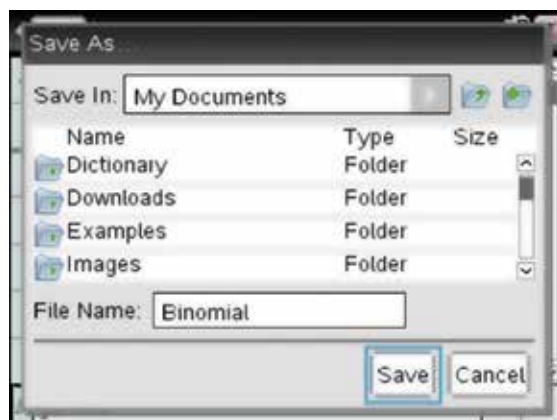


2. We will save this document by pressing:

- **ctrl** **ctrl**
- **S** **S**

In the dialog box for File Name, type 'Binomial'; then press:

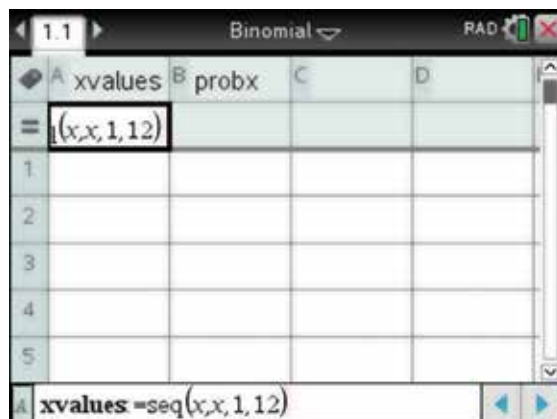
- **tab** **tab**
- **ENTER** **enter**.



3. Move the cursor to the header (grey) cell located under column A. To enter a sequence of numbers from 0 to 12 (12 games played) into column A, complete the entry line as:
= seq(x,x,0,12)

Press the enter key **enter**.

Note: 'seq' can be typed using the letter keys, or it can be found in the catalog.



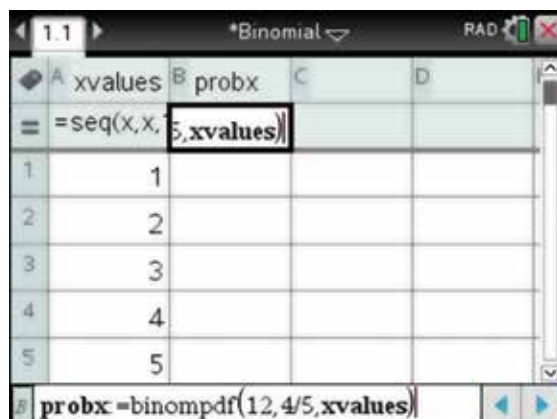
4. Move the cursor to the header (grey) cell located under column B. To calculate the probability for each x-value, complete the entry line as:
= binompdf(12,4/5,xvalues)

Press the enter key **enter**.

Press the enter key **enter**.

Notes

1. 'binompdf' can be typed using the letter keys, or it can be found in the catalog.
2. 12 refers to n , the number of games played, and $p = \frac{4}{5}$ is the probability of winning one game.



5. Use the Touchpad to scroll down to see the value in each cell more clearly. Notice that the document now has the file name *Binomial. The asterisk in front of the file name appears because we have made changes to the document and not yet resaved. To resave this file after making any changes, press:

- **ctrl** (ctrl).
- **S** (S).

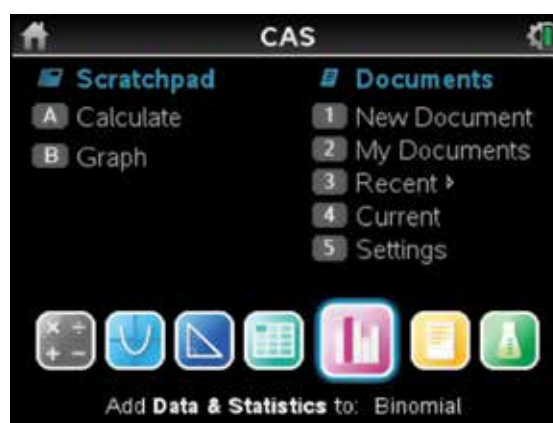
	A xvalues	B probx	C	D
=	=seq(x,x,1,12,0.5)	=binompdf		
1	1	1966080...		
2	2	4325376...		
3	3	2883584...		
4	4	2595225...		
5	5	3321888...		
=1.96608000000001E-7				

6. To draw a scatterplot of the probability distribution, we will add a Data and Statistics page to the document. Press:

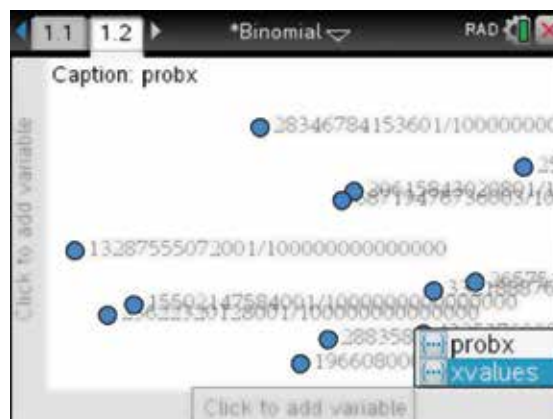
- **HOME** (on).

Using the Touchpad or Tab, select the Data & Statistics icon and then press:

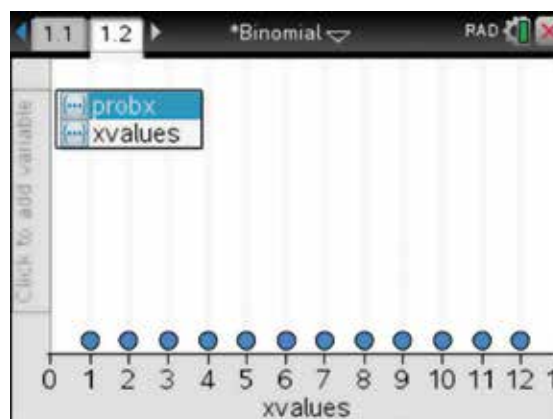
- **ENTER** (enter).






7. Press the tab key (tab) to move to the horizontal axis and select xvalues, and then press the click key (click) or the enter key (enter).





8. Press the tab key (tab) to move to the vertical axis and select probx, and then press the click key (click) or the enter key (enter).

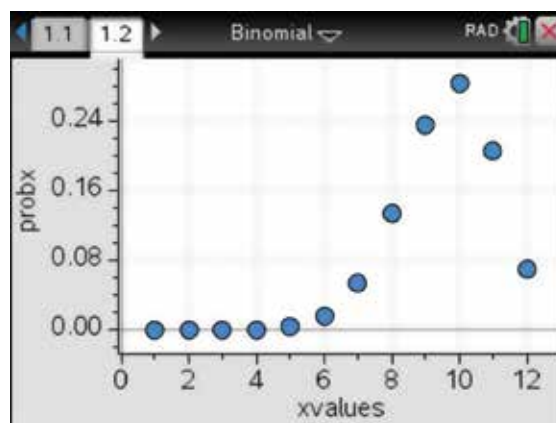





9. This gives the graph of the binomial distribution.
You can colour the points. To do this, press:

- **ctrl** 
- **MENU** 
- **3: Color** 

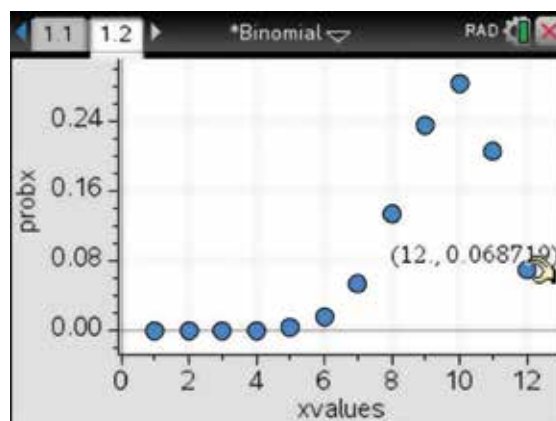
and choose a colour, then resave the document again by pressing:

- **ctrl** 
- **S** 



10. For part (a), to find $\Pr(X = 12)$, you can move the pointer  to the point $x = 12$ on the graph. The pointer  will change into an open hand  and the probability will be displayed.

$$\Pr(X = 12) = 0.06871948$$






11. Alternatively, you can return to the spreadsheet on page 1 by pressing:

- **ctrl** 
- **left arrow** .





Use the Touchpad to scroll to the last entry as shown.

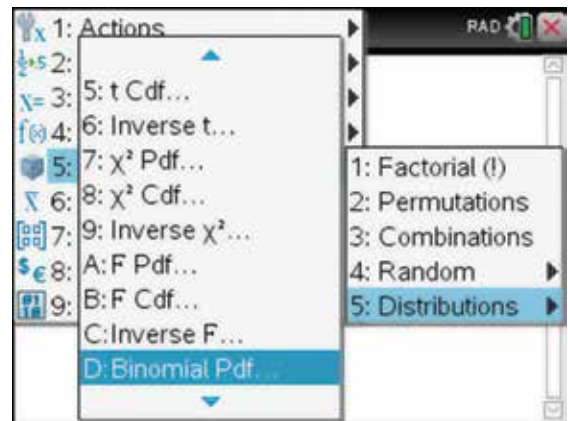
A	xvalues	B	probx	C	D
=	=seq(x,x,1	=binompdf			
9	9.	0.236223			
10	10.	0.283468			
11	11.	0.206158			
12	12.	0.068719			
13					
B12	=0.068719476736003				



12. We can also calculate probabilities on the Calculator page. To insert a Calculator page, press:

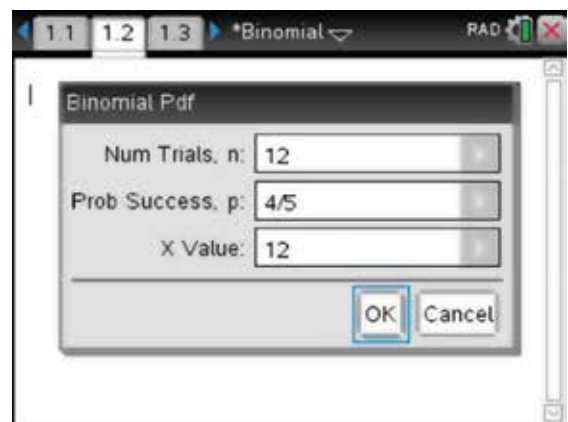
- **ctrl** 
- **I** 
- **1: Add Calculator** .

Now, using an alternative method for finding $\Pr(X = 12)$, press:

- **MENU** 
- **5: Probability** 
- **5: Distributions** 
- **D: Binomial Pdf** .



13. Enter the values as required, using the tab key  to move between fields. Then select OK and press the enter key .

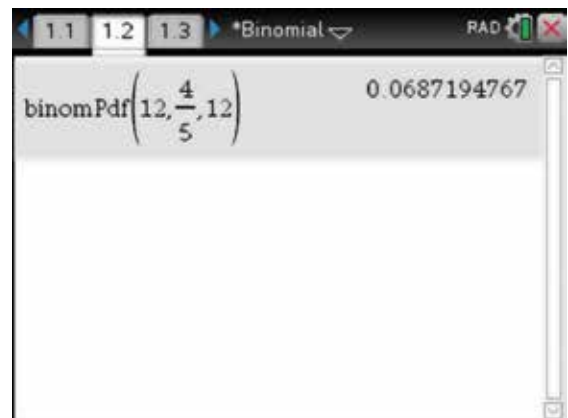


14. Note that we can also locate the binomPdf(in the catalog by pressing:

- **catalog** 
- **B** .

Then use the Touchpad to navigate to the appropriate command.

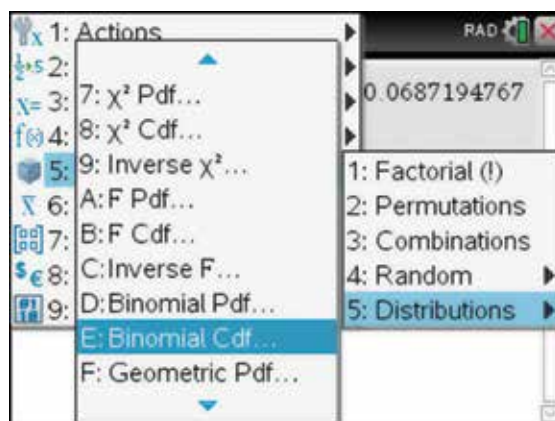
$$\Pr(X = 12) = 0.06871948$$



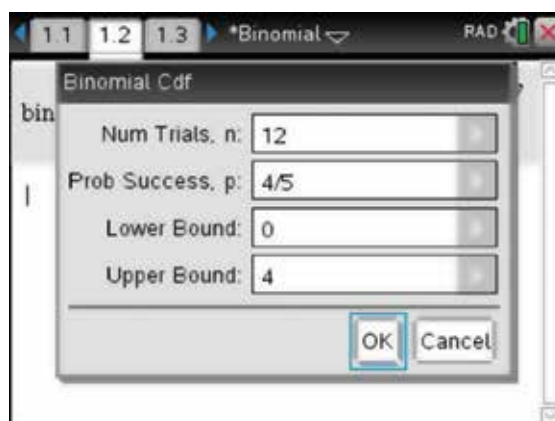
15. For part (b) we need to find the cumulative probability for an interval, and it is necessary to use binomCdf.

To calculate $\Pr(X \leq 4)$, press:

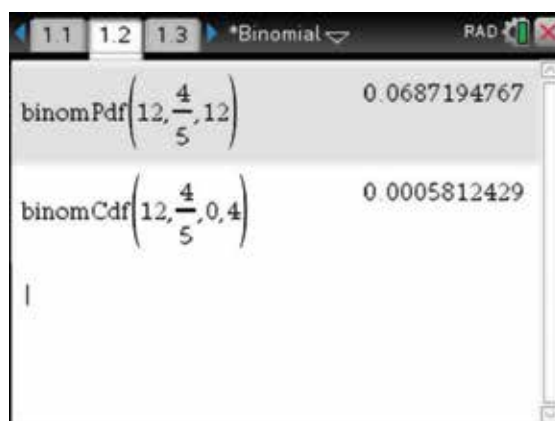
- **MENU** **(menu)**
- **5: Probability** **(5)**
- **5: Distributions** **(5)**
- **E: Binomial Cdf** **(E)**.



16. Enter the values as required, using the tab key **(tab)** to move between fields. Then select OK and press the enter key **(enter)**.



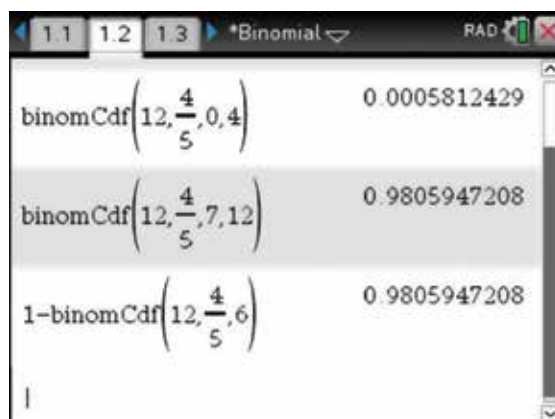
17. $\Pr(X \leq 4) = 0.00058124$



18. For part (c), $\Pr(X > 6) = \Pr(X \geq 7)$.

Repeat the steps above. The lower bound will be 7 and the upper bound will be 12.

Note that $\Pr(X > 6) = \Pr(X \geq 7) = 1 - \Pr(X \leq 6)$, as an alternative method for calculating part (c).






Lauren changes netball teams, and the probability of her team winning a game is now only 0.45. What is the lowest number of games that must be played to ensure that the probability of winning at least twice is more than 0.95?

19. If $\Pr(X \geq 2) > 0.95$, then $\Pr(X < 2) < 0.05$.

$$\Rightarrow {}^n C_0 0.45^0 (0.55)^n + {}^n C_1 0.45^1 (0.55)^{n-1} = 0.05$$

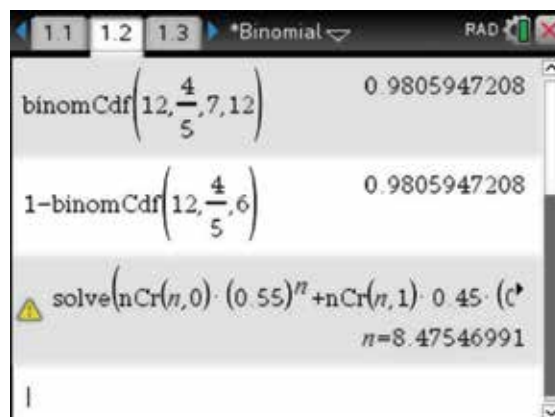
To solve for n , since n is discrete, and $n > 0$, press:

- **MENU** 
- **3: Algebra** 
- **1: Solve** .

Complete on one line as:

$$\text{solve} \left(\begin{array}{l} nCr(n,0) \cdot (0.55)^n \\ + nCr(n,1) \cdot 0.45 \cdot (0.55)^{n-1} = 0.05, n \end{array} \right) | n > 0$$

The answer will need to be rounded to $n = 9$.



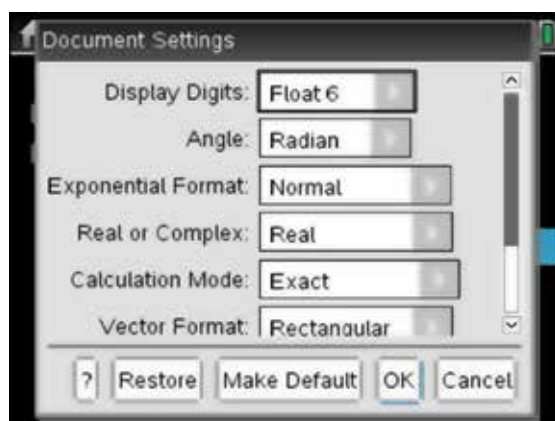
12.3 How to calculate median, quantiles and the IQR

The probability density function of a random variable X is given by:

$$f(x) = \begin{cases} k(x+2)^3, & -2 \leq x \leq 0 \\ 0, & \text{elsewhere} \end{cases}$$

- (a) Find the value of k .
- (b) Calculate the median.
- (c) Calculate the 0.25 quantile.
- (d) Calculate the 0.75 quantile.
- (e) Use the answers to parts (c) and (d) to find the IQR.

1. Begin with a new document, add a Calculator page, and set the Document Settings in the Calculation Mode to Exact. The calculator can be used very efficiently to answer questions such as the one given above by using the instructions that follow.



2. For part (a), to find the value of k , press:

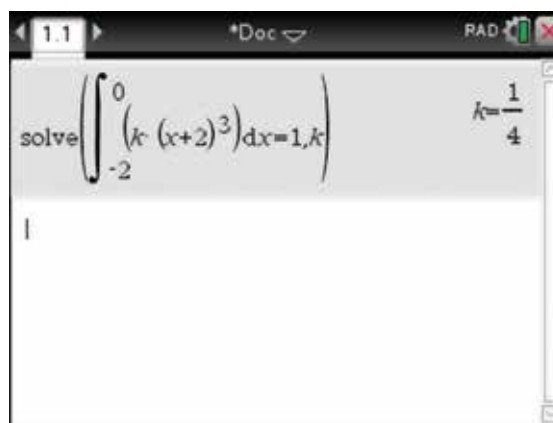
- **MENU** **(menu)**
- **3: Algebra** **(3)**
- **1: Solve** **(1)**.

Complete the entry line as shown:

$$\text{solve}\left(\int_{-2}^0 k \times (x+2)^3 dx = 1, k\right)$$

Press the enter key **(enter)**.

Note: The multiplication sign must be entered between k and the bracket.



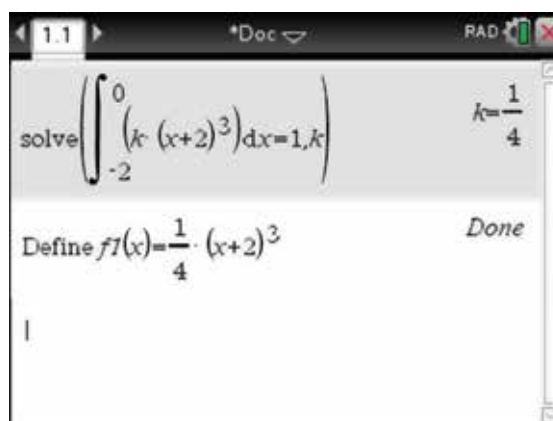
3. For part (b), define the function with its newly found k value. To do this, press:

- **MENU** **(menu)**
- **1: Actions** **(1)**
- **1: Define** **(1)**.

Complete the entry line as:

$$\text{Define } f1(x) = \frac{1}{4}(x+2)^3$$

Press the enter key **(enter)**.

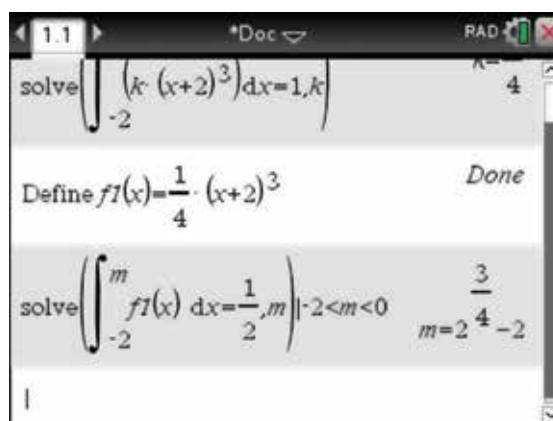


4. Now to find the median for part (b), complete the entry line as:

$$\text{solve}\left(\int_{-2}^m f1(x) dx = \frac{1}{2}, m\right) | -2 < m < 0$$

Press the enter key **(enter)**. The exact correct answer

$$\text{is } m = 2^{\frac{3}{4}} - 2 \text{ as } -2 < m < 0.$$

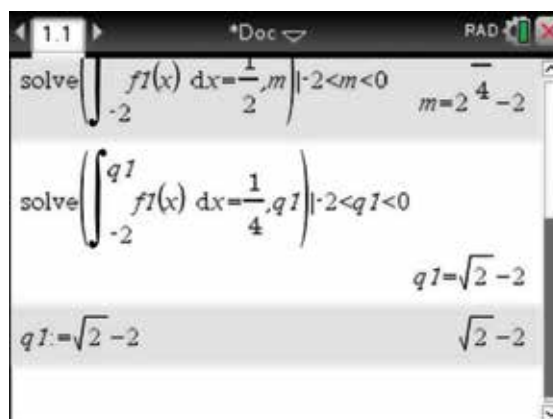


5. Similarly, to answer part (c), complete the entry line as:

$$\text{solve}\left(\int_{-2}^{q1} f1(x) dx = \frac{1}{4}, q1\right) | -2 < q1 < 0$$

Press the enter key **(enter)**. The exact correct answer

$$\text{is then stored as } q1 = \sqrt{2} - 2.$$



6. Similarly, to answer part (d), complete the entry line as:

$$\text{solve}\left(\int_{-2}^{q3} f1(x) dx = \frac{3}{4}, q3\right) | -2 < q3 < 0$$

Press the enter key **enter**. The exact correct answer is then stored as $q3 = 3^{\frac{1}{4}}\sqrt{2} - 2$.

7. To find the exact IQR for part (e), complete the entry line as:

$$q3 - q1$$

Press the enter key **enter**.

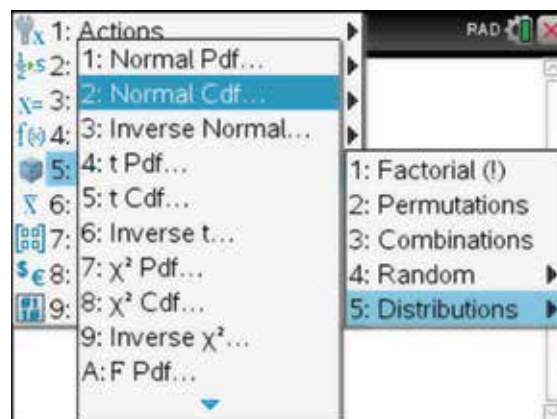
12.4 How to calculate normal distribution probabilities

If X is normally distributed with mean $(\mu) = 50$ and standard deviation $(\sigma) = 8$, find:

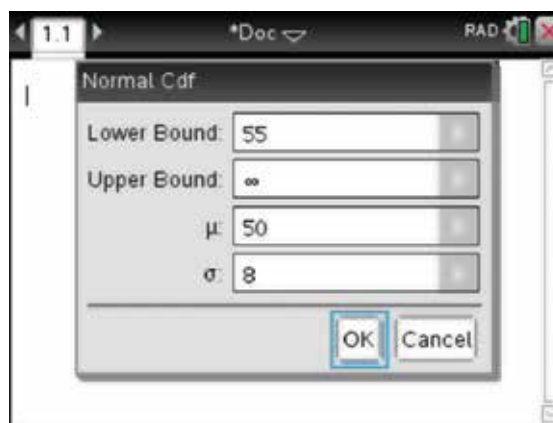
- $\Pr(X > 55)$
- $\Pr(40 < X < 65)$.

1. Open a new document and add a Calculator page (see section 2.1). For part (a), to find $\Pr(X > 55)$, press:

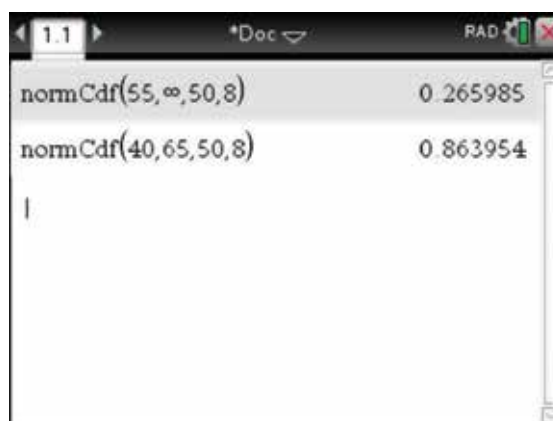
- MENU** **menu**
- 5: Probability** **5**
- 5: Distributions** **5**
- 2: Normal Cdf** **2**.



2. Enter the values as shown. The ∞ symbol can be found in the symbol template, by pressing the ctrl key **ctrl** and **⌵**, or from the palette, by pressing **⌵**. Then select OK and press the enter key **enter**.

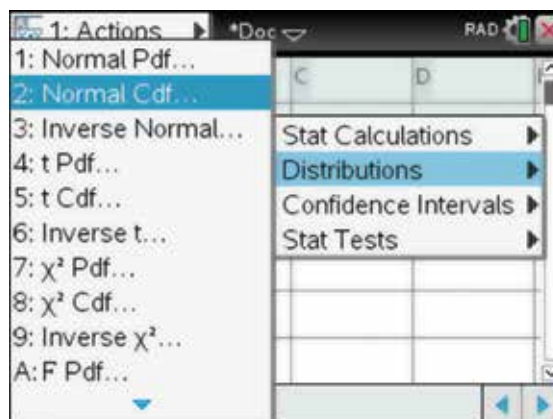


3. For part (b), repeat the above process or simply edit the first expression to find $\Pr(40 < X < 65)$.



4. Alternatively, we can repeat part (b) and shade the corresponding area. Open a Lists and Spreadsheet page, then press:

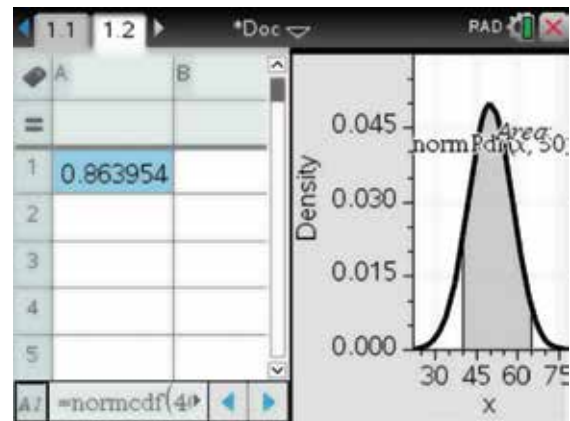
- **MENU** **menu**
- **4: Statistics** **4**
- **2: Distributions** **2**
- **2: Normal Cdf** **2**.



5. Complete as shown, making sure that you check the Shade Area box, then select OK and press the enter key **enter**.

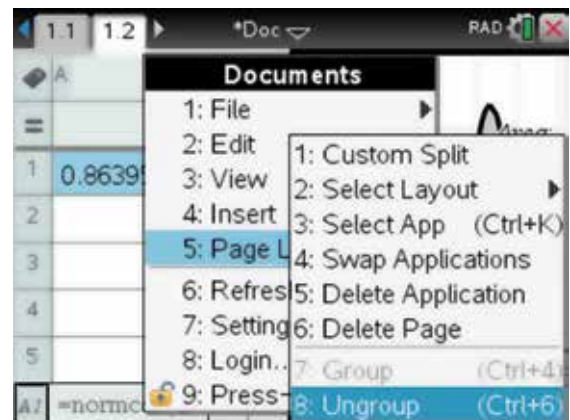


6. The graph of the normal distribution is shown with parameters for the mean and standard deviation, along with the shaded area.

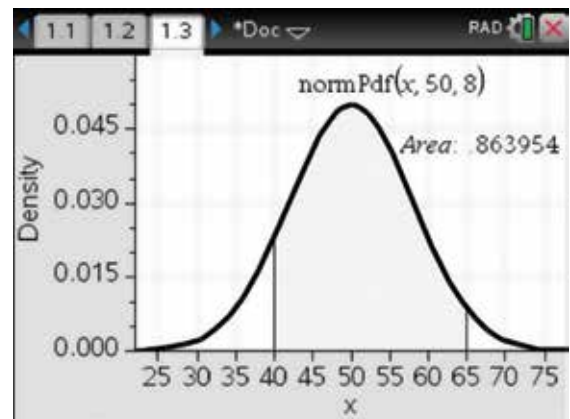


7. We can ungroup these pages and show the graph of the normal distribution on its own page. To do this, press:

- **doc** (doc)
- **5: Page Layout** (5)
- **8: Ungroup** (2).



8. Also we can grab the upper or lower limit and watch as the probabilities change. We can also change the line colour of the graph and the fill colour of the shaded area.

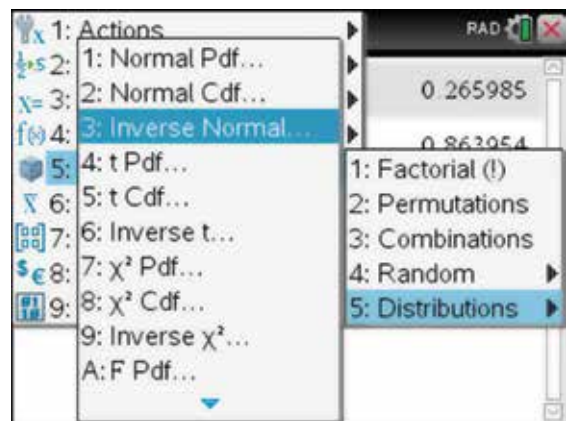


12.5 How to calculate inverse normal distribution probabilities

1. To answer questions such as the one below, use the steps that follow.

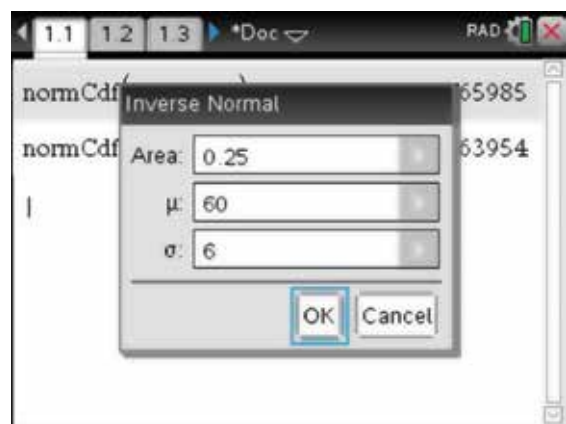
If $\Pr(X > k) = 0.75$ and $X \sim N(60, 36)$, find the value of k . Go back to the Calculator page and press:

- **MENU** **(menu)**
- **5: Probability** **(5)**
- **5: Distributions** **(5)**
- **3: Inverse Normal** **(3)**.

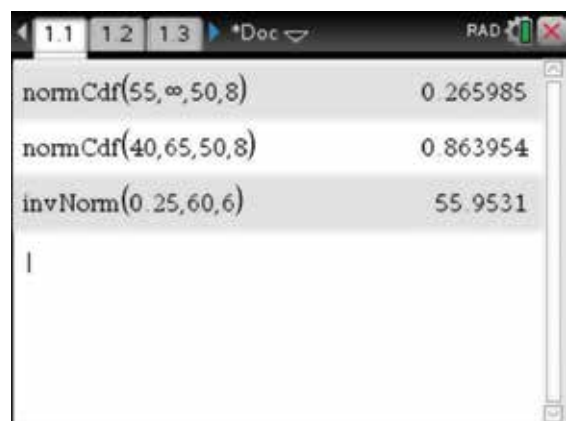


2. Enter the values shown. Then select OK and press the enter key **(enter)**.

Note: The value 0.25 is entered because the area to the left of the value is required. Also note that the variance is 36 and the standard deviation is therefore 6.



3. The answer appears on the screen.







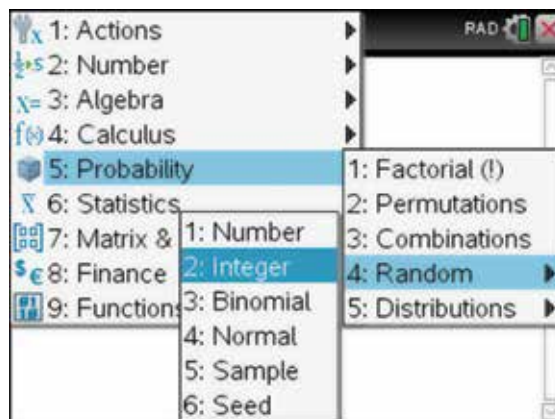
12.6 How to generate random numbers

1. To answer questions such as the one below, use the steps that follow.

- (a) Generate 10 random whole numbers between 1 and 6.
- (b) Generate 5 random whole numbers between 1 and 100.

Clear the history screen, then press:

- **MENU** 
- **5: Probability** 
- **4: Random** 
- **2: Integer** 



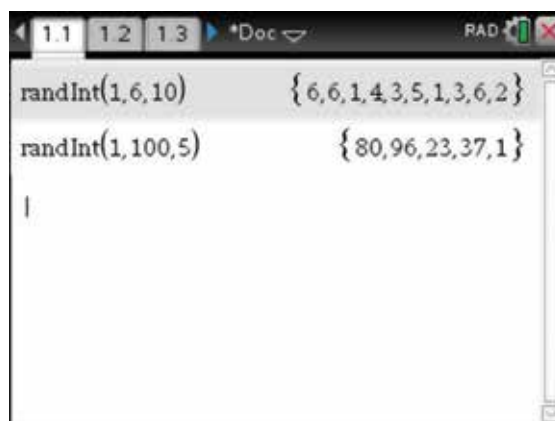
2. Complete the entry lines as:

`randInt(1,6,10)`

`randInt(1,100,5)`

Press the enter key  after each line.

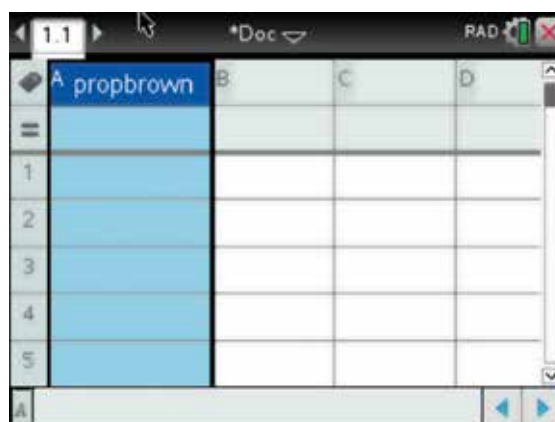
Note: Since the numbers are random, we get different results if we redo the above commands.



12.7 How to generate distributions of sample proportions

Assume 60% of Australians have brown eyes. Use your calculator to illustrate a possible distribution of sample proportions, \hat{p} , that may be obtained when 250 samples, each of size 150, are selected from the population.

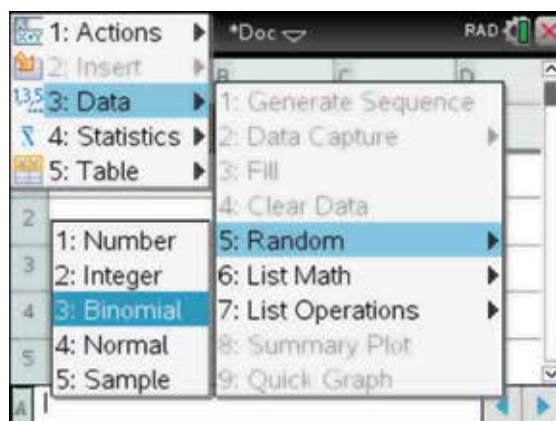
1. Open a new Lists & Spreadsheet page. Type the name 'propbrown' at the top of Column A.



2. Place the cursor in the formula cell and type =.

Then press:

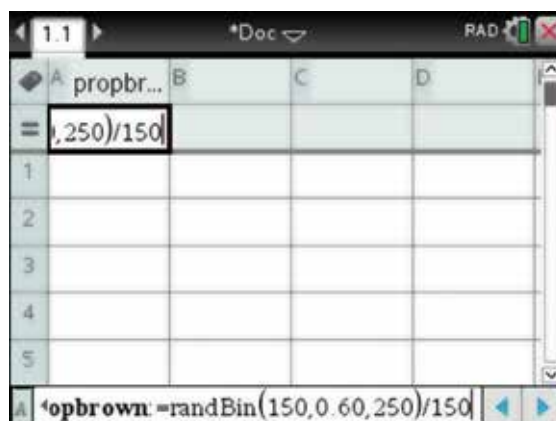
- **MENU** **menu**
- **3: Data** **3**
- **5: Random** **5**
- **3: Binomial** **3**.



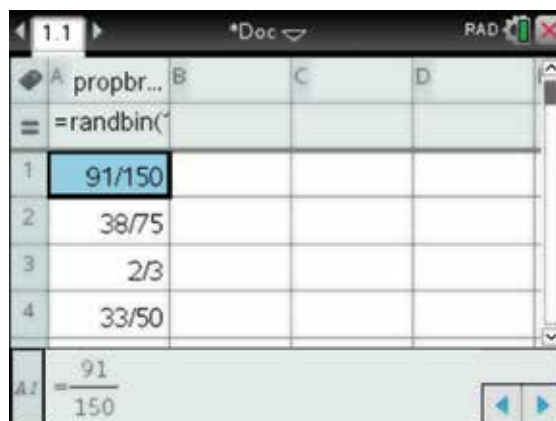
3. Complete the entry line as:

$\text{=randBin}((150,0.60,250)/150)$

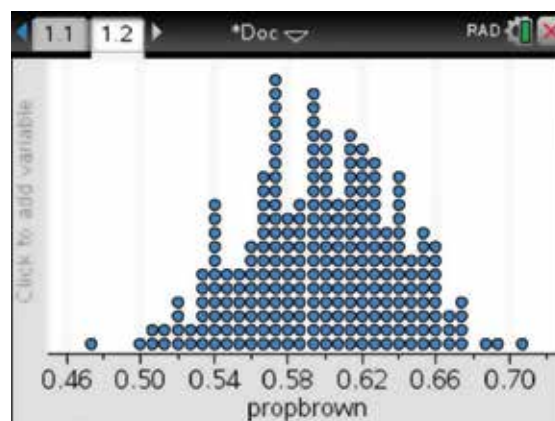
Press ENTER **enter**.



4. The cells will automatically fill.

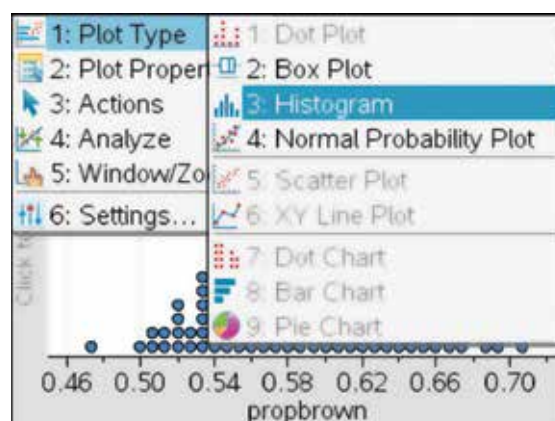


-
5. To display the distribution of the sample proportions, open a new Data & Statistics page.

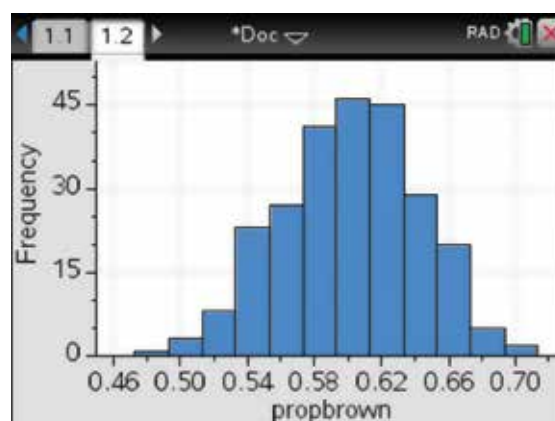


-
6. To change the graph into a histogram, press:

- **MENU** **menu**
- **1: Plot Type** **1**
- **3: Histogram** **3**.

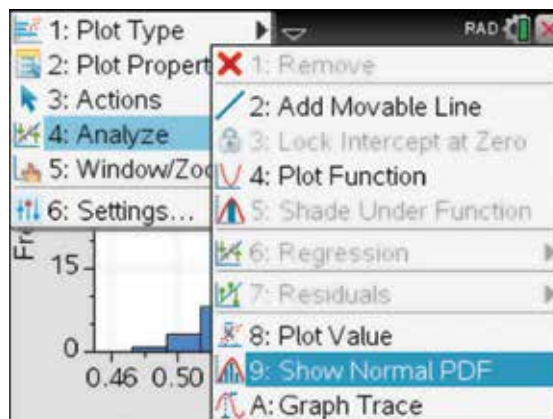


-
7. The histogram will be automatically generated.

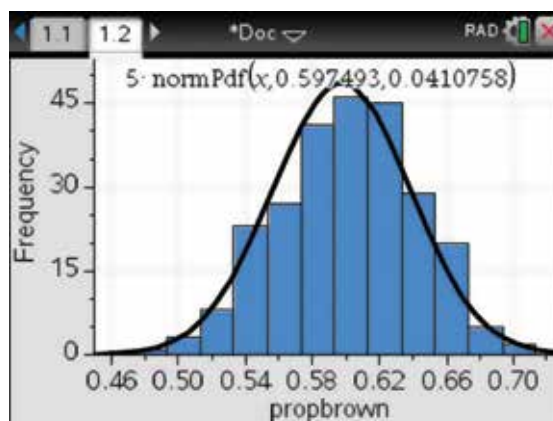


8. To fit a normal curve to the distribution, press:

- MENU **menu**
- 4: Analyze **4**
- 9: Show Normal PDF **9**.



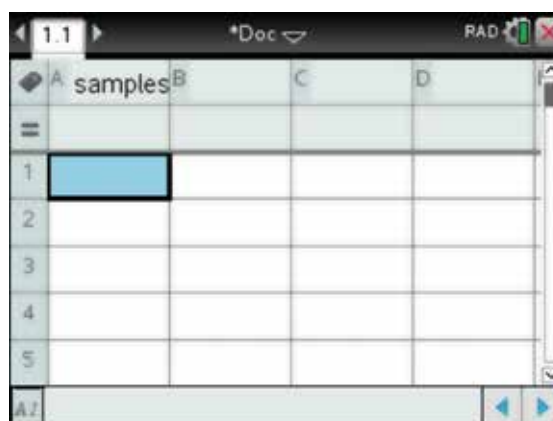
9. The normal curve will be automatically generated, showing the mean and standard deviation for the distribution.



12.8 How to generate sample means for n random samples

Generate sample means for 8 random samples of size 20 from a normal population with a mean of 70 and a standard deviation of 10.

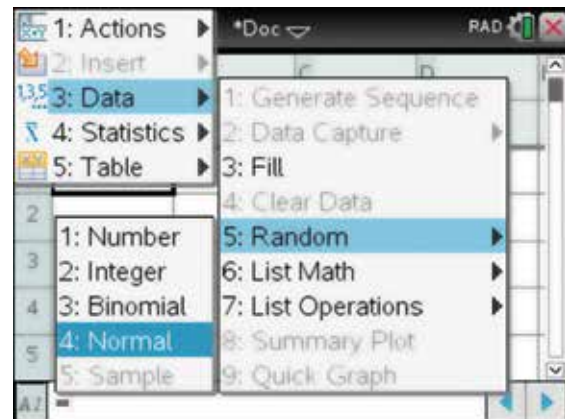
1. Open a new Lists & Spreadsheet page. Type the name 'samples' at the top of Column A.



2. Place the cursor in cell A1 and type =mean(.

Press:

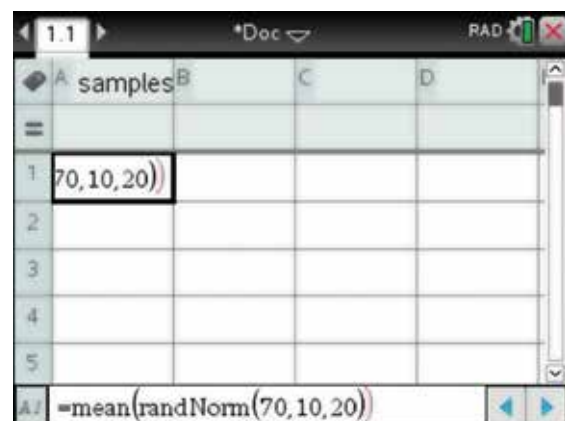
- **MENU** **menu**
- **3: Data** **3**
- **5: Random** **5**
- **4: Normal** **4**.



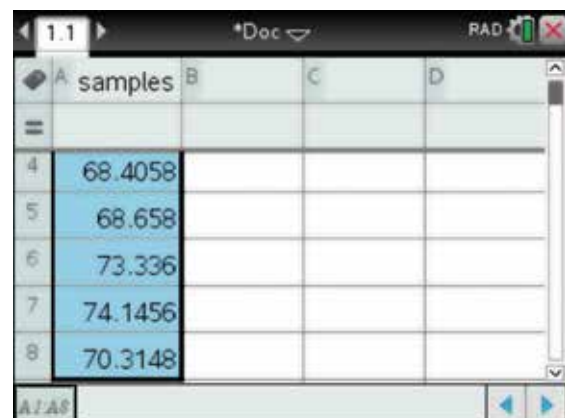
3. Complete the entry line as:

=mean(randNorm(70,10,20)

Press ENTER **enter**.



4. Fill down to obtain the sample means for 8 samples.



12.9 How to conduct a hypothesis test

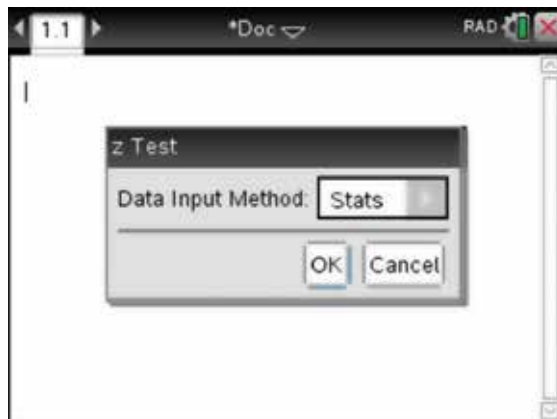
Bi More supermarkets' sales records show that the average monthly expenditure per person on a certain product was \$11 with a standard deviation of \$1.80.

The company recently ran a promotion campaign and would like to know if sales have changed. They sampled 30 families and found their average expenditure to be \$11.50. Determine the p -value for this test.

1. Open a new Calculator page. Press:

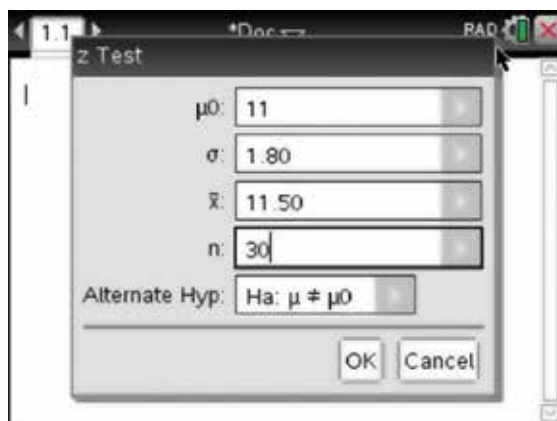
- MENU **menu**
- 6: Statistics **6**
- 7: Stat Tests **7**
- 1: z Test **1**.

Ensure that Data Input Method is set to Stats.

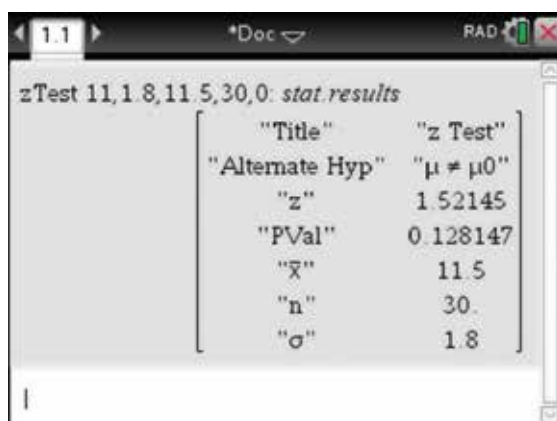


2. Complete the values as shown.

Then press Enter **enter**.



3. The p -value (PVal) is 0.128147.



CHAPTER 13

Statistics

Statistics form an important part of our understanding of the world. This chapter deals with the display and interpretation of univariate and bivariate data.

13.1 How to calculate measures of centre and spread for univariate data

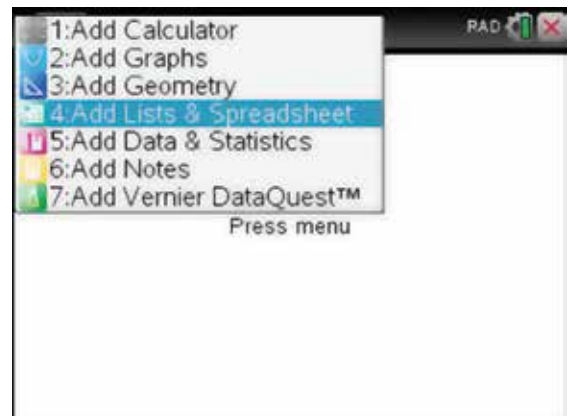
1. Begin with a new document. Unless you wish to save your previous work, press:

- **HOME**  **on**
- **1: New Doc** .

Select No.

Press:


- **ENTER** 
- **4: Add Lists & Spreadsheet** .



2. To calculate measures of centre and spread, complete the instructions below.

In column A, enter the following list of the ages of children attending a birthday party.

2, 3, 4, 4, 1, 2, 6, 7, 8, 4, 6, 8, 9, 1, 3

To do this, place the cursor in cell A1; then enter each data value followed by the enter key .



3. Press:

- **ctrl** (ctrl)
- **7** (7).

This will place the cursor back into cell A1.

To label column A and give it a variable name, use the Touchpad to scroll to the top of the column to the cell next to A, and type 'age'.

Press the enter key (enter).



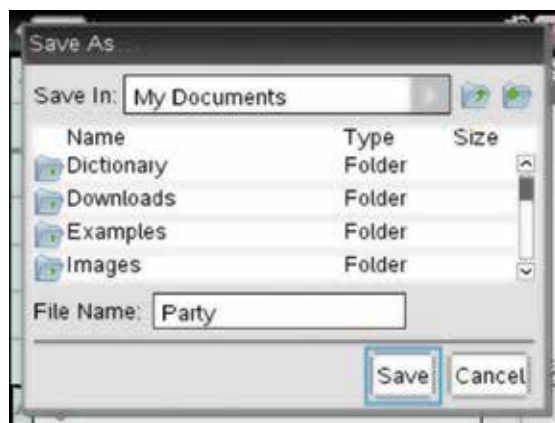
4. Since we will be using this data set throughout this chapter, save the document, give it the name Party. Press:

- **ctrl** (ctrl)
- **save** (save icon).

Type the document name 'Party' in the File Name dialog box. Press the tab key (tab) to select Save and then press the enter key (enter).

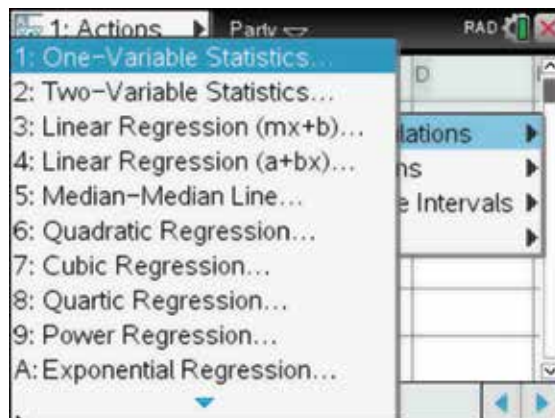
To resave the document at any time, press:

- **ctrl** (ctrl)
- **S** (S).



5. This data set of the ages of 15 children is an example of univariate data (one-variable statistics). You will probably need to know the summary statistics for this data. To determine statistical calculations for this data, press:

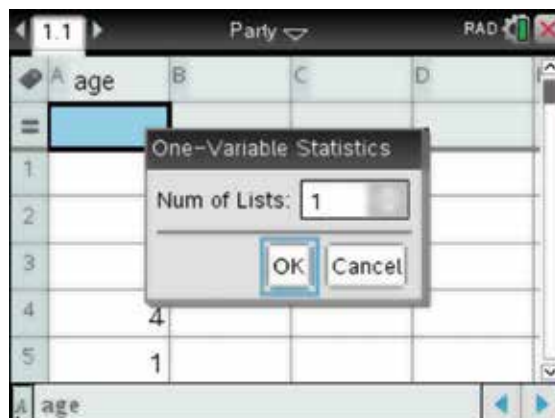
- **MENU** (menu)
- **4: Statistics** (4)
- **1: Stat Calculations** (1)
- **1: One-Variable Statistics** (1).



6. This will return you to the screen shown.


As we are dealing with only one list of data, the Num of Lists should be 1.

Press the tab key (tab) to select OK, then press the enter key (enter).



7. The screen opposite appears.


This allows you to select the list for which you require information. In this case, that list is labelled 'age'.


When the cursor is in a field, pressing the click key  will allow you to see the options.

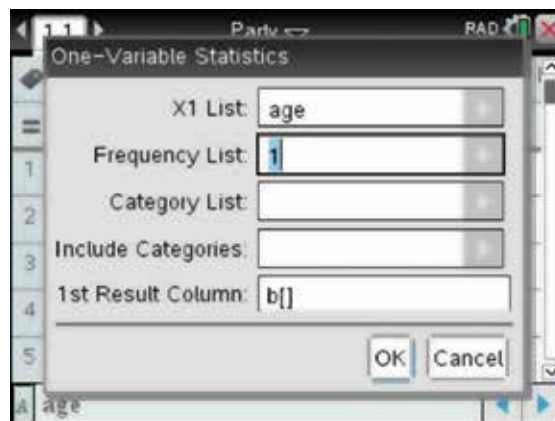
Use the Touchpad arrow keys to select the required list.

Press the enter key .



Note: It is possible to select a[] to represent column A.

Use the tab key  to move through the fields.

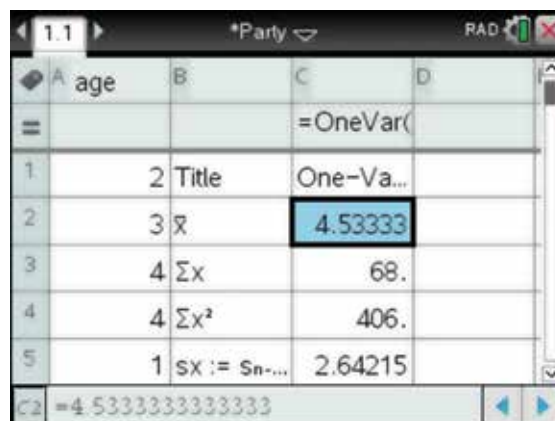
Select OK and press the enter key .



8. The statistical data will be placed in columns B and C. Use the Touchpad to scroll down to view all of the statistical data. For example, the mean age of children attending the birthday party, \bar{x} , is 4.53. To undo the list of statistical calculations when you have finished with them, press:

- **ctrl** 
- **esc** .



Note: The Calculation Mode in the Document Settings is in Approximate.



13.2 How to draw boxplots

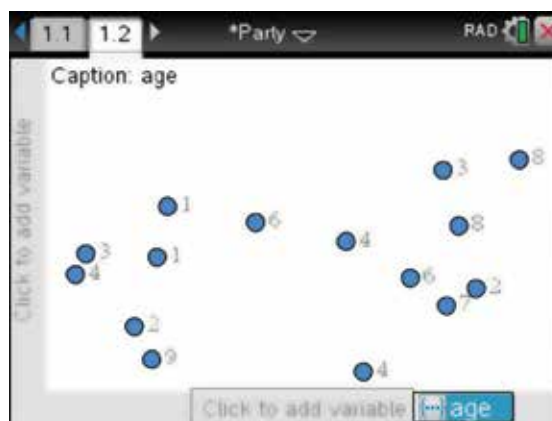
1. We will continue to use the data set from section 13.1 to draw a boxplot.

The ages of children attending a birthday party are 2, 3, 4, 4, 1, 2, 6, 7, 8, 4, 6, 8, 9, 1, 3.

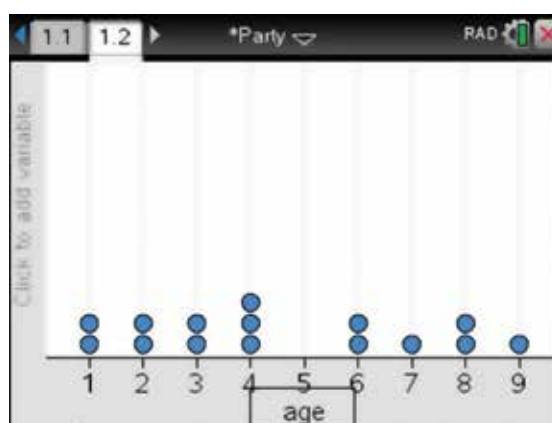
To insert a Data & Statistics page, into the document, press HOME , select the Data & Statistics icon and press the enter key .



- The adjacent screen appears. Use the tab key (tab) to move to the horizontal axis. Select the variable age.

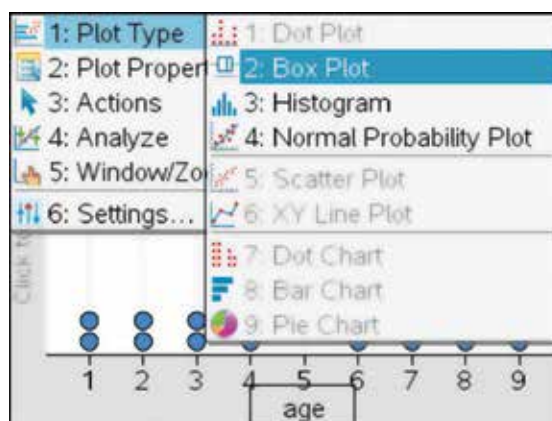


- The graph will appear as shown; it is a scatterplot. Follow the instructions below to draw the boxplot.



- To draw a boxplot, press:

- MENU (menu)
- 1: Plot Type (1)
- 2: Box Plot (2).

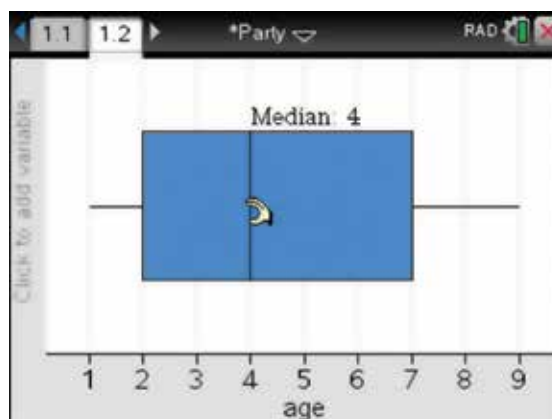


- The boxplot appears. You can use the pointer to move around to display relevant information on the boxplot, such as the median.

Note: The default settings for boxplots are the modified boxplot, which will automatically display any outliers.

If you wish to remove the outliers and draw a standard boxplot, press:

- ctrl (ctrl)
- MENU (menu)
- 4: Extend Box Plot Whiskers (4).

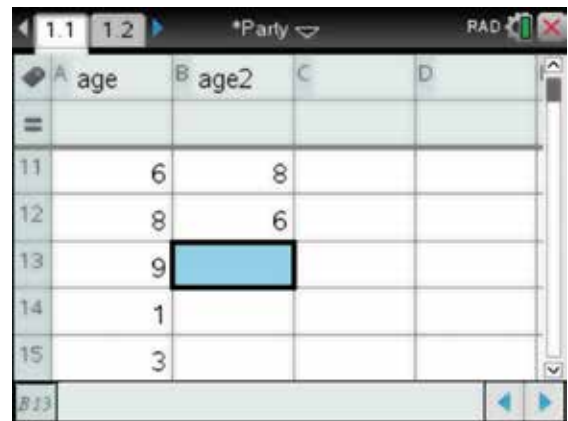


6. To compare two boxplots, return to the Lists & Spreadsheet page by pressing:

- **ctrl** (ctrl)
- **left arrow** ◀.

Move to the Lists & Spreadsheet page, name column B as age2 and enter the following data for the ages of children at a second birthday party.

6, 3, 4, 7, 6, 5, 4, 4, 4, 8, 8, 6



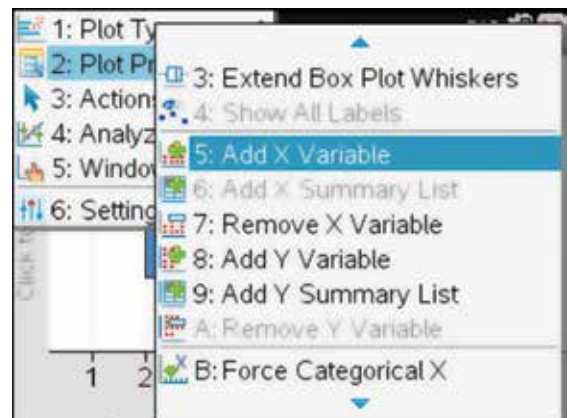
	A age	B age2	C	D
11		6	8	
12		8	6	
13		9		
14		1		
15		3		

7. To draw the second boxplot on the same Data & Statistics page, first return to the Data and Statistics page. To do this, press:

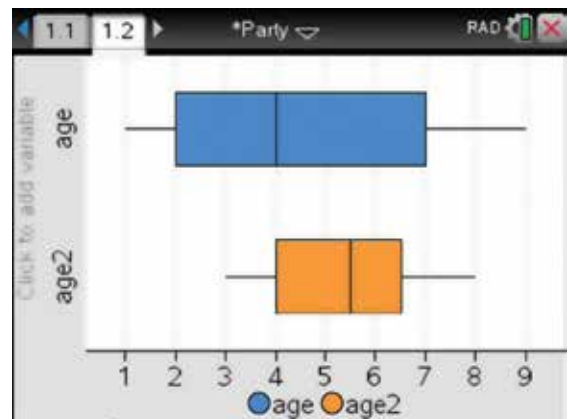
- **ctrl** (ctrl)
- **right arrow** ▶.

Now press:

- **MENU** (menu)
- **2: Plot Properties** (2)
- **5: Add X Variable** (5).



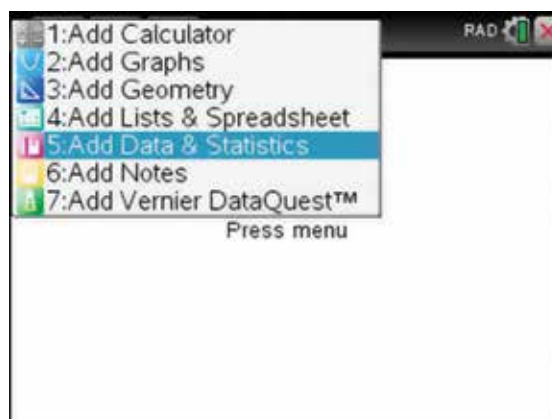
8. Select age2 and the boxplots will appear.



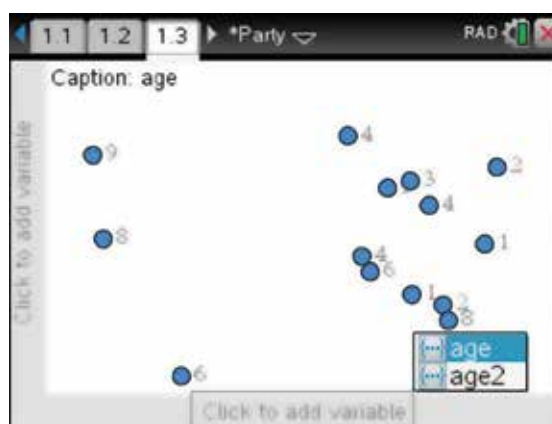
13.3 How to draw histograms

1. We will continue to use the same data set from section 13.1 to draw a histogram. To insert another Data & Statistics page into this document, press:

- **ctrl** **(ctrl)**
- **I** **(I)**
- **5: Add Data & Statistics** **(5)**.

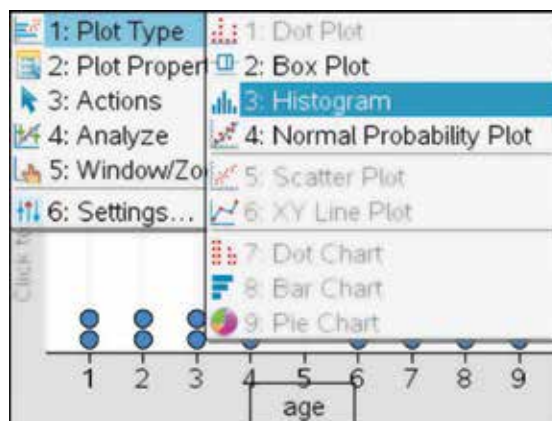


2. The adjacent screen appears. Use the tab key **(tab)** to move to the horizontal axis. Select the variable age.



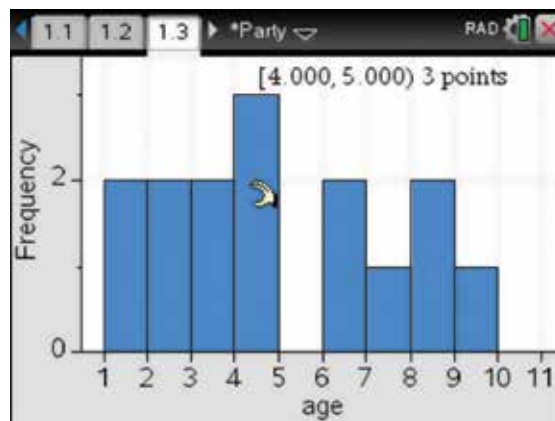
3. To draw a histogram, press:

- **MENU** **(menu)**
- **1: Plot Type** **(1)**
- **3: Histogram** **(3)**.



- The histogram is displayed as shown on the screen. If you move the pointer to any of the columns, the values of the boundaries of the columns and the number of data values within that column will be displayed.

For example, the column highlighted in the screen opposite shows that there were 3 children between the ages of 3.5 and 4.5 at the party.



- It is possible to split categorical data. A small set of data is used here; normally, a larger set would be used.

Open a new document and add a new Lists & Spreadsheet page. Enter the data and name the columns as shown. Adjust the fill colour of the columns.

Gender	Height (cm)	Weight (kg)
Male	171	73
Male	165	66
Female	156	72
Male	158	82
Female	159	64
Female	173	61
Female	179	84
Male	167	102
Male	160	71
Female	151	48

	gender	height	weight
7	female	179	84
8	male	167	102
9	male	160	71
10	female	151	48
11			

- Add a Data & Statistics page to the document. A graph will appear as shown.

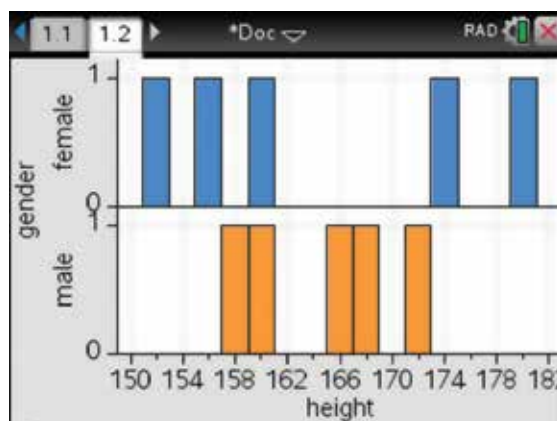


7. To select the variables, press the tab key **(tab)** and then the enter key **(enter)** to select, as shown.
To draw a histogram or a boxplot, press:

- **MENU** **(menu)**
- **1: Plot Type** **(1)**.

Then select the graph type.

The data set used here is so small that the graphs look distorted. However, they are sufficient to show the functionality of these features. Create new boxplots of weight versus gender and height versus gender on new separate Data & Statistics pages.

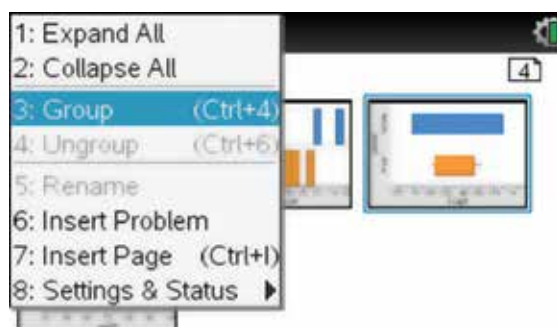


8. Press:

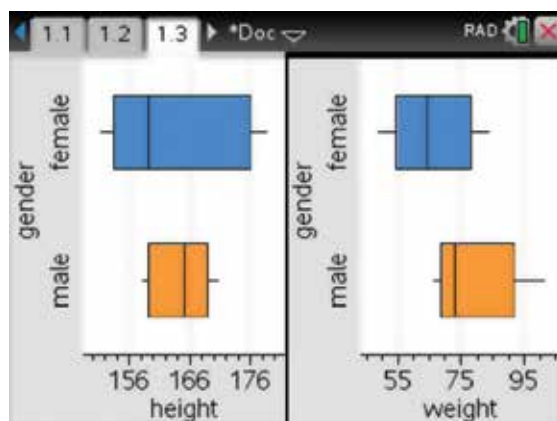
- **ctrl** **(ctrl)**
- **up arrow** **▲**

to see the page sorter view, then press:

- **MENU** **(menu)**
- **3: Group** **(3)**.



9. Boxplots comparing gender, for males and females, for both heights and weights are shown on the one page. Save the document.



13.4 How to draw scatterplots using bivariate data

The following beep-test data was obtained from a Year 11 Physical Education student. The table below shows the heart rate recorded each minute.

Time	0	1	2	3	4	5	6	7	8	9	10	11	12
Heart rate	78	137	150	157	170	179	183	190	194	197	203	207	208


1. Open a new document and add a new Lists & Spreadsheet page. Enter the data and name the columns as shown. Colour the columns and save the document, giving it the file name Physical.

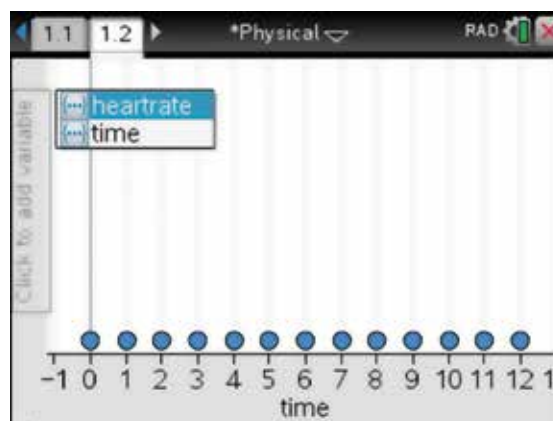


2. To draw a scatterplot of the data, press:

- **HOME** 

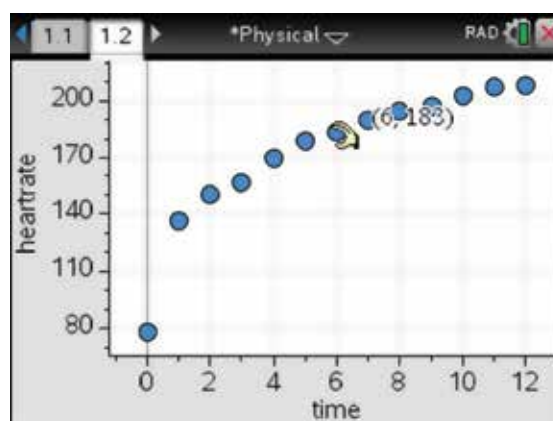
and add a Data & Statistics page to the document.

Use the tab key **tab** to move to each axis. Press the click key  or the enter key **enter** to place time on the horizontal axis and heartrate on the vertical axis.



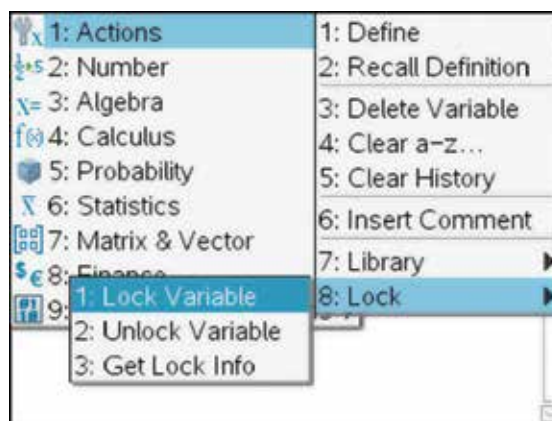
3. The graph will appear.

If you move the pointer over any point, the coordinates for that point will be displayed. If you grab one of the points and move it, the values in the Lists & Spreadsheet would also change. Since we do not want to accidentally change these values, this is a good example of locking the data. See section 7.6.



4. Now add a Calculator page to this document.
To do this, press:

- **MENU** **menu**
- **1: Actions** **1**
- **8: Lock** **8**
- **1: Lock Variable** **1**.



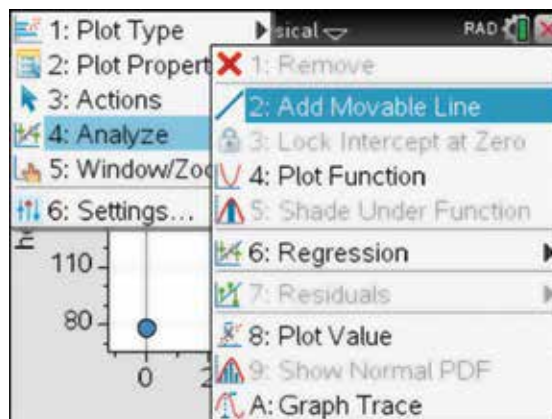
5. Complete as shown. If we now return to the Data & Statistics page of the document, we find that we cannot change or move the data points.



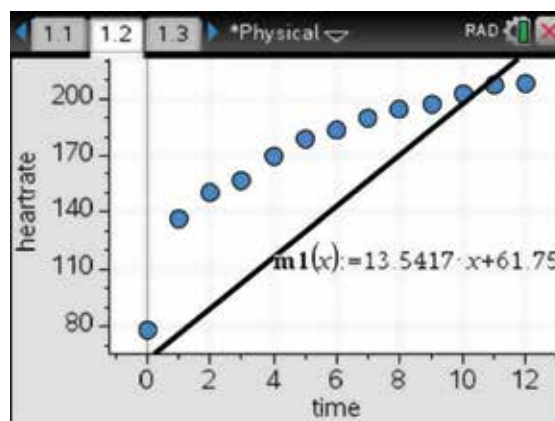
13.5 How to construct lines of best fit and regression lines

1. To construct a line of best fit by eye on the scatterplot in section 13.4, press:

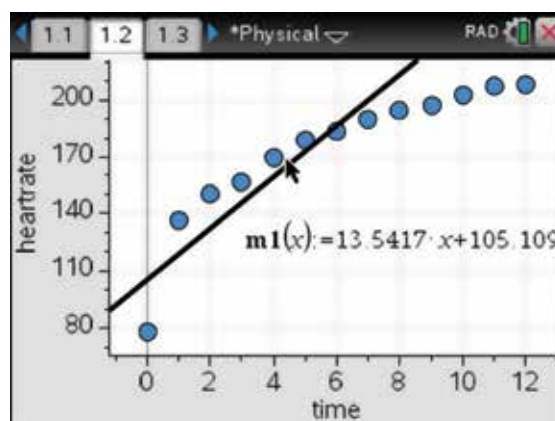
- **MENU** **menu**
- **4: Analyze** **4**
- **2: Add Movable Line** **2**.



2. A line and its equation appear automatically on the graph as shown.

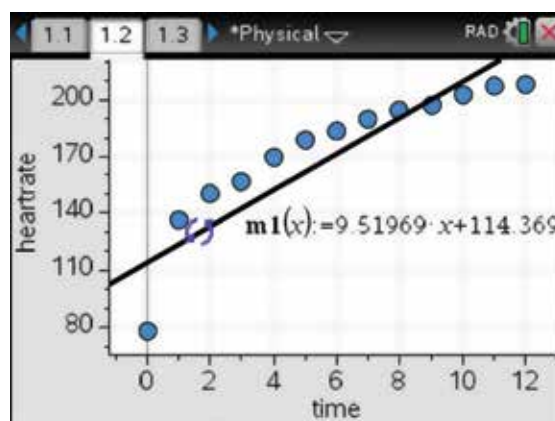


3. Manually repositioning the line is done in two steps by moving the position of the y-intercept and altering the gradient. To change the y-intercept, move the pointer until it rests somewhere around the middle of the line and the symbol changes to . Press the click key , then use the Touchpad to move the line parallel to itself, until the y-intercept is in an appropriate position.



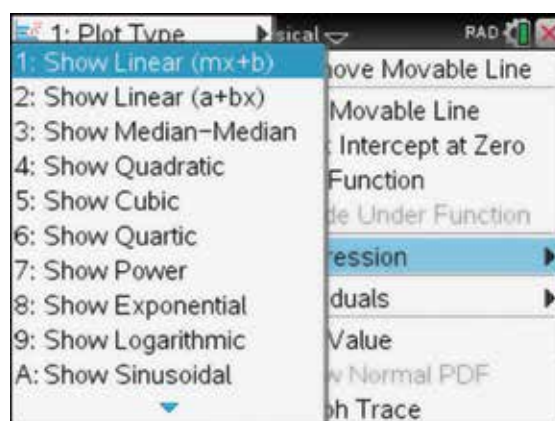
4. To change the gradient, move the pointer until it rests somewhere near the line ends of the line and the symbol changes to . Press the click key , then use the Touchpad to rotate the line and change the gradient.

Continue to use these tools until you are satisfied with the line of best fit by eye.

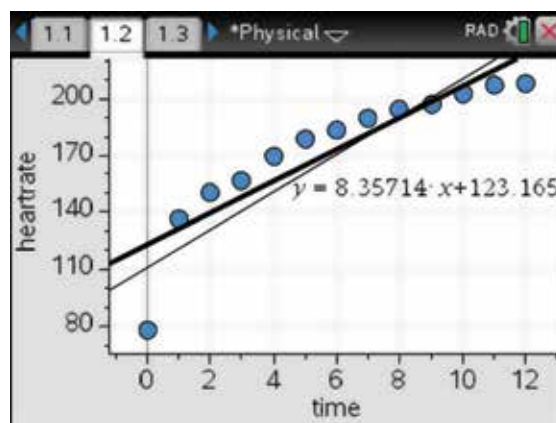


5. To find the least squares regression line, press:

- MENU
- 4: Analyze
- 6: Regression
- 1: Show Linear (mx+b) .

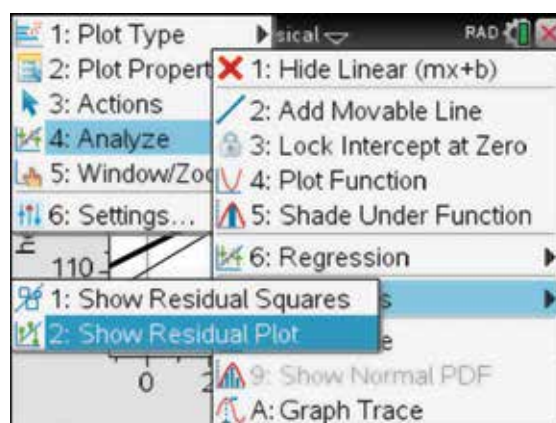


6. The thicker line is the least squares regression line. The corresponding equation is shown. The movable line is thinner. The gradient and the y-intercept of the two lines are very similar.

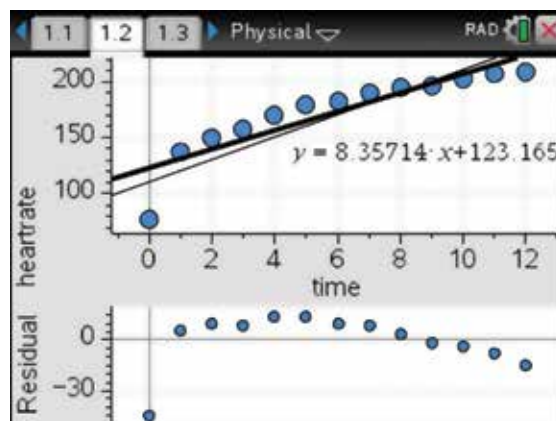


7. To draw a residual plot, press:

- MENU **menu**
- 4: Analyze **4**
- 7: Residuals **7**
- 2: Show Residual Plot **2**.



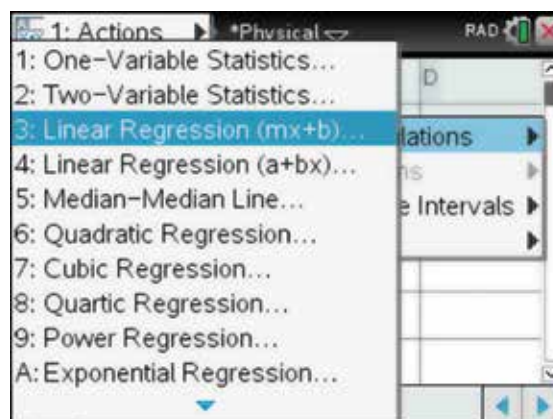
8. The residual plot will appear just below the scatterplot. Resave the document.



13.6 How to make predictions and calculate r and r^2

1. To make predictions (interpolate or extrapolate) using the data provided in section 13.4, it is necessary to write the least squares regression line as a function. To do this, return to the Lists & Spreadsheet page, then press:

- **MENU** **menu**
- **4: Statistics** **4**
- **1: Stat Calculations** **1**
- **3: Linear Regression (mx+b)** **3**.



2. The screen opposite appears. Enter the values shown. Remember to press the tab key **tab** to move between fields and press the click key **click** to drop down the options for a particular field. Select OK, then press the enter key **enter**.



3. The Lists & Spreadsheet page opposite appears with the bivariate statistical calculations. Pearson's product-moment correlation coefficient (r) and the coefficient of determination (r^2) appear in this list.


	1.1	1.2	1.3	*Physical	RAD
	time	heartrate			
=					=LinRegV
2	1	137	RegEqn	m*x+b	
3	2	150	m	8.35714	
4	3	157	b	123.165	
5	4	170	r ²	0.802639	
6	5	179	r	0.895901	
D6	=0.89590142194573				

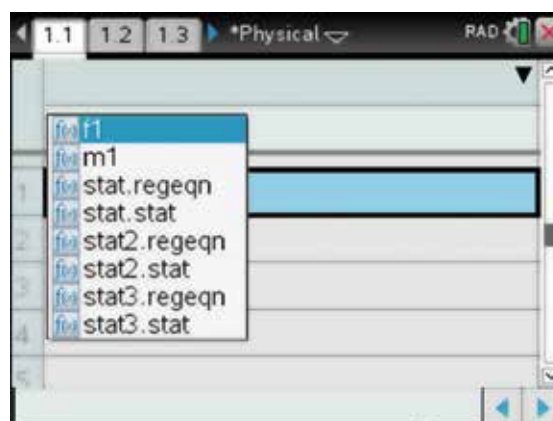
4. While on the Lists & Spreadsheet page, press:

- **MENU** 
- **5: Table** 
- **1: Switch to Table** .



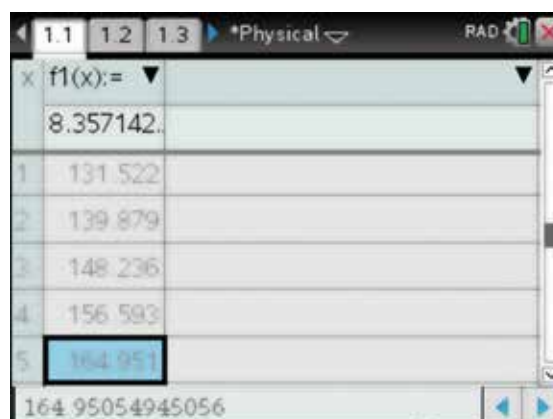
3	2	150	m	8.35714
4	3	157	b	123.165
5	4	170	r^2	0.802639
6	5	179	r	0.895901

5. With the function f1 selected, press the enter key .





f1	m1
stat.regeqn	stat.stat
stat2.regeqn	stat2.stat
stat3.regeqn	stat3.stat


6. This function table gives a heart rate of 164.95 after 5 minutes.

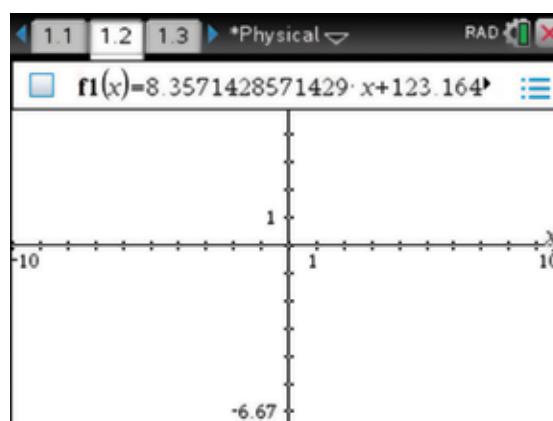


x	f1(x) =
8.357142	
1	131.522
2	139.879
3	148.236
4	156.593
5	164.95

7. An alternative method to using the Function Table to predict a value from the regression equation is to add a Graphs page to the document. To do this, press:

- **ctrl** 
- **I** 

and add a Graphs page. Notice that the least squares regression equation has already been inserted to $f1(x)$. Use the up arrow  to see it.

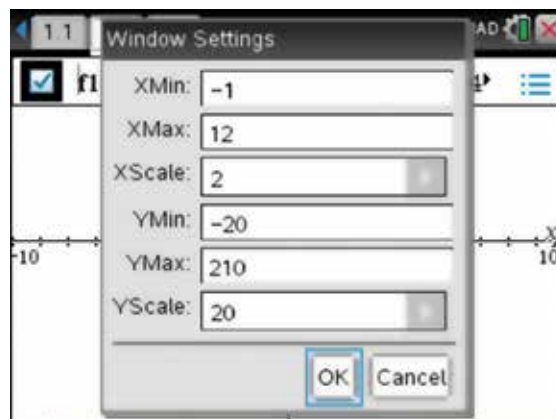


8. Press the enter key (enter). The graph will appear.
Adjust the Window Settings as shown by pressing:

- MENU (menu)
- 4: Window / Zoom (4)
- 1: Window Settings (1).

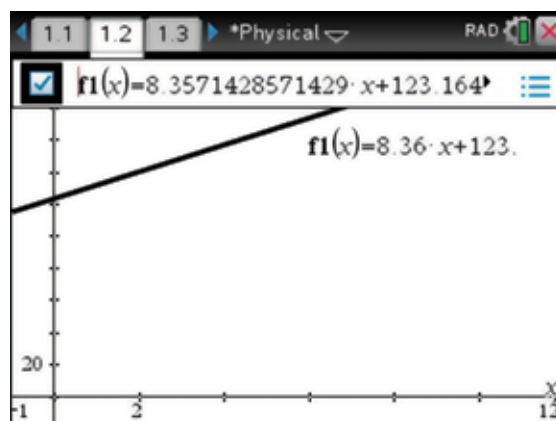
Select OK, then press the enter key (enter).

Note that these settings were chosen so we can see the axes as well.

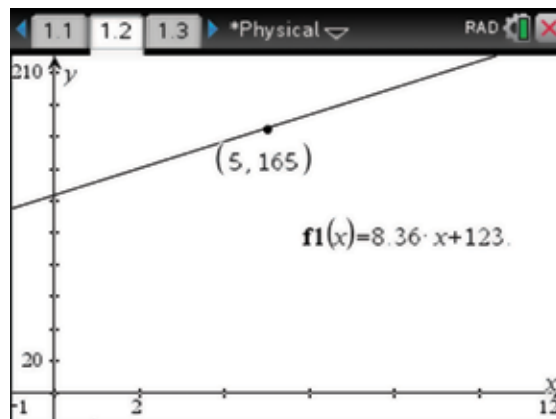


9. We can now use this graph to find any values using the defined function $f1(x)$. For example, to predict the heart rate after 5 minutes, place a point on the line. Press:

- MENU (menu)
- 7: Points & Lines (7)
- 2: Point On (2).



10. Move over the line and press the enter key (enter) twice. Press the esc key (esc) to anchor the point on the line. Move the point or edit the x -value of its coordinates to $x = 5$.



13.7 How to display time-series plots and smoothed data

1. The temperature of a sick patient is measured and recorded every 2 hours. Create a 3-point moving average of the data and plot both the original and smoothed data.

Time	2	4	6	8	10	12	14	16
Temp	36.5	37.2	36.9	37.1	37.3	37.2	37.5	37.8

In a new document, add a Lists & Spreadsheet page. Enter the time values in column A and the temperature values in column B. Label the columns accordingly and colour the columns. Save the document with the file name Smoothing.

	A time	B temp	C	D
1	2	36.5		
2	4	37.2		
3	6	36.9		
4	8	37.1		
5	10	37.3		

2. Label column C as threepointsmooth. To enter the rule for the 3-point moving average, $\frac{t_1 + t_2 + t_3}{3}$, start in cell C2 and complete the entry line as:

$$= \frac{\text{sum}(b1 : b3)}{3}$$

Fill down to C[7] of the data list and colour the column.

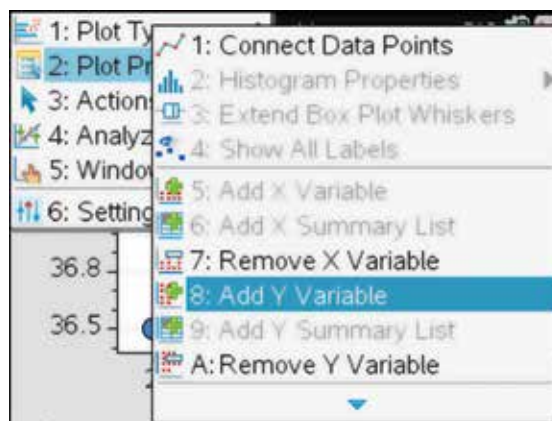
	A time	B temp	C threep...	D
5	10	37.3	37.2	
6	12	37.2	37.3333	
7	14	37.5	37.5	
8	16	37.8		

3. Add a Data & Statistics page. Press the tab key **(tab)** to move to the horizontal axis and select time. Press the tab key **(tab)** to move to the vertical axis and select temp.

To add the second y-variable threepointsmooth, press:

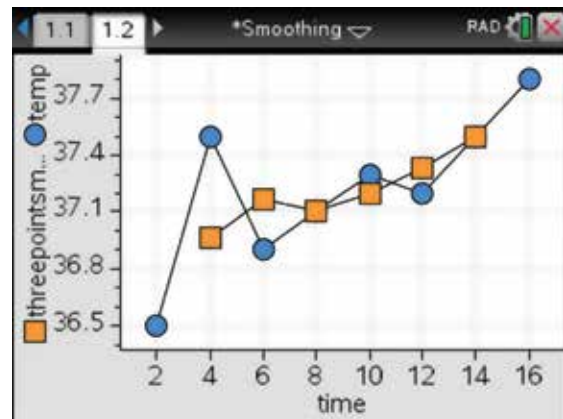
- **MENU** **(menu)**
- **2: Plot Properties** **(2)**
- **8: Add Y Variable** **(8)**.

Select threepointsmooth, then press the enter key **(enter)**.



4. To connect the points, press:

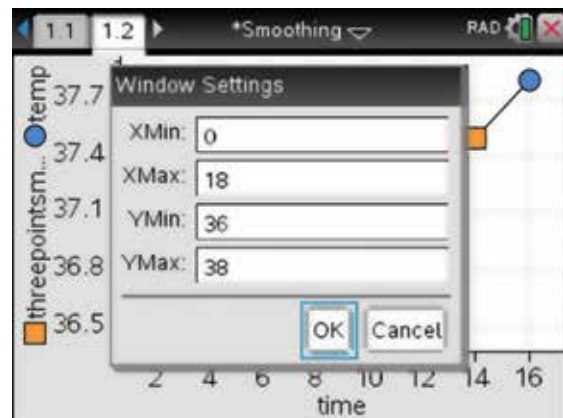
- MENU **menu**
- 2: Plot Properties **2**
- 1: Connect Data points **1**.



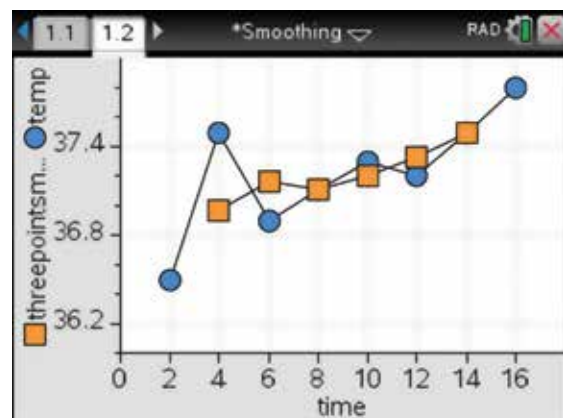
5. To alter the scale to a suitable viewing window, press:

- MENU **menu**
- 5: Window / Zoom **5**
- 1: Window Settings **1**.

Complete as shown and select OK.



6. The time series with the 3-point smoothing is displayed in orange; the original data points of the temperature are displayed in blue. Resave the document.



CHAPTER 14

Matrices and vectors

This chapter deals with basic operations involving matrices and vectors.

14.1 How to add, subtract and multiply matrices

Given the following matrices,

$$A = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}, B = \begin{bmatrix} 5 & 0 \\ 1 & -2 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{ find:}$$

(a) $5A$

(b) $-2B$

(c) $2A + 3B$


(d) $\det A$

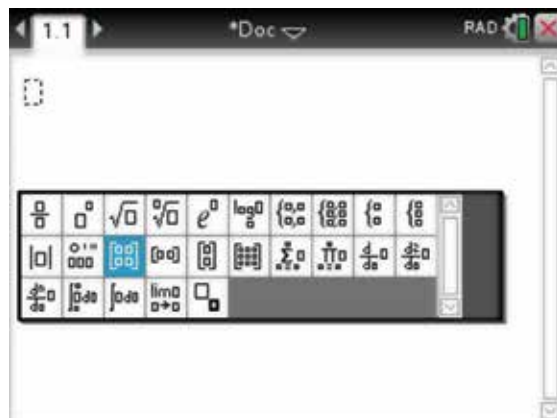
(e) B^{-1}



(f) $B.I.$

- To answer a question such as the one above, use the steps that follow. Begin with a new document and add a Calculator page (see section 2.1). There are templates in the Maths expression palette that can be used for matrices of order 2×2 , 1×2 , 2×1 and 3×3 . The Maths expression palette can be found by pressing:

- .

Use the arrow keys on the Touchpad to select the 2×2 matrix, and then press the enter key .



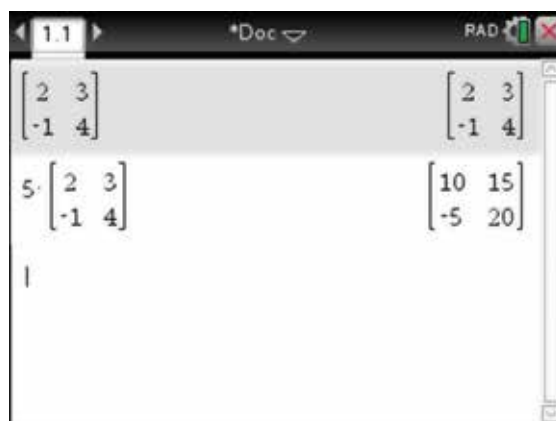
- Using the tab  key, enter the numbers in the matrix as shown. Then press the enter key .



3. For part (a), to calculate $5A$, complete the entry line as:

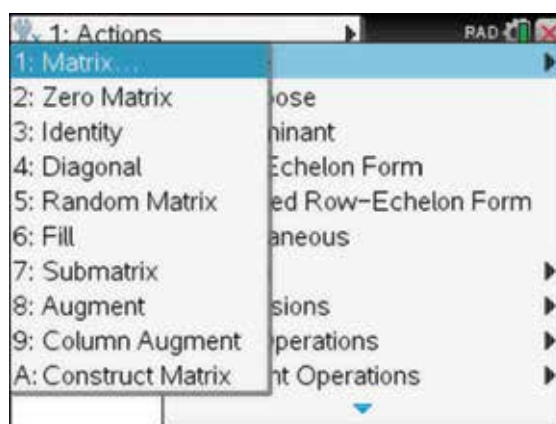
$$5 \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

To do this, type 5 **5**, scroll back to matrix A as shown and press the enter key **enter**. This places matrix A in the entry line. Press the enter key **enter** to get the answer.

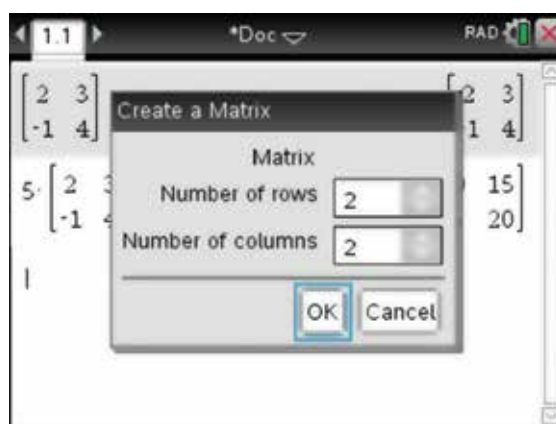


4. Alternatively, matrices of any order can be entered by pressing:

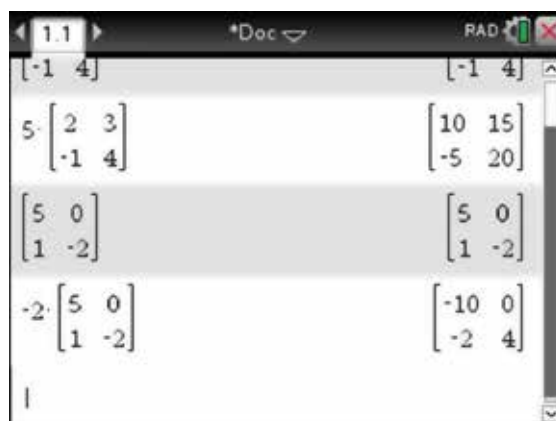
- **MENU** **menu**
- **7: Matrix & Vector** **7**
- **1: Create** **1**
- **1: Matrix** **1**.



5. Select the number of rows and columns (both 2 in this case) and use this to enter the matrix B . Tab **tab** to OK and press the enter key **enter**.

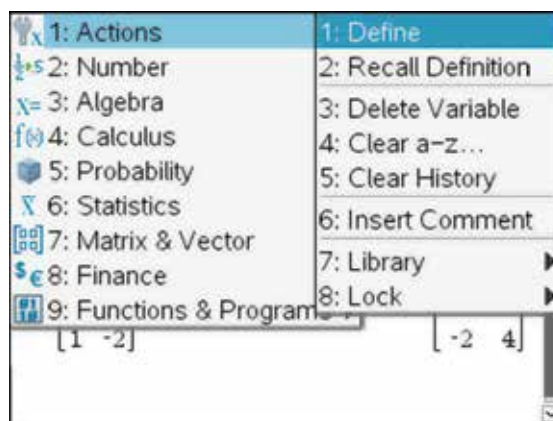


6. For part (b), to calculate $-2B$, repeat the process outlined above.



7. Another way of dealing with matrices is to define each matrix before carrying out operations. To do this, press:

- **MENU** **(menu)**
- **1: Actions** **(1)**
- **1: Define** **(1)**.

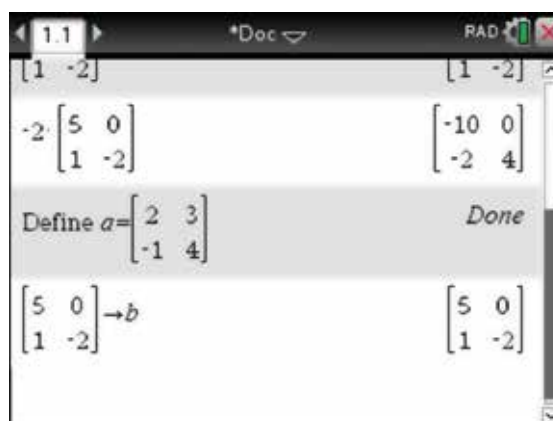


8. Complete the entry line as:

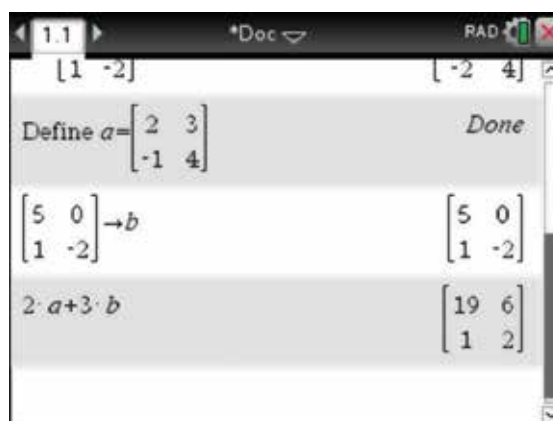
$$\text{Define } a = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix}$$

Press the enter key **(enter)**. This stores the matrix as variable a . Alternatively, you can press the ctrl key **(ctrl)** and **(var)** to store matrix b .

$$b = \begin{bmatrix} 5 & 0 \\ 1 & -2 \end{bmatrix}$$

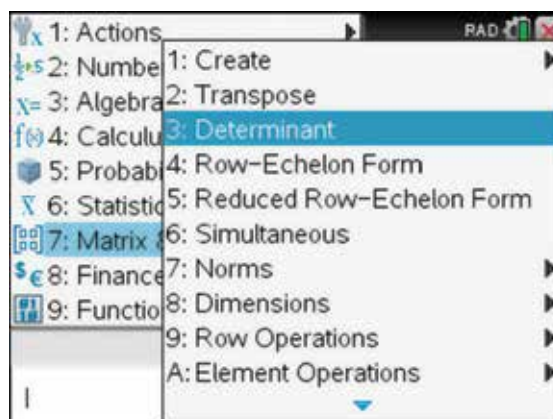


9. The required calculations can now be carried out using the defined variables. For part (c), to calculate $2A+3B$, simply type ' $2a+3b$ ' and press the enter key **(enter)**.



10. To find the determinant of matrix A ($\det A$) for part (d), complete the following steps. Press:

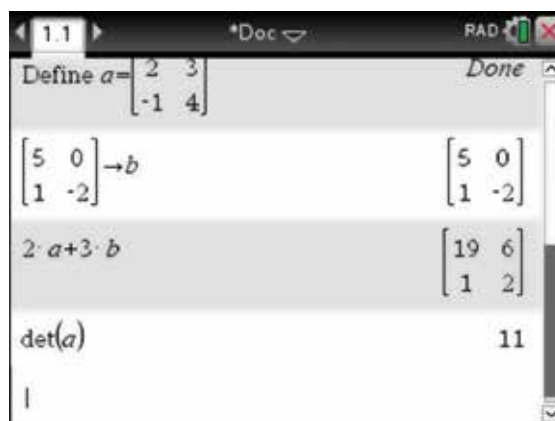
- **MENU** **(menu)**
- **7: Matrix & Vector** **(7)**
- **3: Determinant** **(3)**.



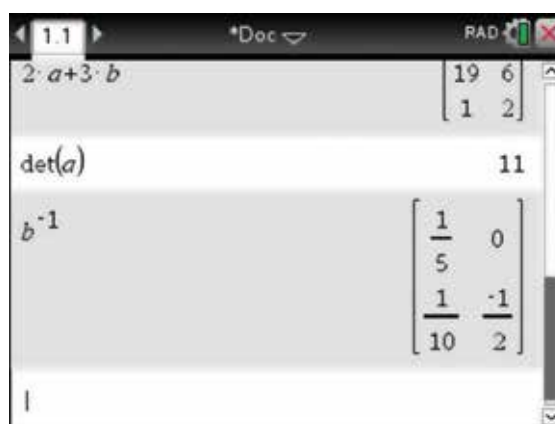
11. Complete the entry line as:

$\det(a)$

Press the enter key **(enter)**.

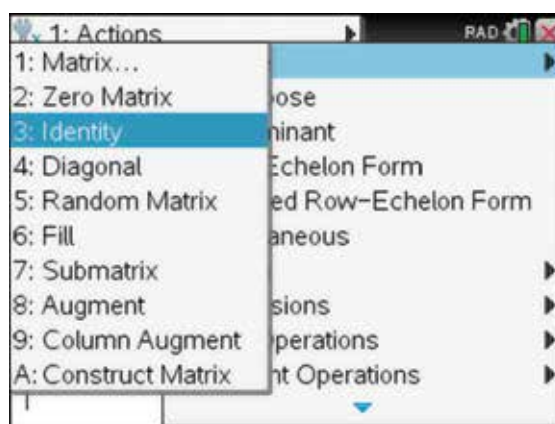


12. For part (e), to find the inverse of matrix B (Inv B), simply type ' b^{-1} ' as shown. Press the enter key **(enter)**.



13. For part (f), to enter a 2×2 identity matrix, press:

- **MENU** **(menu)**
- **7: Matrix & Vector** **(7)**
- **1: Create** **(3)**
- **3: Identity** **(3)**.

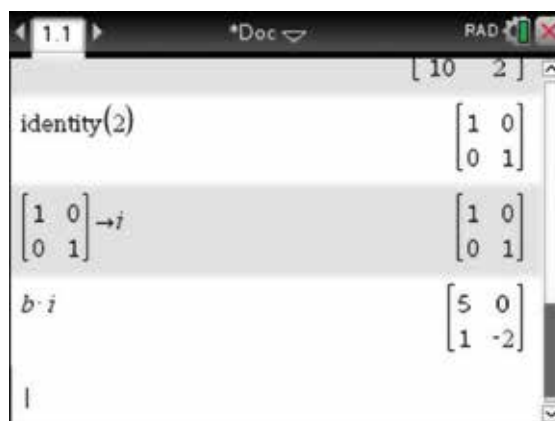


14. Complete as shown:

$\text{identity}(2)$

and store this matrix to i . Now, to find the matrix $B \cdot I$ for part (f), simply type ' $b \cdot i$ ' as shown.

Press the enter key **(enter)**. Note that an identity matrix must be a square matrix, so $\text{identity}(2)$ will give a 2×2 identity matrix. Furthermore, identity matrices satisfy $BI = IB = B$.



14.2 How to solve matrix equations

- To solve matrix equations such as the one below, use the steps that follow. Begin with a new document and add a Calculator page.

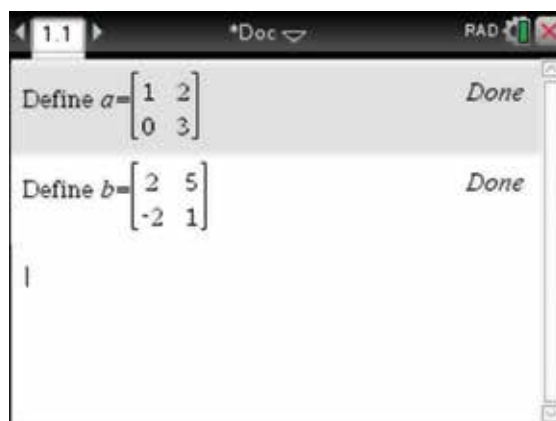
$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 5 \\ -2 & 1 \end{bmatrix}$$

Find X if:

(a) $AX = B$

(b) $XA = B$.

First, define these new matrices as A and B (see step 7 or step 8 in section 14.1).



- For part (a), to find X , we know that:

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

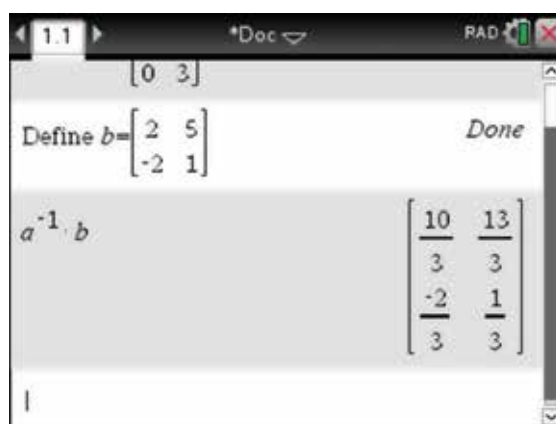
$$X = A^{-1}B.$$

where I is the 2×2 identity matrix.

Complete the entry line as:

$$a^{-1} \times b$$

Press the enter key **enter**.



- For part (b), to find X , we know that:

$$XA = B$$

$$XAA^{-1} = BA^{-1}$$

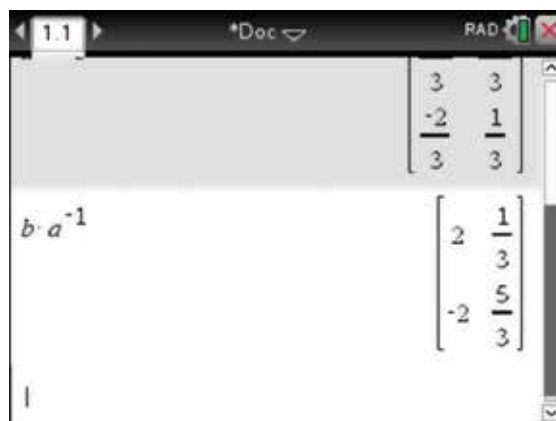
$$XI = BA^{-1}$$

$$X = BA^{-1}.$$

Complete the entry line as:

$$b \times a^{-1}$$

Press the enter key **enter**.



14.3 How to solve linear simultaneous equations using matrices

- To solve the following 2×2 linear simultaneous equations by matrix methods, complete the steps that follow. Begin with a new document and add a Calculator page.

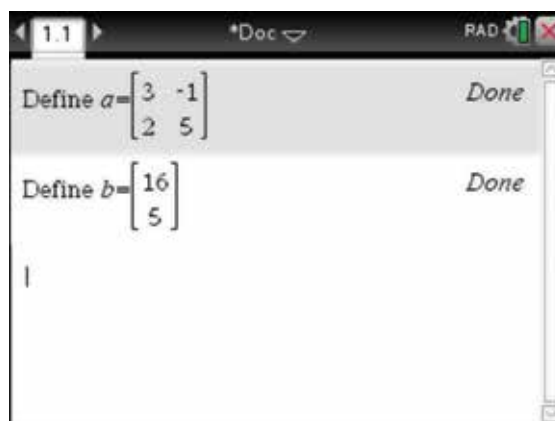
$$3x - y = 16$$

$$2x + 5y = 5$$

Express these equations in matrix form as

$$\begin{bmatrix} 3 & -1 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 16 \\ 5 \end{bmatrix}, \text{ where } AX = B.$$

First, define these new matrices as A and B (see step 7 or 8 in section 14.1).



- We know that if

$$AX = B$$

$$A^{-1}AX = A^{-1}B$$

$$IX = A^{-1}B$$

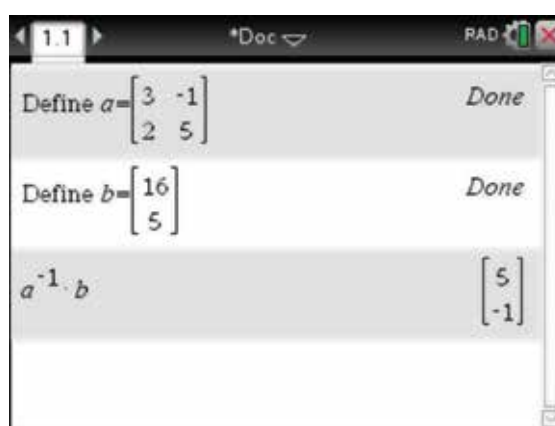
$$X = A^{-1}B.$$

To find X , complete the entry line as:

$$a^{-1} \times b$$

Press the enter key **enter**.

Therefore, $x = 5$ and $y = -1$.



- To solve the following 3×3 linear simultaneous equations by matrix methods, complete the steps that follow.

$$2a + b - 3c = 6$$

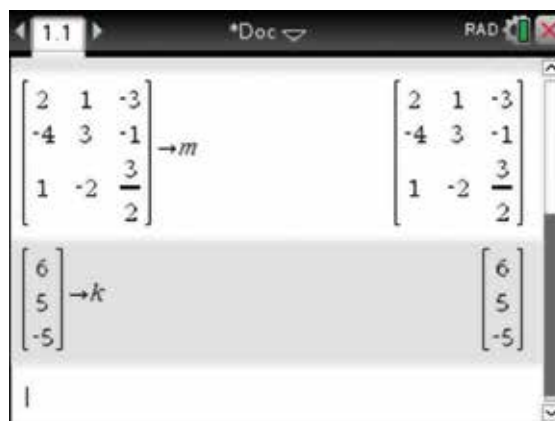
$$-4a + 3b - c = 5$$

$$a - 2b + \frac{3}{2}c = -5$$

Express these equations in matrix form as

$$\begin{bmatrix} 2 & 1 & -3 \\ -4 & 3 & -1 \\ 1 & -2 & \frac{3}{2} \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 6 \\ 5 \\ -5 \end{bmatrix}, \text{ where } MX = K.$$

Define the matrices M and K as shown.

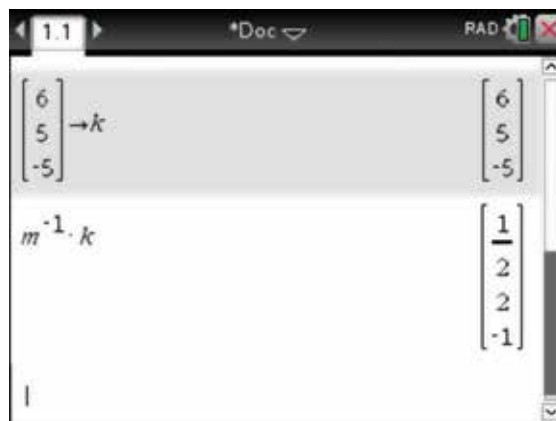


4. To find X , complete the entry line as:

$$m^{-1} \times k$$

Press the enter key **enter**.

Therefore, $a = \frac{1}{2}$, $b = 2$ and $c = -1$.



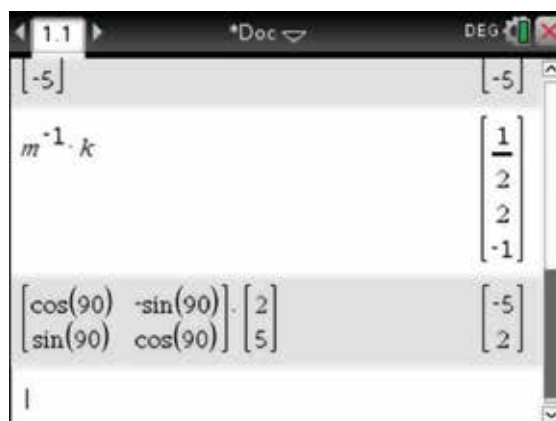
5. Matrices can be used to find the images of points under various transformations. For example:

Find the image of the point $(2, 5)$ following an anticlockwise rotation of 90° about the origin.

Complete the entry line as:

$$\begin{bmatrix} \cos(90) & -\sin(90) \\ \sin(90) & \cos(90) \end{bmatrix} \times \begin{bmatrix} 2 \\ 5 \end{bmatrix}$$

Press the enter key **enter**. The General Settings for the Angle should be in degrees. This can be checked by using the Touchpad to move over the Settings icon; the Angle settings will be shown.



14.4 How to work with vectors

Given the vectors $\underline{a} = 2\underline{i} + 3\underline{j} - \underline{k}$, and $\underline{b} = \underline{i} - 2\underline{j} + 2\underline{k}$, find:

- a unit vector in the direction of \underline{a}
- the scalar resolute of \underline{a} in the direction of \underline{b}
- the vector resolute of \underline{a} in the direction of \underline{b}
- the angle in degrees and minutes between the vectors \underline{a} and \underline{b} .

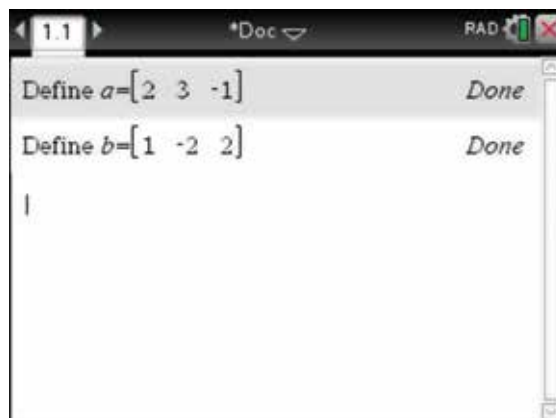
1. Open a new document and add a Calculator page. To define the vectors, press:

- MENU** **menu**
- 1: Actions** **1**
- 1: Define** **1**.

Complete the entry line as:

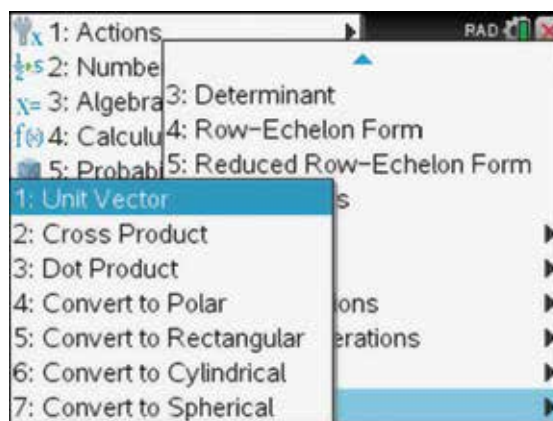
Define $a = [2, 3, -1]$

Press the enter key **enter**. You must put commas between the coefficients of the vectors in the square brackets. The results will be as shown when you press the enter key **enter**. The defined variable will appear bold at first. Repeat this process for vector \underline{b} .



2. For part (a), to find a unit vector in the direction of \underline{a} , press:

- **MENU** 
- **7: Matrix & Vector** 
- **C: Vector** 
- **1: Unit Vector** .

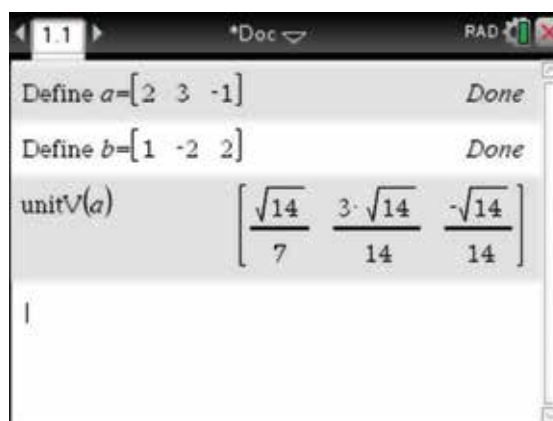


3. Complete the entry line as:

`unitV(a)`

Press the enter key .

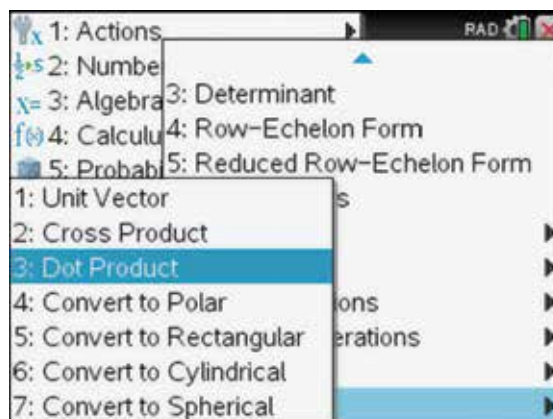
$$\hat{a} = \frac{\sqrt{14}}{7}\underline{i} + \frac{3\sqrt{14}}{14}\underline{j} - \frac{\sqrt{14}}{14}\underline{k}$$



4. For part (b), to find the scalar resolute of \underline{a} in the direction of \underline{b} , we need the dot product of \underline{a} and $\hat{\underline{b}}$.

To get this, press:

- **MENU** 
- **7: Matrix & Vector** 
- **C: Vector** 
- **3: Dot Product** .

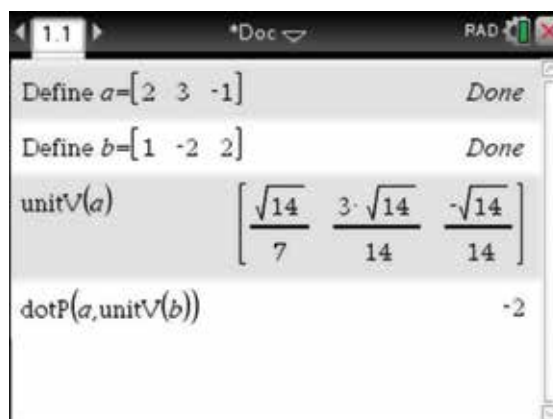


5. Complete the entry line as:

`dotP(a,unitV(b))`

Press the enter key .

$$\underline{a} \cdot \hat{\underline{b}} = -2$$

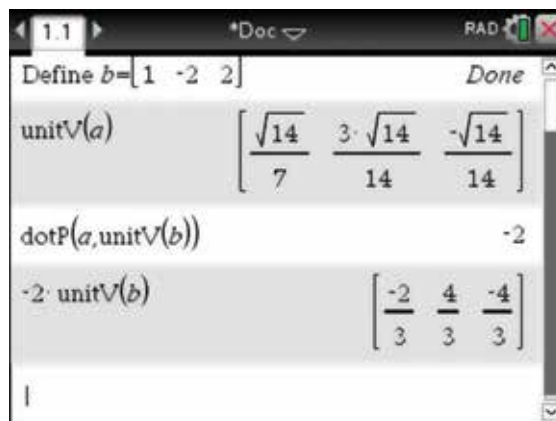


6. For part (c), to find the vector resolute of a in the direction of b . Complete the entry line as:

$$-2 \times \text{unitV}(b)$$

Press the enter key **enter**.

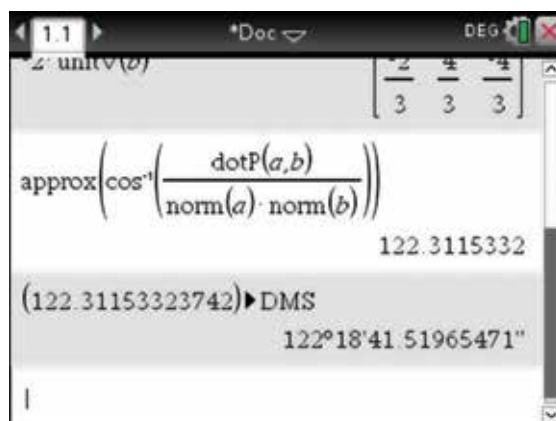
$$(a \cdot \hat{b})\hat{b} = \frac{1}{3}(-2\hat{i} + 4\hat{j} - 4\hat{k})$$



7. For part (d), to find the angle between the vectors a and b , complete the entry line as:

$$\cos^{-1}\left(\frac{\text{dotP}(a,b)}{\text{norm}(a) \cdot \text{norm}(b)}\right)$$

Note that norm(a) is used to find the length of a vector. We have also used the approx command to get an approximate answer, and the angle setting is in Degrees mode. The angle between the vectors is $122^\circ 19'$.



CHAPTER 15



Differential calculus

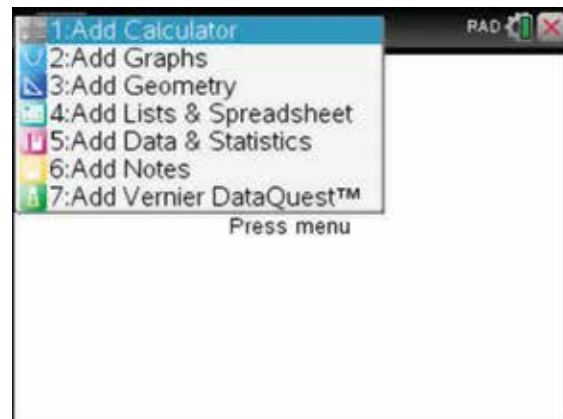
This chapter deals with limits and derivatives by first principles, by rules and applications.

15.1 How to use a spreadsheet to demonstrate the limit of a function




1. To investigate the limit of a function using a spreadsheet, we will find $\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}$ as x approaches 1 from the left and from the right.

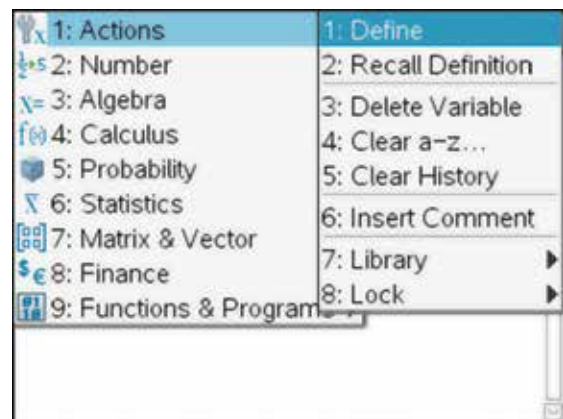
Start with a new document and add a Calculator page.
To do this, press:

- **MENU** 
- **1: Add Calculator** .



2. Press:

- **MENU** 
- **1: Actions** 
- **1: Define** .





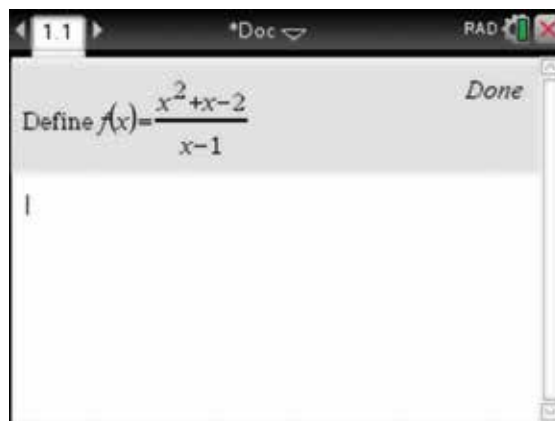
3. Complete the entry line as:

$$\text{Define } f(x) = \frac{x^2 + x - 2}{x - 1}$$

Press:

- **ENTER** .

Note: Use the fraction template by pressing the ctrl key  and .



4. Now add a Lists & Spreadsheet page to the document. To do this, press:



- **HOME** .

Select the Lists & Spreadsheet icon, then press:

- **ENTER** .



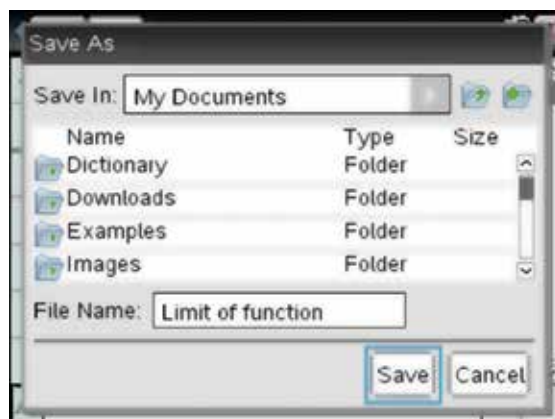
5. To save and name this document, press:

- **ctrl** .
- **save** .

Type in the file name as 'limit of function'.

Tab  to Save, then press:

- **ENTER** .



6. Label the columns as indicated below.

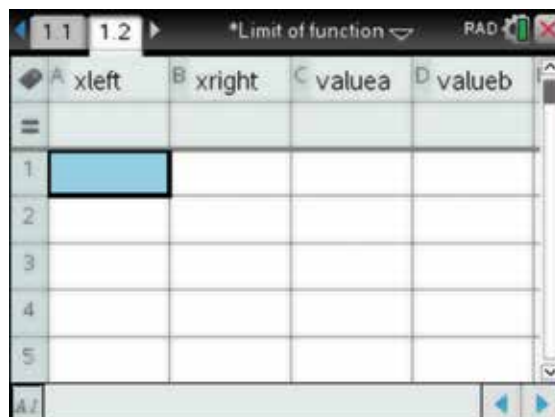
Label column A as xleft.

Label column B as xright.

Label column C as valuea.

Label column D as valueb.

To do this, simply type the labels into the cells as shown.



7. Enter the data values approaching $x = 1$ in xleft and xright as shown.

	xleft	xright	valuea	valueb
1	0.99	1.01		
2	0.999	1.001		
3	0.9999	1.0001		
4	0.99999	1.00001		
5				

8. Using the Touchpad, move the cursor to the top of cells A and B and select both these columns. Now press:

- **ctrl** (ctrl)
- **MENU** (menu)
- **A: Color** (A)
- **2: Fill Color** (2).

	xleft	xright	valuea	valueb
1	0.99	1.01		
2	0.999	1.001		
3	0.9999	1.0001		
4	0.99999	1.00001		
5				

9. Using the Touchpad, choose a colour to fill the background of the cells. Leave the text colour as black.

	xleft	xright	valuea	valueb
1	0.99	1.01		
2	0.999	1.001		
3	0.9999	1.0001		
4	0.99999	1.00001		
5				

10. In the header cell of column C, labelled valuea, complete the rule for the cells as $\text{valuea} := f(\text{xleft})$.

Press the enter key (enter).

	xleft	xright	valuea	valueb
1	0.99	1.01		
2	0.999	1.001		
3	0.9999	1.0001		
4	0.99999	1.00001		
5				

11. In the header cell of column D, labelled valueb, complete the rule for the cells as $\text{valueb} := f(x_{\text{right}})$.

Using the method described above in step 8, colour the background of the columns C and D in another colour.

	A xleft	B xright	C valuea	D valueb
=			=f(xleft)	=f(xright)
1	0.99	1.01	2.99	
2	0.999	1.001	2.999	
3	0.9999	1.0001	2.9999	
4	0.99999	1.00001	2.99999	
5				

Formula bar: valueb = f(xright)

12. Notice that, as x approaches 1 from the left and from the right, the value of the function approaches 3.

Resave the document by pressing:

- **ctrl**
- **S**

Note: The asterisk in front of the file name no longer appears if a file is saved and no further changes are made.

	A xleft	B xright	C valuea	D valueb
=			=f(xleft)	=f(xright)
1	0.99	1.01	2.99	3.01
2	0.999	1.001	2.999	3.001
3	0.9999	1.0001	2.9999	3.0001
4	0.99999	1.00001	2.99999	3.00001
5				

Formula bar: valueb = f(xright)

15.2 How to calculate the value of a limit

1. Continue working with the document used in section 15.1. Go back to the Calculator page by pressing:

- **ctrl**
- **left arrow**


Define $f(x) = \frac{x^2 + x - 2}{x - 1}$ Done

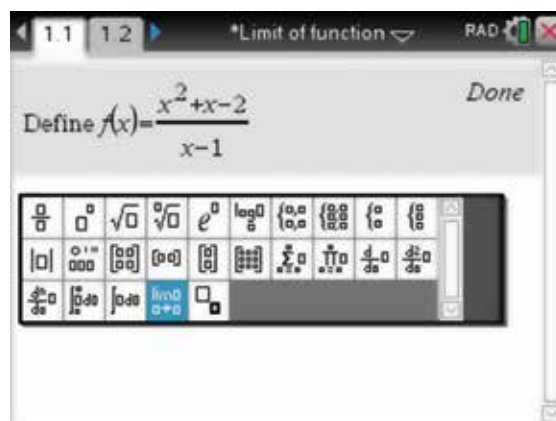
2. To calculate the limit of the function below, complete the steps that follow.


$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1}$$

Use the limit template from the Maths Expression palette by pressing:

- .

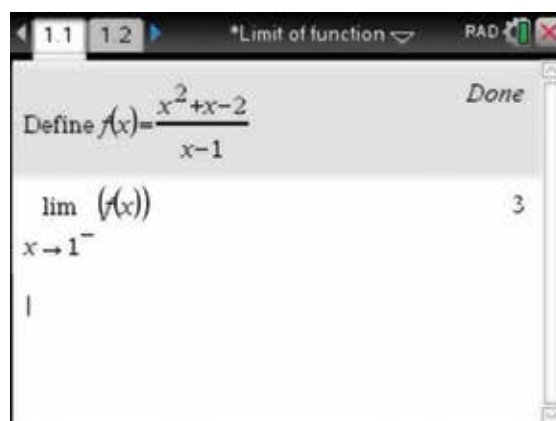
Select the template highlighted on the screen opposite, then press the enter key .



3. Type in the values required. Press the tab key  to move between fields. To find the limit as x approaches 1 from the left, complete the entry line as:

$$\lim_{x \rightarrow 1^-} f(x)$$

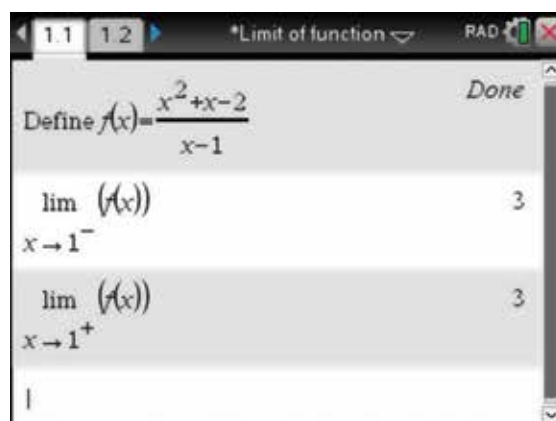
Press the enter key  to find the limit.



4. To find the limit as x approaches 1 from the right, complete the entry line as:

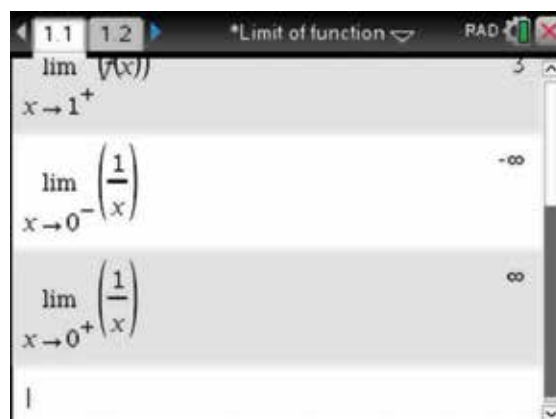
$$\lim_{x \rightarrow 1^+} f(x)$$

Press the enter key  to find the limit.





5. These ideas are especially useful in cases such as the one shown below.

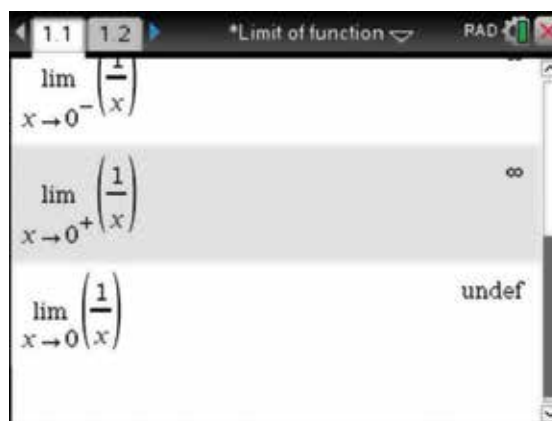
Find $\lim_{x \rightarrow 0} \left(\frac{1}{x} \right)$ from the left and from the right as described above.



6. If the + or – is omitted, the limit at $x = 0$ is calculated.

The actual limit for this function at $x = 0$ does not exist, as the left and right limits are not the same. Therefore, the calculator returns an answer of undefined. To resave the document, press:

- **ctrl** .
- **save** .



15.3 How to differentiate from first principles

Find the gradient of $f(x) = x^2 + 1$ at the point (1, 2) by first finding the gradients of the secants that join the x -values listed below.

$x = 1$ and $x = 1.1$


$x = 1$ and $x = 1.01$

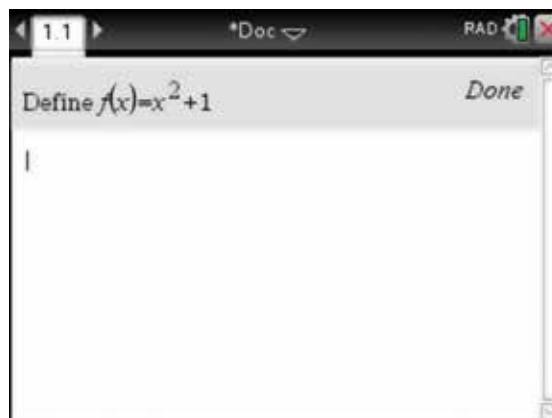
$x = 1$ and $x = 1.001$

$x = 1$ and $x = 1.0001$

$x = 1$ and $x = 1.00001$

1. Start with a new document and add a Calculator page, then define the function $f(x) = x^2 + 1$.

Complete the entry line as shown and press the enter key . Then add a Lists & Spreadsheet page (see section 15.1).



2. Label column A as hvalue and label column B as gradient, then enter the values of the difference as shown on the screen.

	A hvalue	B gradient	C	D
=				
1	0.1			
2	0.01			
3	0.001			
4	0.0001			
5	0.00001			
B	gradient			

3. We know that the gradient at the point at $x = 1$ is equal to $\frac{f(1+h) - f(1)}{h}$.

In the header (grey) cell of column B, labelled gradient, complete the rule for the cells as

$$\text{gradient} := \frac{f(1+\text{hvalue}) - f(1)}{\text{hvalue}}$$

Note: Use the fraction template by pressing the ctrl key $\text{\textcircled{ctrl}}$ and $\text{\textcircled{\div}}$.

	A	B	C	D
	hvalue	gradient		
=		$\frac{f(1+\text{hvalue}) - f(1)}{\text{hvalue}}$		
1	0.1			
2	0.01			
3	0.001			
4	0.0001			

Formula bar: $\text{gradient} = \frac{f(1+\text{hvalue}) - f(1)}{\text{hvalue}}$

4. Press the enter key $\text{\textcircled{enter}}$.

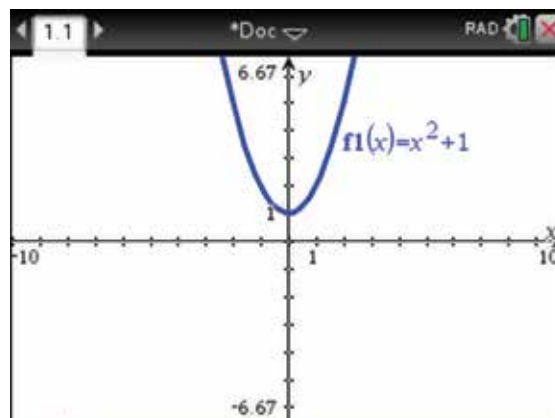
For the two selected background columns, set the fill colour as described in section 15.1 step 8.

Notice that as h (hvalue) $\rightarrow 0$, the gradient $\rightarrow 2$.





	A	B	C	D
	hvalue	gradient		
=		$=f(1+\text{hvalue}) - f(1)$		
1	0.1	2.1		
2	0.01	2.01		
3	0.001	2.001		
4	0.0001	2.0001		
5	0.00001	2.00001		

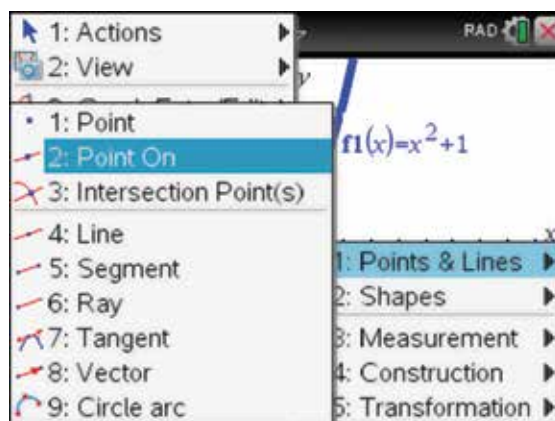
15.4 How to draw a tangent to a curve and calculate its gradient







1. Start with a new document and add a Graphs page. To draw the graph of $f(x) = x^2 + 1$, type the equation into the function entry line. Press the enter key $\text{\textcircled{enter}}$.

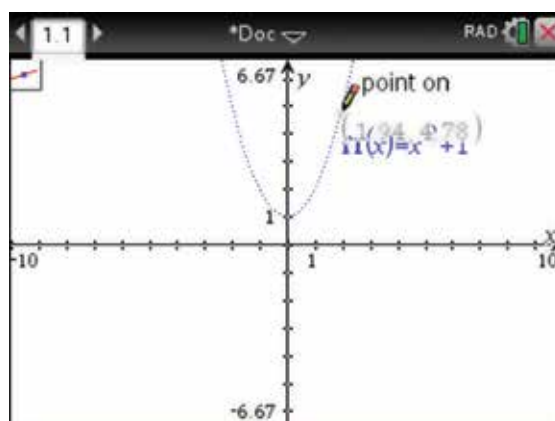



2. To draw the equation of the tangent to the curve $f(x) = x^2 + 1$ at $x = 1$, press:

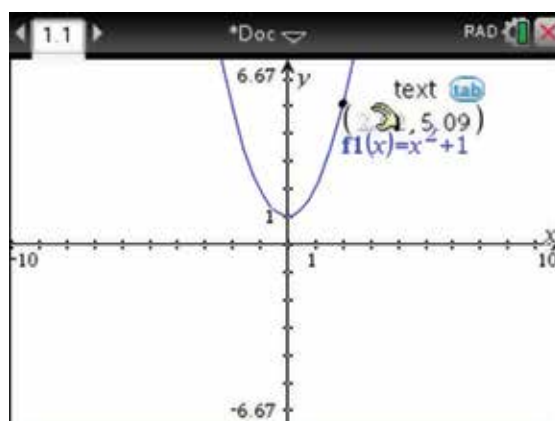
- **MENU** 
- **8: Geometry** 
- **1: Points & Lines** 
- **2: Point On** .





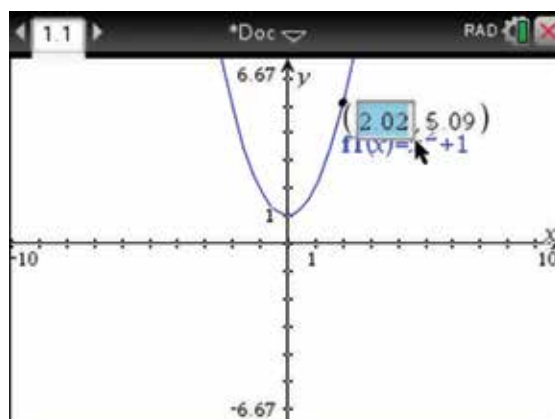
3. Use the Touchpad to move the pointer  to any point on the graph. When the point is over the graph, the graph will appear bold and the pointer will change to the pointing finger . Press the click key . A pencil  will appear with the coordinates of the point displayed in grey. Press the click key  twice to anchor the coordinates. Press the esc key  to stop adding points to the graph.



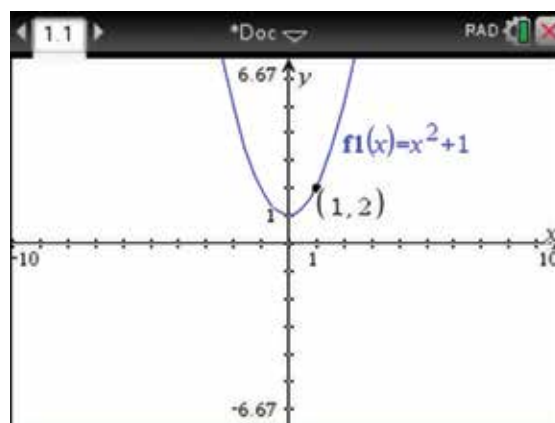
4. To specify the x -coordinate at the point of tangency, move the pointer over the x -coordinate shown. An open hand  will appear.



5. Press the click key  twice. The x -coordinate will be displayed in a text box ready to edit. Delete the existing value and type the number 1, then press the enter key .

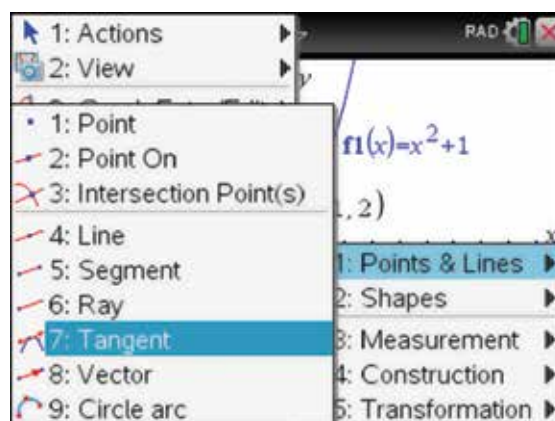


6. The new coordinates (1, 2) will be displayed in the appropriate position.

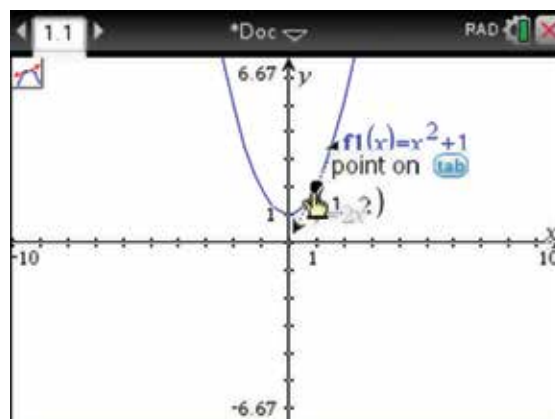


7. To draw the tangent to the curve at this point, complete the following steps. Press:

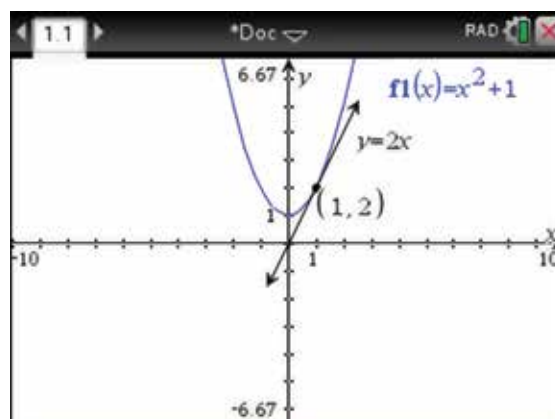
- **MENU** **menu**
- **8: Geometry** **8**
- **1: Points & Lines** **1**
- **7: Tangent** **7**.



8. Use the Touchpad to move the pointer to the point (1, 2) on the graph. A dotted tangent line will appear on the graph with the pointing finger. Press the enter key **enter** to anchor the tangent to the work area. The tangent line will now appear as a solid line.



9. The equation of the tangent will also appear.



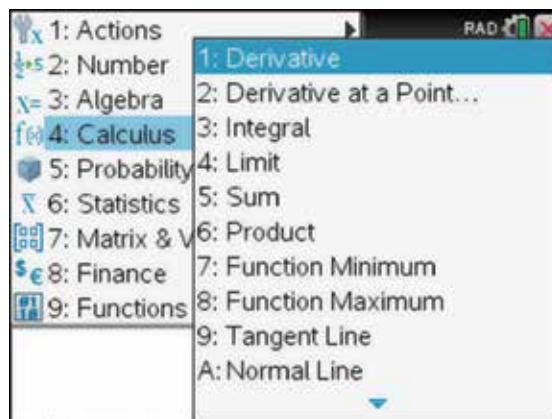
15.5 How to find the derivative by rule

1. Start with a new document and add a Calculator page. To differentiate each of these functions below, complete the steps that follow.

- (a) $y = x^8$
 (b) $y = 3x^2$
 (c) $y = 2x^5 + \frac{3}{5}x^2 - 6x$

Press:

- **MENU** **menu**
- **4: Calculus** **4**
- **1: Derivative** **1**.

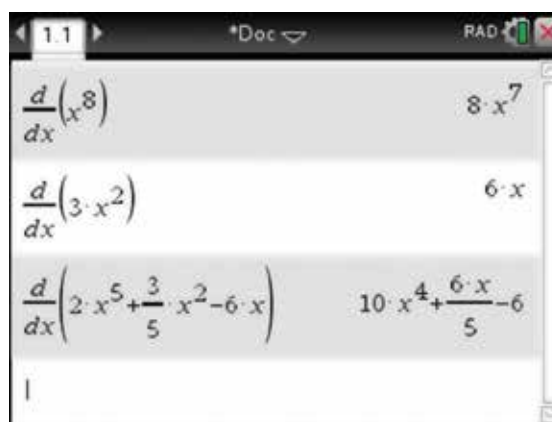


2. To answer part (a), complete the entry line as:

$$\frac{d}{dx}(x^8)$$

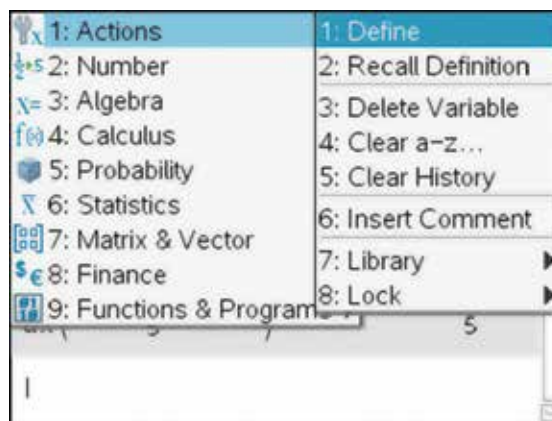
Press the enter key **enter**.

Repeat this process for parts (b) and (c).



3. To differentiate a function such as $f(x) = 6\sqrt{x}$ using the Define application, press:

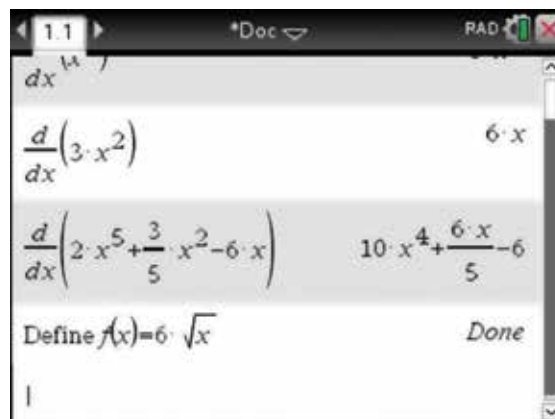
- **MENU** **menu**
- **1: Actions** **1**
- **1: Define** **1**.



4. Complete the entry line as:

Define $f(x) = 6 \times \sqrt{x}$

Press the enter key **enter**.



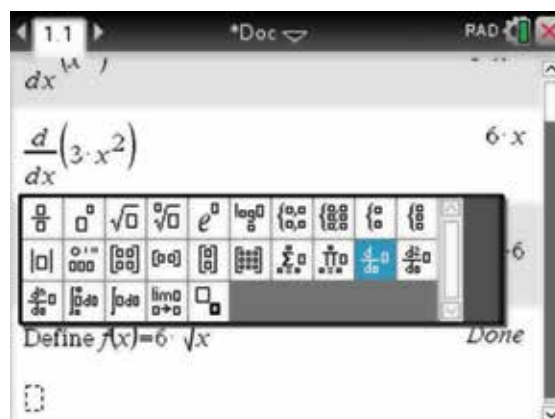
5. Alternatively the derivative template can be found in the Maths Expression palette.

Press:

-

Select the template highlighted and press:

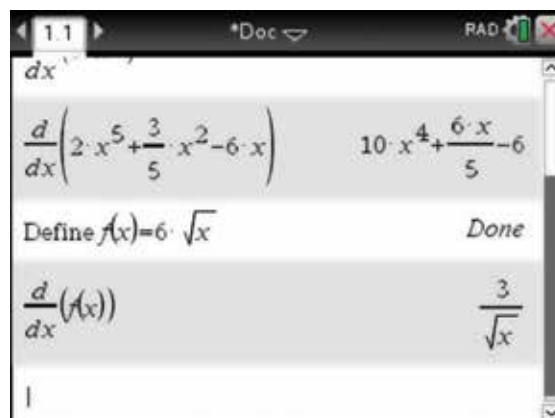
- **ENTER** **enter**.



6. Complete the entry line as:

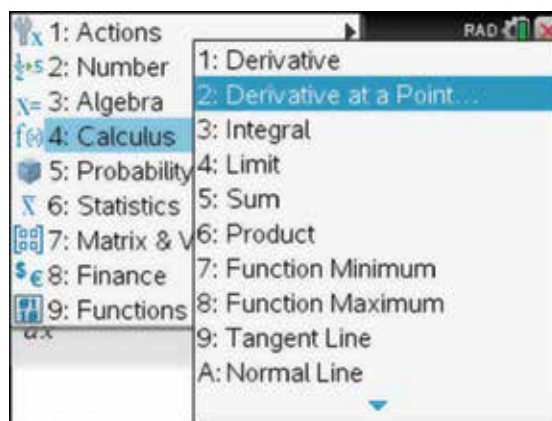
$\frac{d}{dx}(f(x))$

Press the enter key **enter**.

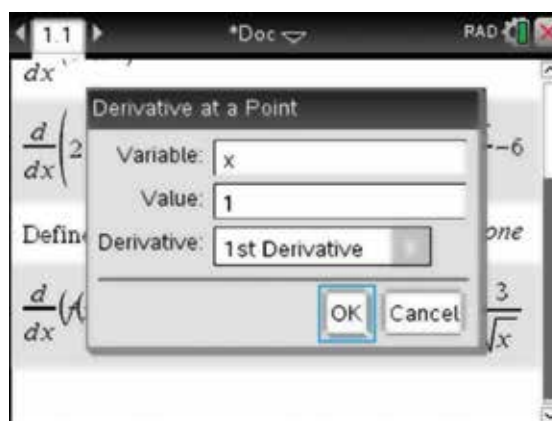


7. To find the gradient of $f(x) = 6\sqrt{x}$ at $x = 1$, press:

- **MENU** **(menu)**
- **4: Calculus** **(4)**
- **2: Derivative at a Point** **(2)**.



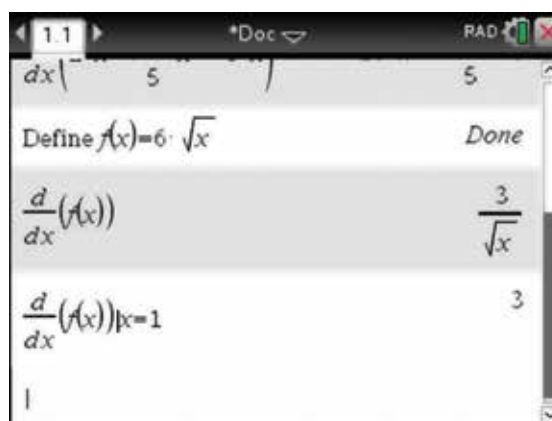
8. Type in the values as shown, tab **(tab)** to OK and press the enter key **(enter)**.



9. Complete the entry line as shown

$$\frac{d}{dx}(f(x))|_{x=1}$$

Press the enter key **(enter)**.

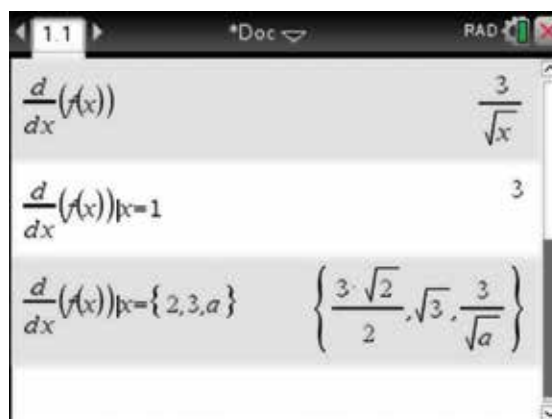


10. To find the gradient of $f(x) = 6\sqrt{x}$ at multiple points such as $x = \{2, 3, a\}$, complete the entry line as:

$$\frac{d}{dx}(f(x))|_{x=\{2,3,a\}}$$

Press the enter key **(enter)**.

Note: Another shortcut key to find the derivative template is to press the shift key **(shift)** and **(-)**.

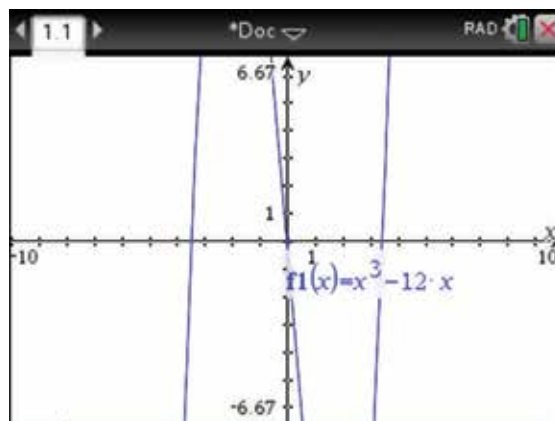


15.6 How to sketch derivatives

1. To sketch the graphs of $f(x) = x^3 - 12x$ and its gradient function, or $f'(x)$, on the one set of axes, complete the steps that follow. Start with a new document and add a Graphs page. Complete the function entry line as:

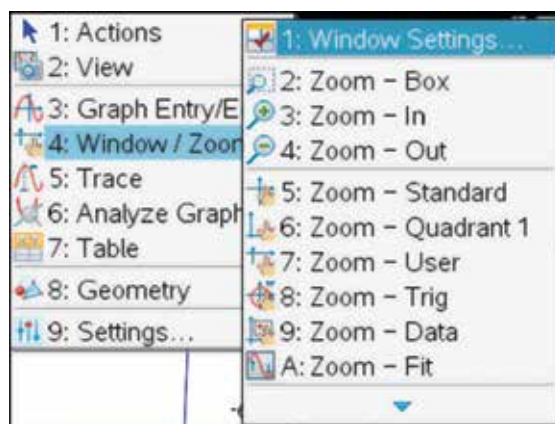
$$f1(x) = x^3 - 12x$$

Press the enter key **enter**.

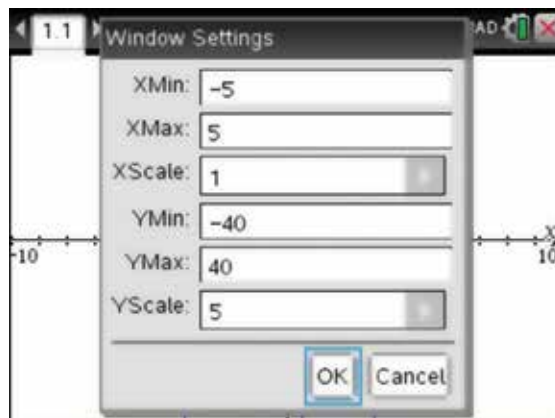


2. To change the window settings, press:


- **MENU** **menu**
- **4: Window / Zoom** **4**
- **1: Window Settings** **1**.



3. Enter the values shown using the tab key **tab** to move between fields. Select OK, then press the enter key **enter**.



4. To return to the function entry line, press:

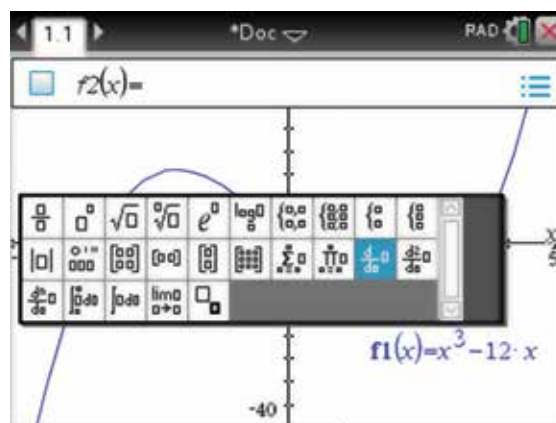
- **tab** .

To draw the derivative graph, $f'(x)$, we need the derivative template. Press:

- **catalog** .

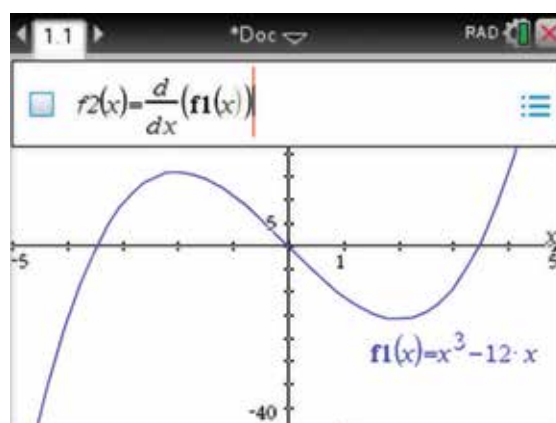
Select the template highlighted, then press:


- **ENTER** .

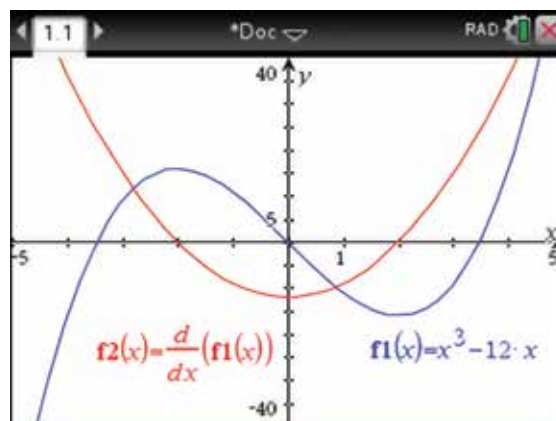


5. Complete the function entry line as:

$$f2(x) = \frac{d}{dx}(f1(x))$$



6. Press the enter key , and the graph of the gradient or the derivative is now displayed.



15.7 How to calculate implicit differentiation

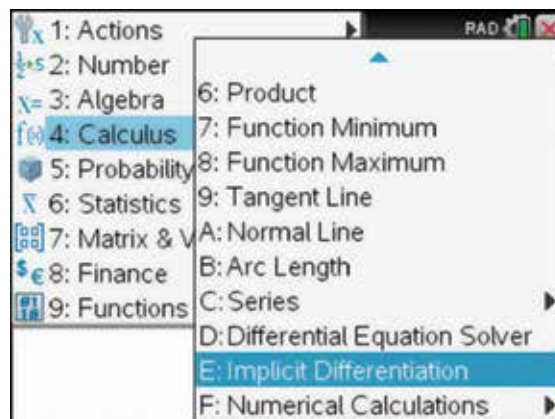
1. To use implicit differentiation to find $\frac{dy}{dx}$ for the equations below, complete the steps that follow on a new Calculator page.

(a) $2x + y = 4$

(b) $9x^2 + 36x + 4y^2 - 8y + 4 = 0$

Press:

- **MENU** **menu**
- **4: Calculus** **4**
- **E: Implicit Differentiation** **E**.



2. (a) Complete the entry line as:

`impDif(2·x+y=4,x,y)`

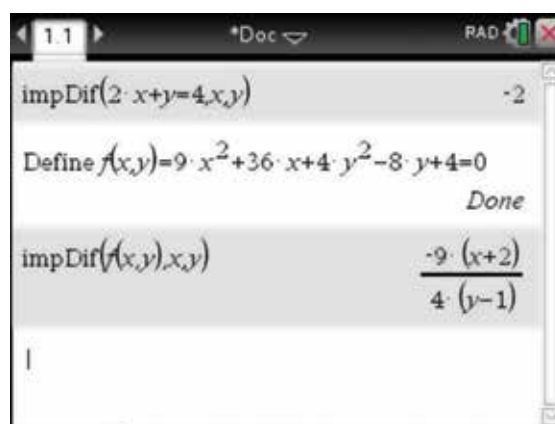
Press the enter key **enter**.

- (b) Complete the entry line as:

Define $f(x,y) = 9x^2 + 36x + 4y^2 - 8y + 4 = 0$

`impDif(f(x,y),x,y)`

Press the enter key **enter**.



CHAPTER 16

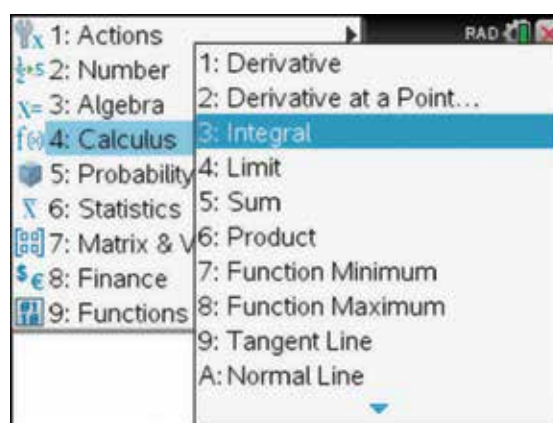
Integral calculus

This chapter deals with indefinite integrals, definite integrals, graphing anti-derivatives, finding the area under a curve and differential equations.

16.1 How to find indefinite integrals

1. Open a new document and add a Calculator page (see section 2.1). To find the antiderivative of $5x^3$, press:

- **MENU** **menu**
- **4: Calculus** **4**
- **3: Integral** **3**.



2. Notice that there are provisions for lower and upper bounds. These are not necessary in this indefinite integral, so use the tab key **tab** to move into the required field and enter the rule.



3. Complete the entry line as:

$$\int 5x^3 dx$$

Press the enter key **(enter)**. Do not type in the brackets, as they will be inserted automatically.

Note: The answer given is one of an infinite number of antiderivatives that can be obtained. The answer would normally be written as $\int 5x^3 dx = \frac{5x^4}{4} + c$.

4. To antidifferentiate $4t^2 + 5t - 7$ with respect to t , complete the entry line as:

$$\int (4t^2 + 5t - 7) dt$$

Press the enter key **(enter)**.

To antidifferentiate $s(4s + 5)^2$ with respect to s , complete the entry line as:

$$\int (s(4s + 5)^2) ds$$

Press the enter key **(enter)**.

To antidifferentiate $d^2 - 3d + 1$ with respect to d , complete the entry line as:

$$\int (d^2 - 3d + 1) dd$$

Press the enter key **(enter)**.

16.2 How to find definite integrals

1. The template for the definite integral can be found using the menu as shown in section 16.1 or by using the Maths Expression palette. Press:



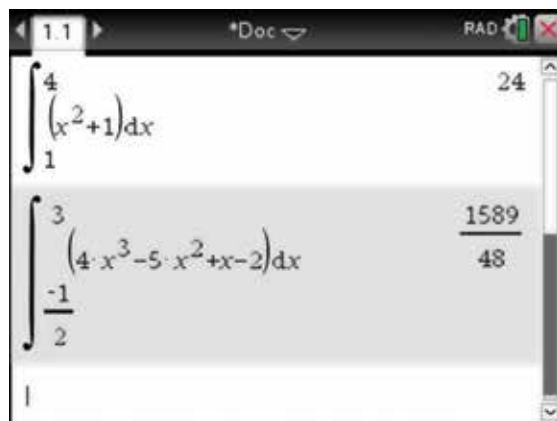
Select the template highlighted in the screen opposite, then press the enter key **(enter)**.

2. (a) To find:

$$\int_1^4 (x^2 + 1) dx$$

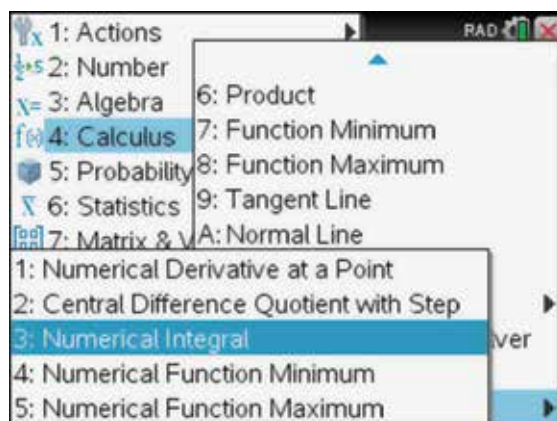
complete the entry line as shown, then press the enter key **enter**.

(b) Repeat this process for $\int_{-\frac{1}{2}}^3 (4x^3 - 5x^2 + x - 2) dx$.



3. A numerical integral can also be found by pressing:

- **MENU** **menu**
- **4: Calculus** **4**
- **F: Numerical Calculations** **F**
- **3: Numerical Integral** **3**.



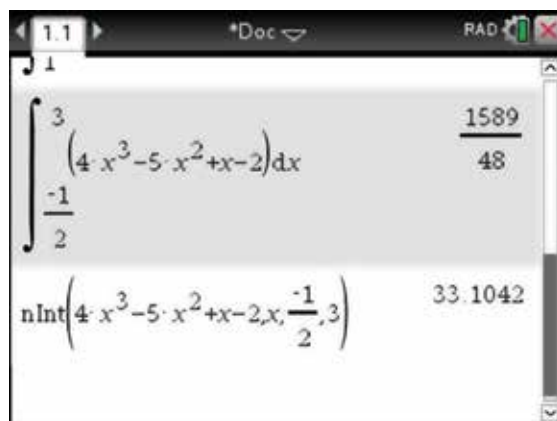
4. To find a numerical value for

$$\int_{-\frac{1}{2}}^3 (4x^3 - 5x^2 + x - 2) dx, \text{ complete the entry line as:}$$

$$\text{nInt}\left(4x^3 - 5x^2 + x - 2, x, \frac{-1}{2}, 3\right)$$

Press the enter key **enter**.

Note: For another shortcut key to find the integral template, press the shift key **shift** and **+**.



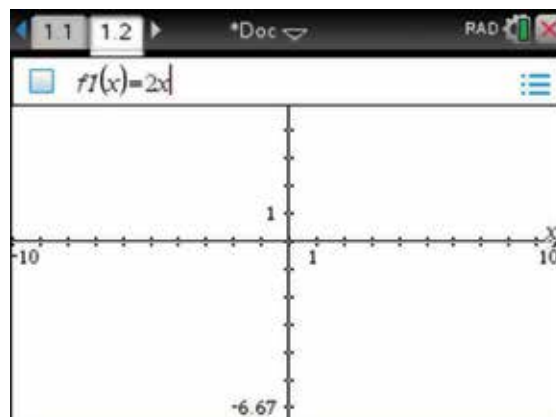
16.3 How to graph a possible original function from a gradient function

1. Open a new document and add a Graphs page (see section 3.4). To sketch a possible graph for $f(x)$ if $f'(x) = 2x$, complete the steps that follow.

Complete the function entry line as:

$$f1(x) = 2x$$

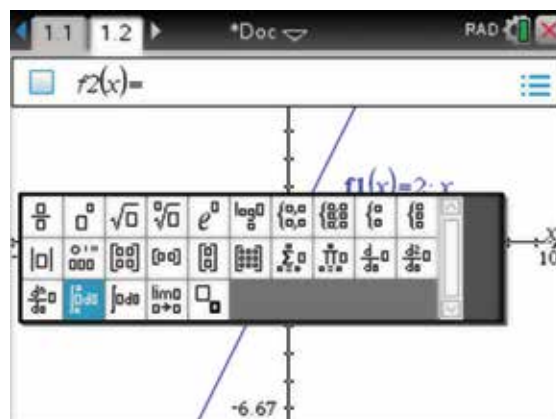
Press the enter key **enter**.



2. Press the tab key **tab** to get back to the function entry line. Access the Maths Expression palette by pressing:

-

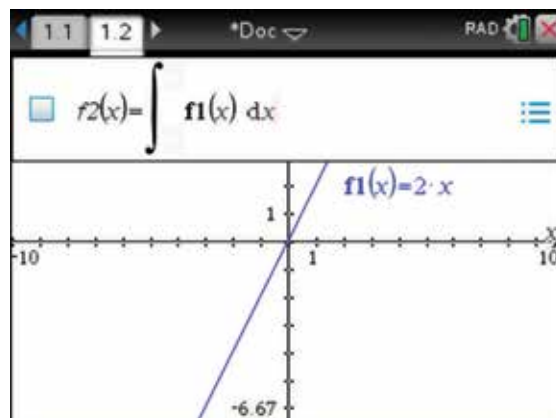
Select the template highlighted in the screen opposite, then press the enter key **enter**.



3. Complete the function entry line as:

$$f2(x) = \int f1(x) dx$$

Press the enter key **enter**.

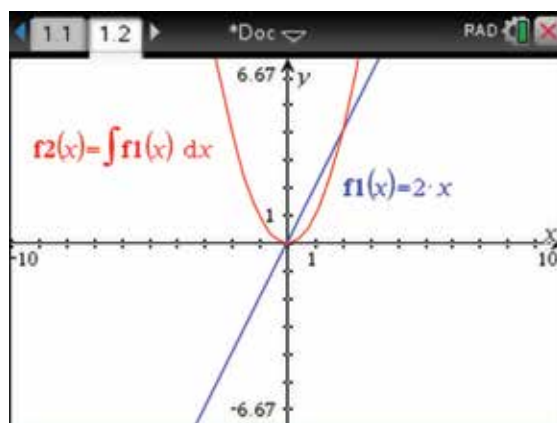



4. This is one of an infinite number of possible graphs of an antiderivative function of $f'(x)$.

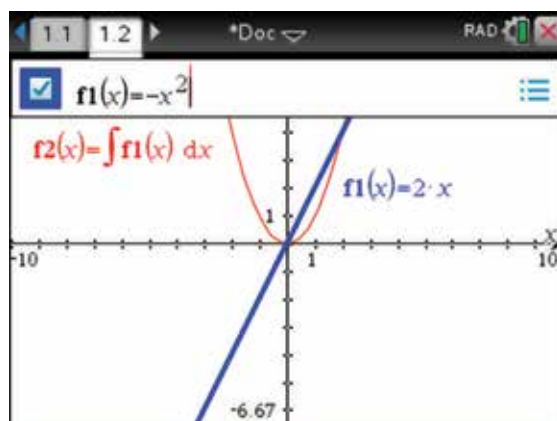
Remember that $f1(x) = f'(x) = 2x$ and


$$f2(x) = f(x).$$

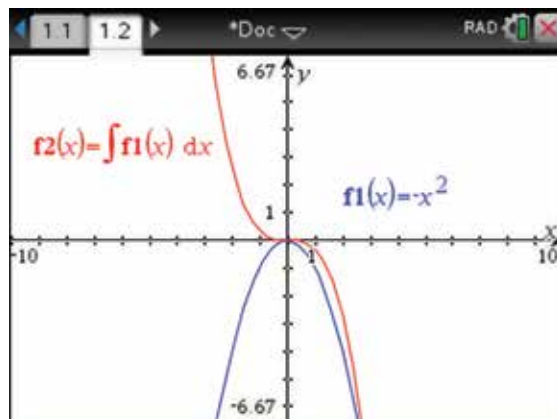
Note: You may need to move the labels to get a better view of the graph.



5. To change the rule for $f(x)$ and to see the antiderivative of $f1(x)$ change, complete the steps that follow. Move the pointer until it is over $f1(x)$. An open hand icon appears. Then press the click key  twice. Edit the equation to $f1(x) = -x^2$.



6. Now when we press the enter key , both graphs change to reflect the changes in $f1(x)$.

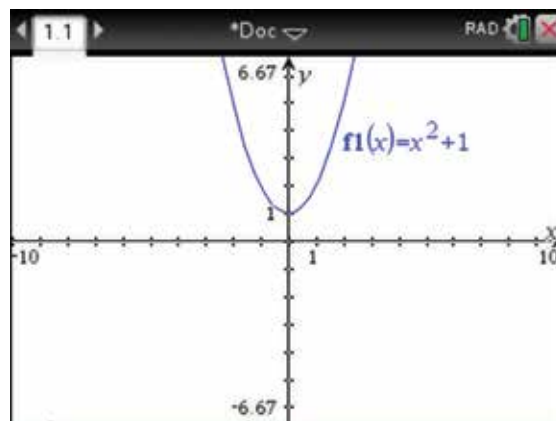


16.4 How to calculate the area under the curve

1. To find the area bounded by the curve $y = x^2 + 1$, the x -axis and the lines $x = 1$ and $x = 4$, complete the steps that follow. Open a new document and add a Graphs page. Complete the function entry line as:

$$f1(x) = x^2 + 1$$

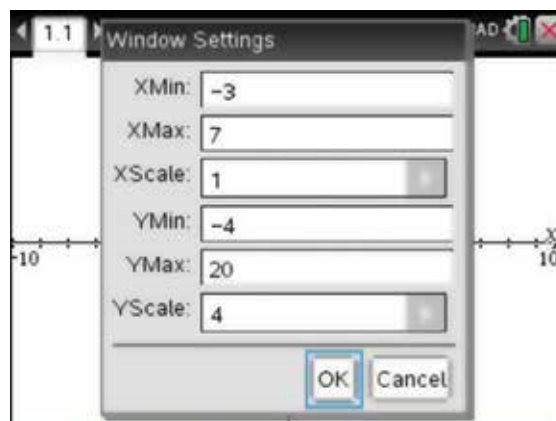
Press the enter key **enter**.



2. To get a clear view of the area being calculated, press:




- **MENU** **menu**
- **4: Window/Zoom** **4**
- **1: Window Settings** **1**.

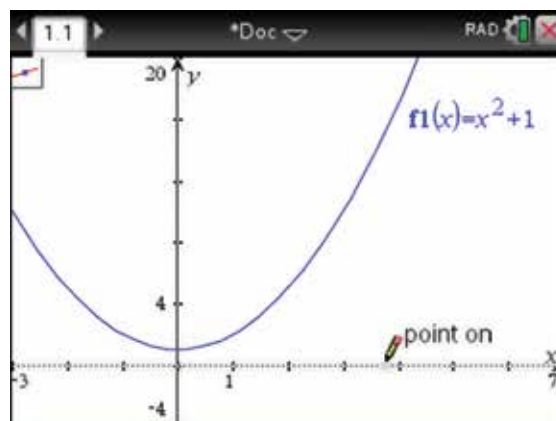
Complete the window settings as shown, then select OK and press the enter key **enter**.






3. Place two points on the x -axis. Try to place them close to $x = 1$ and $x = 4$. To do this, press:




- **MENU** **menu**
- **8: Geometry** **8**
- **1: Points & Lines** **1**
- **2: Point On** **2**.

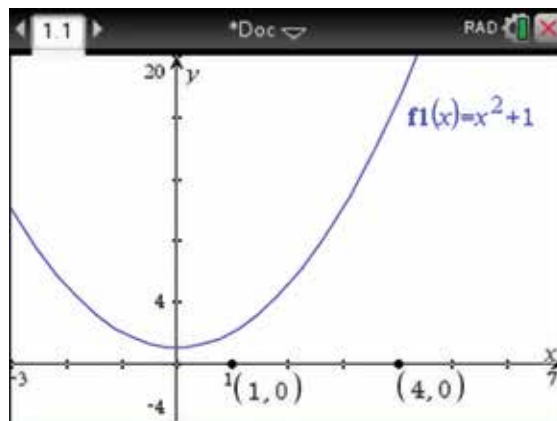
The pointer  will change to a pointing finger  as it is moved over the x -axis. When you are satisfied with the location of the point $x = 1$, press the enter key **enter**. The pencil  and the point will appear. Press the enter key **enter** to anchor the point to the x -axis. Repeat these steps for the point at $x = 4$. Press escape **esc** to stop adding more points.






4. To label the points with their coordinates, press:

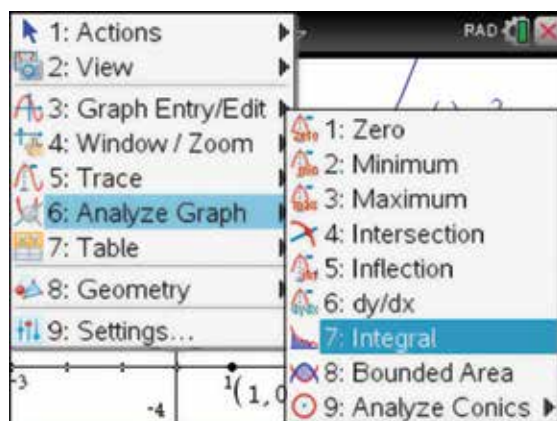
- **MENU** 
- **1: Actions** 
- **8: Coordinates and Equations** .



Move the pointer  over the point. A pointing finger  appears with the word 'point' and the coordinates dimmed. Press the **CLICK** key  twice; the coordinates of the points will appear. Repeat for the other point. The coordinates may be moved for a better view. If the coordinates are not (1, 0) and (4, 0), they can be edited in the same way that the rule was changed for the function in section 16.3.

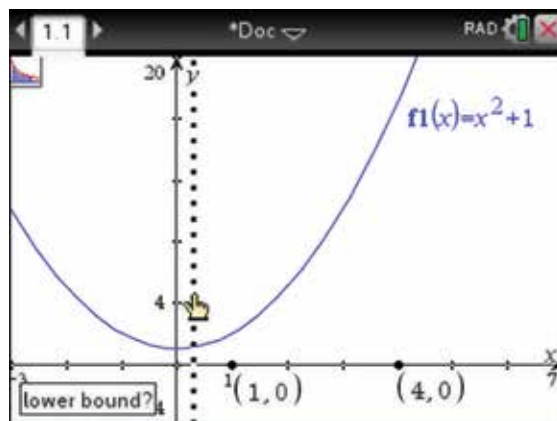




5. To shade and find the required area, press:


- **MENU** 
- **6: Analyze Graph** 
- **7: Integral** .






6. With the lower bound prompt, move the pointing finger  so that the dotted vertical line is over the point (1, 0) and press the enter key .

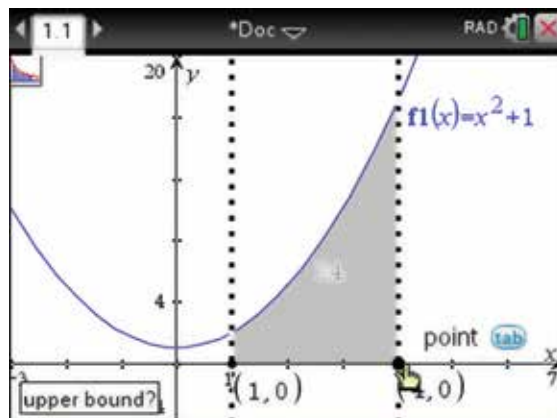


7. With the upper bound prompt, move the pointing finger  so that the dotted vertical line is over the point (4, 0) and press the enter key .

It is also possible to colour the shaded area. To do this, move the pointer over the integral until the open hand  appears with the label 'integral'. Then press

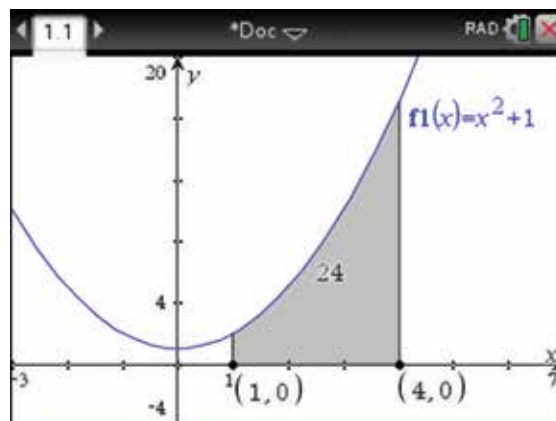
- **ctrl** 
- **MENU** 
- **B: Color** 

and choose the line and fill colours.



8. The shaded area appears; in this case, it is 24 square units. This area agrees with the result in step 2 of section 16.2, which is the value of the definite integral as a measure of the required area
- $$\int_1^4 (x^2 + 1) dx = 24.$$

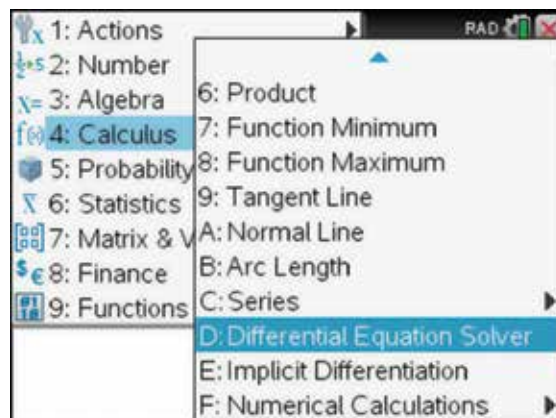
It was not really necessary to place points on the x -axis before finding the area, as the points will be added automatically when an area is found. However, doing this has the added advantage of using the precise x -values; furthermore, the points can be moved to change the value of the area.



16.5 How to find the general solution of a differential equation

1. To find the general solution of the differential equation $\frac{dy}{dx} = 2x - 3y$, open a new document and add a Calculator page, then press:

- **MENU**
- **4: Calculus**
- **D: Differential Equation Solver** .

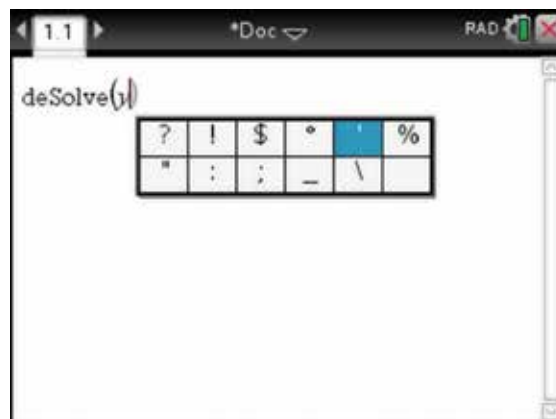


2. Complete the entry line as:

$\text{deSolve}(y' = 2x - 3y, x, y)$

Press the enter key .

Note: The prime is used for the derivative. It can be accessed by pressing (this key is located to the right of the **G** key) and selecting the highlighted symbol as shown.



3. The solution is given in terms of an arbitrary constant $c1$. Note that for the solution shown, the calculation mode in the General settings must be set to Auto or Exact mode.

TI-Nspire CAS calculator screen showing the solution to the differential equation $y' = 2x - 3y$. The command `deSolve(y'=2*x-3*y,x,y)` is entered, and the result is displayed as $y = c1 \cdot e^{-3 \cdot x} + \frac{2(3 \cdot x - 1)}{9}$.

16.6 How to find the particular solution of a differential equation

1. To find the particular solution of the initial value problem $\frac{dy}{dx} = 2x - 3y$, $y(1) = 2$, press:

- MENU
- 4: Calculus
- D: Differential Equation Solver

and complete the entry line as:

`deSolve(y' = 2x - 3y and y(1) = 2, x, y)`

TI-Nspire CAS calculator screen showing the command `deSolve(y'=2*x-3*y and y(1)=2,x,y)` entered. The previous solution is still visible above it.

2. Press the enter key .


Note: Use the catalog key to find 'and', or it can simply be typed. The solution can be expanded to express it into a simpler form as shown.


TI-Nspire CAS calculator screen showing the expanded solution. The command `expand(2*e^(-3*x)*((3*x-1)*e^(3*x)+7*e^3)/9)` is entered, resulting in the simplified expression $\frac{14 \cdot e^3}{9 \cdot (e^x)^3} + \frac{2 \cdot x}{3} - \frac{2}{9}$.

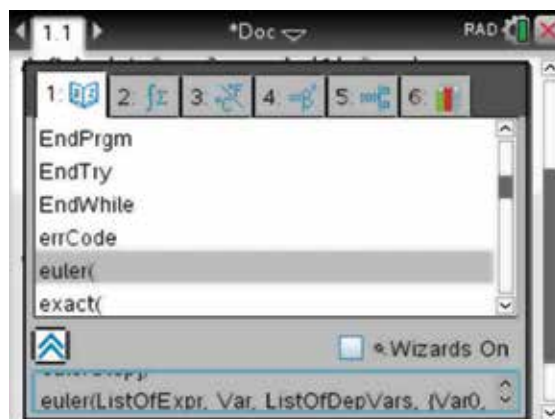
16.7 How to use Euler's method to tabulate the solution of a differential equation


Euler's method can be used to tabulate the solution to the differential equation $\frac{dy}{dx} = f(x, y)$, $y(x_0) = y_0$ in equally spaced step sizes of h , for x from $x = x_0$ up to x_n where $x_i = x_0 + ih$ and $y_{i+1} = y_i + h f(x_i, y_i)$. There is now a built-in function for using Euler's method. The syntax is `euler($f(x, y), x, y, \{x_0, x_n\}, y_0, h$)`.

- To use Euler's method to tabulate the solution of the differential equation $\frac{dy}{dx} = 2x - 3y$, $y(1) = 2$ with a step size of 0.25 up to $x = 3$, complete the following. Press:

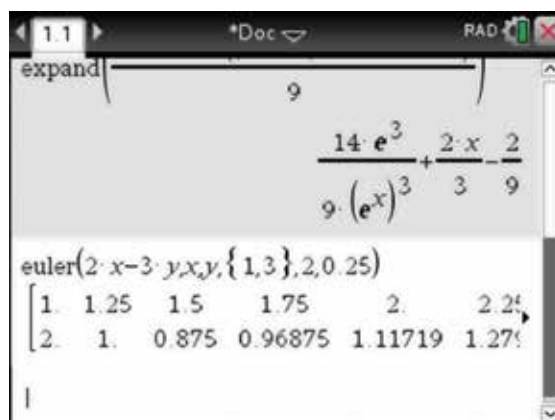
- **catalog** 
- **1:** **1**
- **E:** **E**.

Scroll to find `euler(`, then press the enter key .



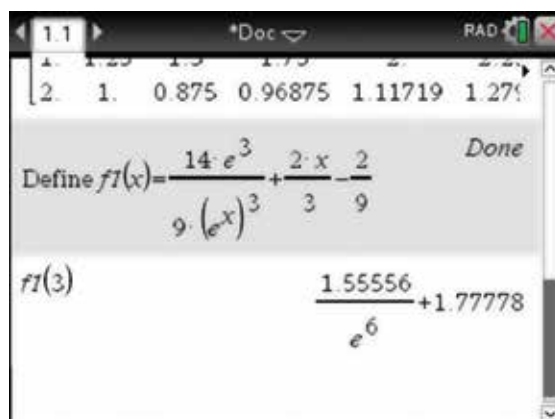
- Complete as shown:
`euler(2x - 3y, x, y, {1, 3}, 2, 0.25)`
then press the enter key .

Note that the solution is given as a matrix; the first row contains the x -values and the second row contains the y -values.



- Using the Touchpad, we can scroll across to see more of the matrix and compare the value from Euler's method to the exact value of the solution in this case. Define the function $f1(x)$ as the solution of the differential from section 16.6, as

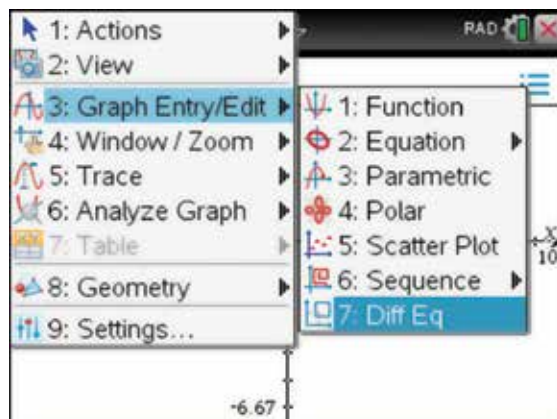
$$f1(x) = \frac{14e^3}{9(e^x)^3} + \frac{2x}{3} - \frac{2}{9} \text{ and complete as shown.}$$



16.8 How to plot slope fields for a differential equation

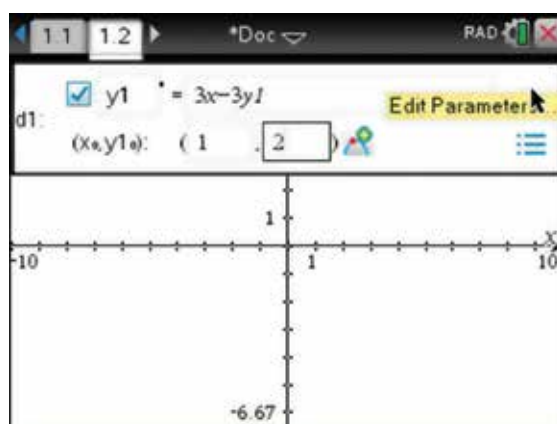
1. To sketch the slope fields for a differential equation, insert a Graph page and press:

- MENU **menu**
- 3: Graph Entry/Edit **3**
- 7: Diff Eq **7**.

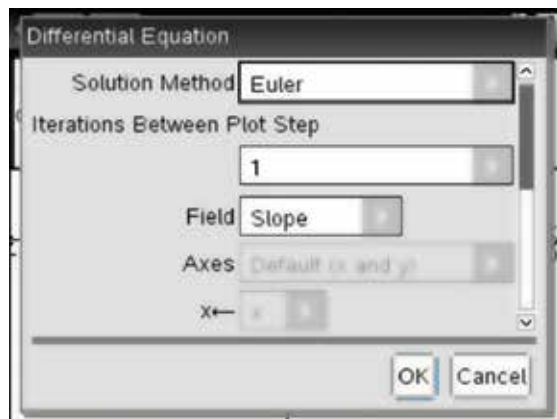


2. To sketch the slope fields for $\frac{dy}{dx} = 2x - 3y$, $y(1) = 2$, complete as shown, then click on the ellipsis (three dots, ...).

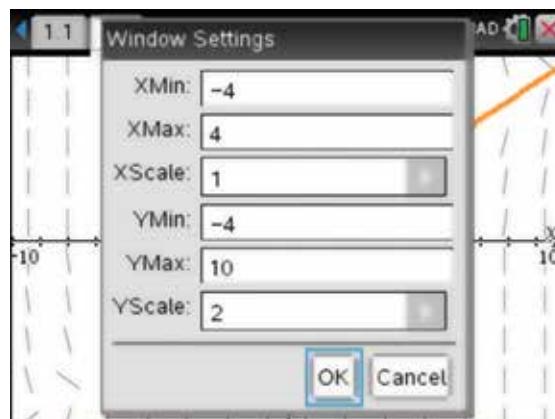
Note the syntax: $y1' = 2x - 3y1$ with the initial conditions $(x_0, y_0) = (1, 2)$.



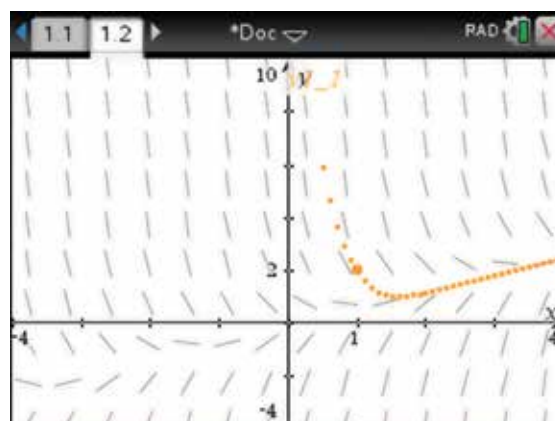
3. Choose the solution method as Euler and tab **tab** down to fill in the parameters as shown.



4. Set the window settings as shown, press the tab key **tab**, select OK and press the enter key **enter**.



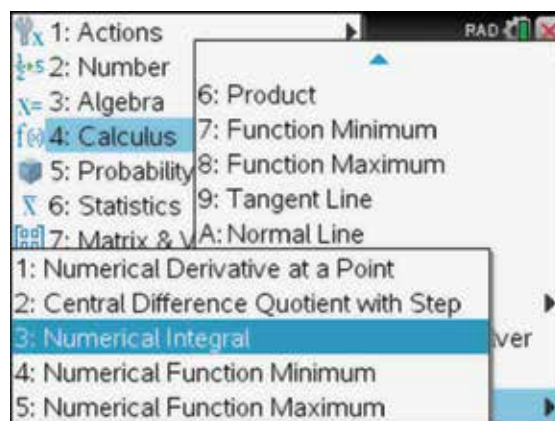
5. The slope field appears along with the particular curve passing through the initial conditions.



16.9 How to find the numerical solution of a differential equation

1. Add another Calculator page to the document. To find the value of y when $x = 3$ for the differential equation $\frac{dy}{dx} = \cos\left(\frac{1}{x}\right)$, $y(1) = 2$, press:

- **MENU** **menu**
- **4: Calculus** **4**
- **F: Numerical Calculations** **F**
- **3: Numerical Integral** **3**.

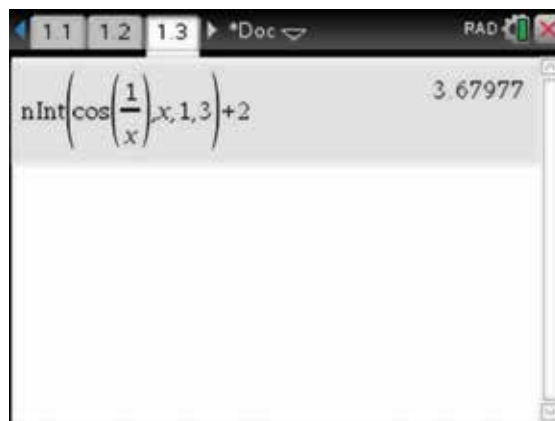


2. Complete the entry line as:

$$\text{nInt}\left(\cos\left(\frac{1}{x}\right), x, 1, 3\right) + 2$$

Press the enter key **enter**.

Note: The General Settings for Angle must be in Radians.



CHAPTER 17

Vernier Dataquest

This chapter deals with using the Vernier Dataquest.

17.1 How to set up, run and view experimental data




The Vernier Dataquest is used to collect, view, analyse and set up mathematical models for real-life data. To use the Vernier Dataquest you must connect a sensor or probe to the handheld device. There are different types of sensors, such as temperature, light, pH, voltage, motion detectors and radiation monitors. In the experiment that follows, we will place a motion detector in front of a swinging pendulum as shown below and record the position of the bob over a period of time.

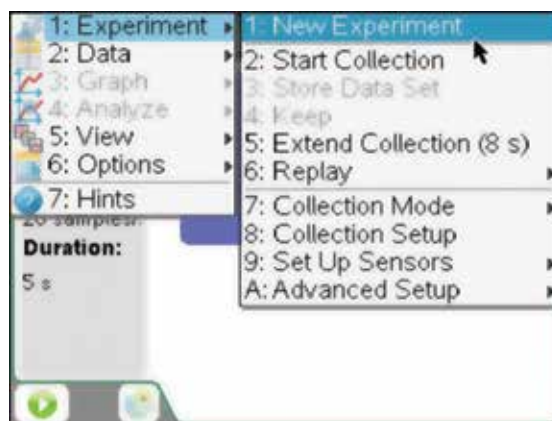


1. Start a new document and select the Vernier DataQuest and press ENTER **enter**. Alternatively, when a probe or sensor is attached, the Vernier application opens automatically.







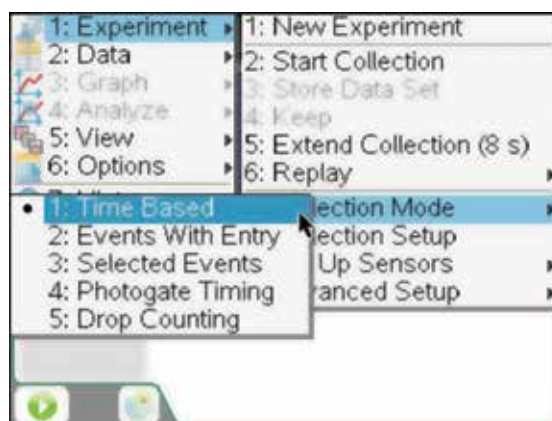
2. To begin a new experiment, press:

- MENU 
- 1: Experiment 
- 1: New Experiment 

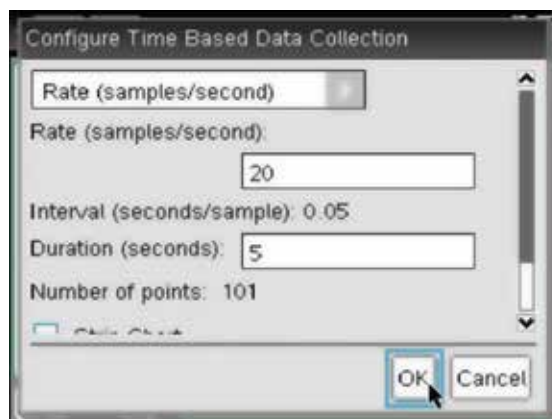


3. Before actually starting the experiment, we must decide how often we will collect the data. For this motion detector, we will use a time-based collection. To do this, press:

- MENU 
- 1: Experiment 
- 7: Collection Mode 
- 1: Time Based 



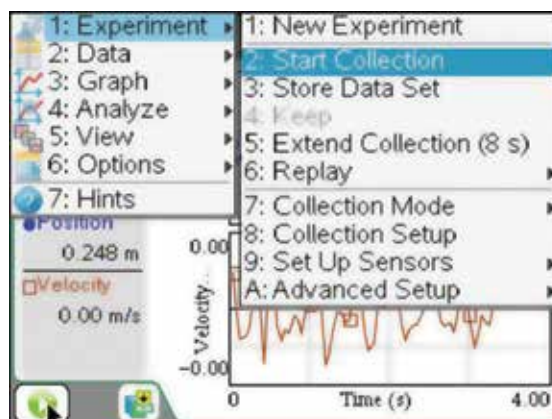
4. Here we can choose between the rate (samples/second) or interval (seconds/sample) and the duration in seconds of the experiment. Note the rate and interval are just reciprocals of each other. For this experiment we will collect the position of the bob at a rate of 20 samples per second, for 5 seconds, so overall we will collect 101 data points.



5. We are now ready to start the experiment. Position the bob of the pendulum, when hanging vertically at rest, approximately 25 cm from the motion detector. Set the bob in motion (as shown in the picture on page 209). Then press:

- MENU 
- 1: Experiment 
- 2: Start Collection 

Alternatively, click on the green start button in the bottom left corner of the screen.

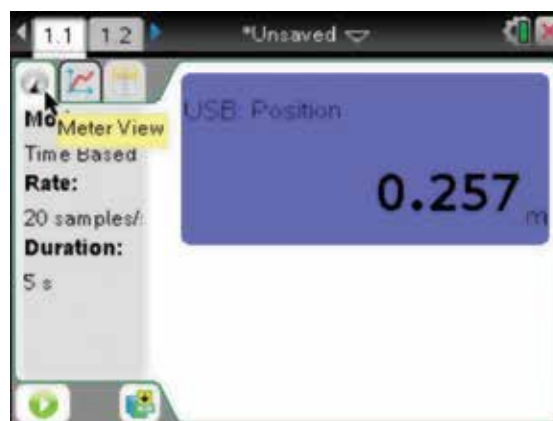


6. There are three different ways to view the results of the experiment over time: a metre, a graph or a table view. To see the metre view, press:

- MENU **menu**
- 5: View **5**
- 1: Metre **1**.

Alternatively, click on the Metre View icon.

Notice that the position of the bob is measured in metres, and its velocity is measured in m/s.

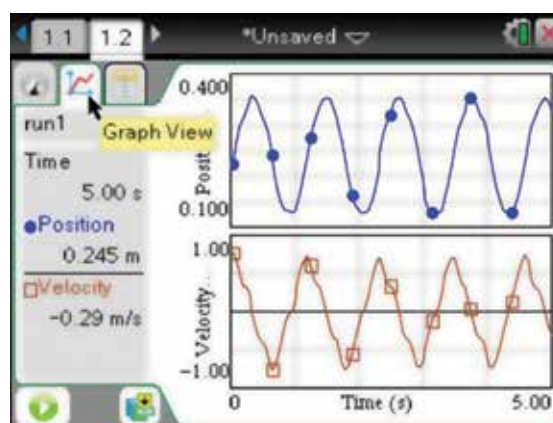


7. As time progresses, these views automatically update. To view the graph view, press:

- MENU **menu**
- 5: View **5**
- 2: Graph **2**.

Alternatively, click on the Graph View icon.

Note that if a temperature probe was attached, the Vernier application would automatically measure the temperature. It automatically measures quantities and displays appropriate units for the connected probe. There are however options to choose between appropriate measurable units, for example feet or metres.



8. To view the table view, press:

- MENU **menu**
- 5: View **5**
- 3: Table **3**.

Alternatively, click on the Table View icon.

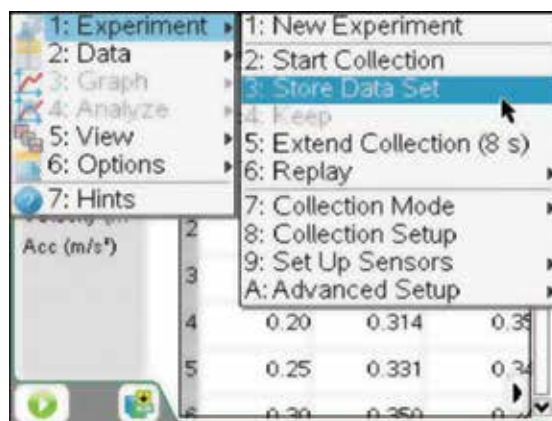
Note that the time, position, velocity and acceleration of the bob has been recorded. This data has been stored in variables called run1.time, run1.position, run1.velocity and run1.acceleration. We could perform another run of the experiment, and if we did, this new data set would be stored as run2... for each of these four quantities. The current time and position are stored in the variables metre.time and metre.position respectively.



17.2 How to store and analyse the collected data

- Note that there are options to extend, replay or stop the experiment. To store the data set, press:

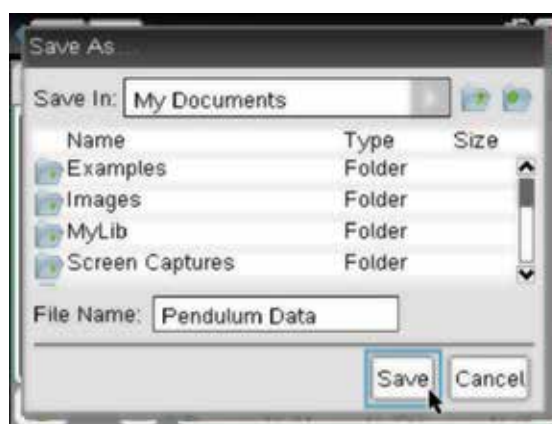
- **MENU** **(menu)**
- **1: Experiment** **(1)**
- **3: Store Data Set** **(3)**.



- To save the file, press:

- **doc** **(doc)**
- **1: File** **(1)**
- **5: Save As** **(5)**.

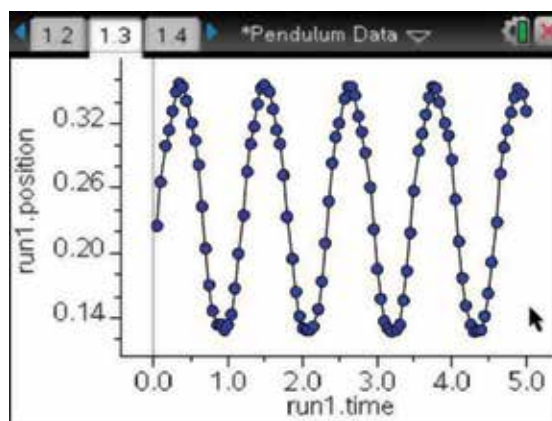
Give it the file name 'Pendulum Data' as shown.



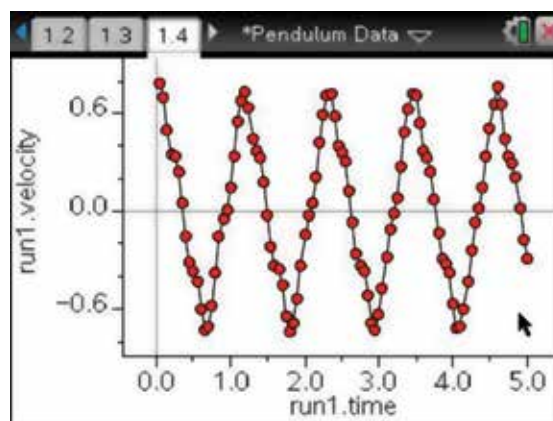
- The Vernier DataQuest has already graphed the data and has provisions for performing regression analysis. We can also graph the data on a Data & Statistics page. Open a Data & Statistics page by pressing:

- **ctrl** **(ctrl)**
- **doc** **(doc)**
- **5: Data & Statistics** **(5)**.

Tab **(tab)** to the horizontal axis and select run1.time. Tab **(tab)** to the vertical axis and select run1.position. Colour and connect the data points as described in chapter 13.

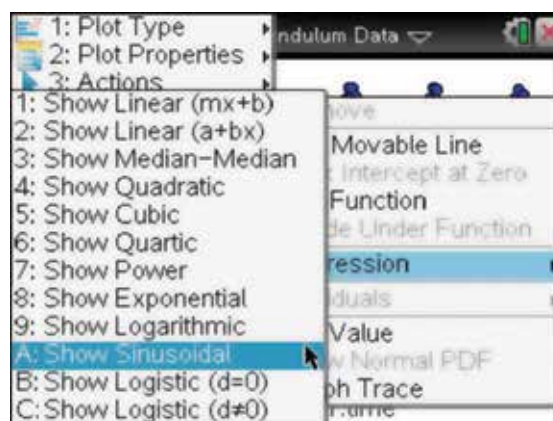


4. Open another Data & Statistics page. This time, draw the velocity–time graph. Colour and connect the data points.



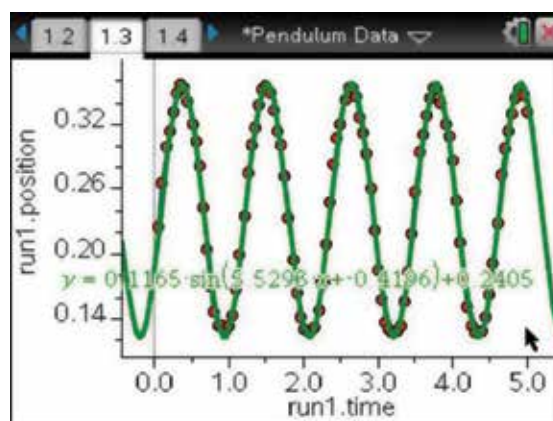
5. These graphs look like sine waves. Go back to the page for the position–time graph and press:

- **MENU**
- **4: Analyze**
- **6: Regression**
- **A: Show Sinusoidal**



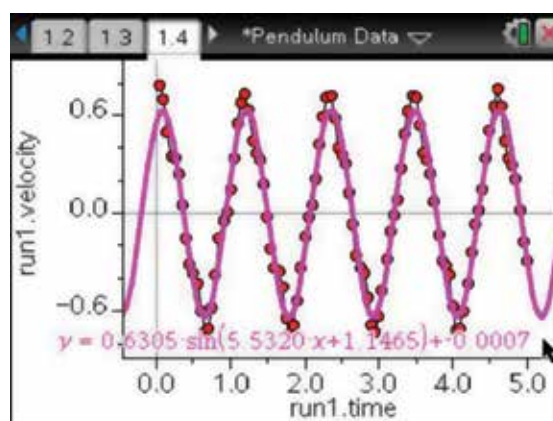
6. The green graph shown is
 $y = 0.1165 \sin(5.5293x - 0.4196) + 0.2405$.

What is the significance of the numbers 0.1165, 5.5293 and 0.2404 in relation to the graph and to the motion of the bob? In this experiment the length of the pendulum was 32 cm; how is this related to the numbers above?



7. Repeat the sinusoidal regression analysis on the velocity–time graph. The pink graph shown is
 $y = 0.6305 \sin(5.5320x + 1.1465) - 0.0007$.

What is the significance of these numbers and what is the relationship between these graphs? Further investigations and a graph of time versus acceleration can also be carried out as described above. What is the relationship between these graphs and what is the relationship between position, velocity and acceleration?



CHAPTER 18

Further graphing techniques

This chapter deals with further graphing techniques.

18.1 How to plot families of functions

1. Start a new document and add a Graphs page. To graph a family of functions where each graph differs by one or more parameters, complete the function entry line as:

$$f1(x) = x + \{-4, -2, 0, 2, 4\}$$

Notice that there are five graphs plotted. These are the graphs of:

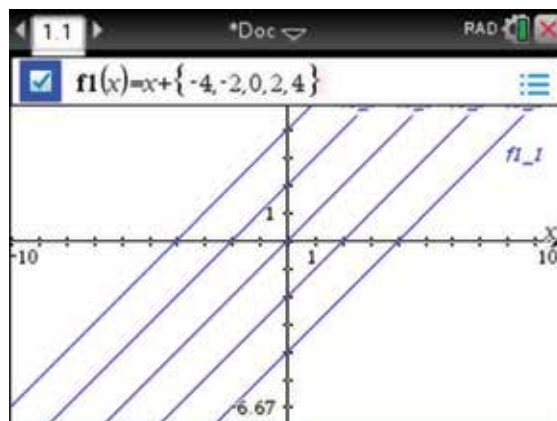
$$y = x - 4$$

$$y = x - 2$$

$$y = x$$

$$y = x + 2$$

$$y = x + 4.$$



2. On a new Graphs page, complete the function entry line as:

$$f1(x) = \{-2, -1, 1, 2\}x^2 + \{-3, -1, 1, 3\}$$

Notice that this time four graphs are plotted. These are the graphs of

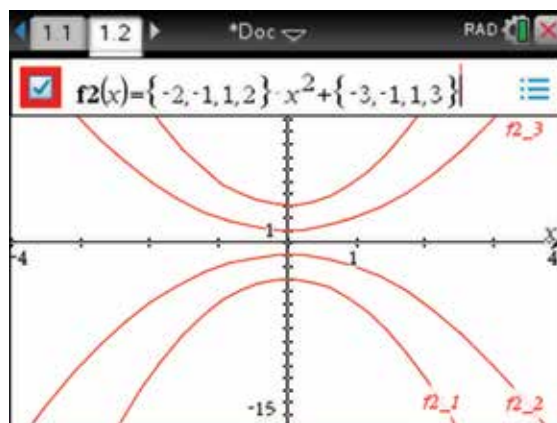
$$y = -2x^2 - 3$$

$$y = -x^2 - 1$$

$$y = x^2 + 1$$

$$y = 2x^2 + 3$$

Note also that the graphing window has been changed to show more of the graphs.

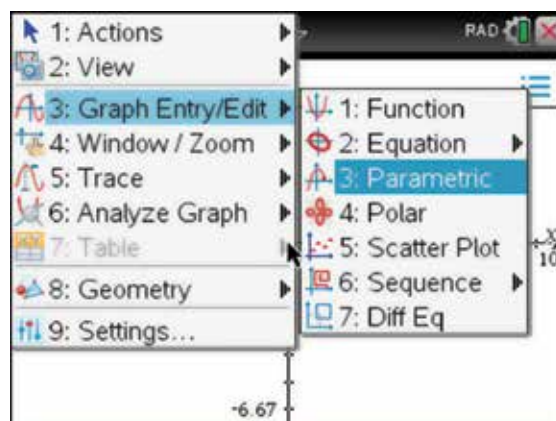


18.2 How to plot parametric graphs

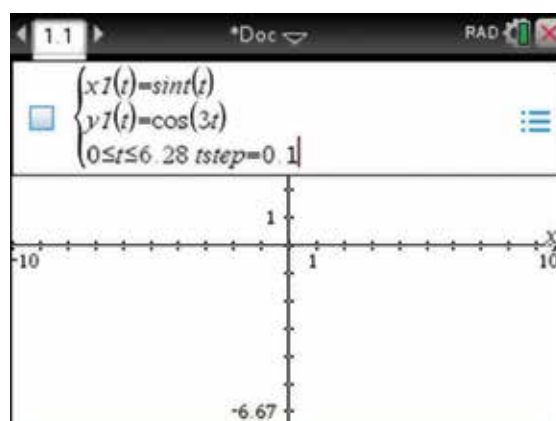
Parametric graphs are described by $x = f(t)$ and $y = g(t)$, where t is called a parameter. As the value of t changes, both x and y change; this then traces out a curve in the Cartesian plane.

1. Start with a new document and add a Graphs page.
To sketch parametric graphs, press:

- MENU **menu**
- 3: Graph Entry/Edit **3**
- 3: Parametric **3**.

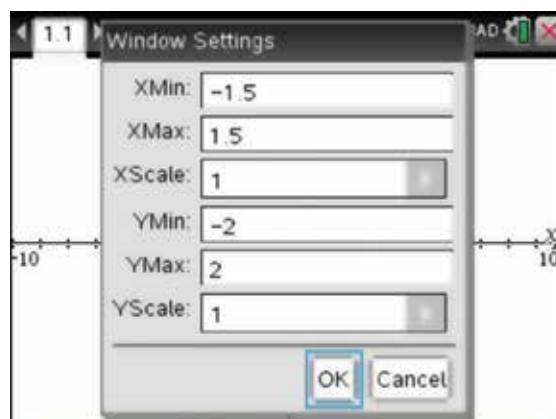


2. As an example, to plot the graph of $x = \sin(t)$ and $y = \cos(3t)$, complete the entry line as shown, then press the enter key **enter**.

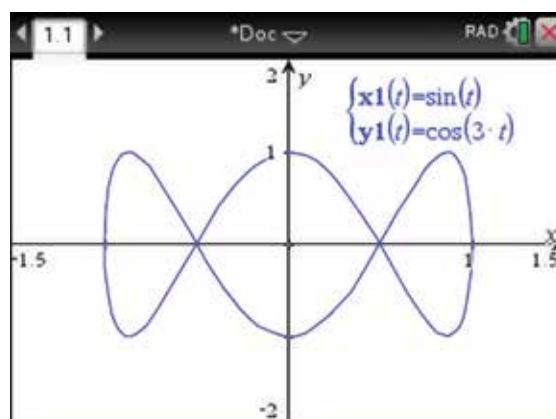


3. Change to the viewing window as shown.

Note that the Graphing Angle in the Graphs and Geometry Settings must be set to Radians.



4. The graph is shown. Move the label to a suitable position.



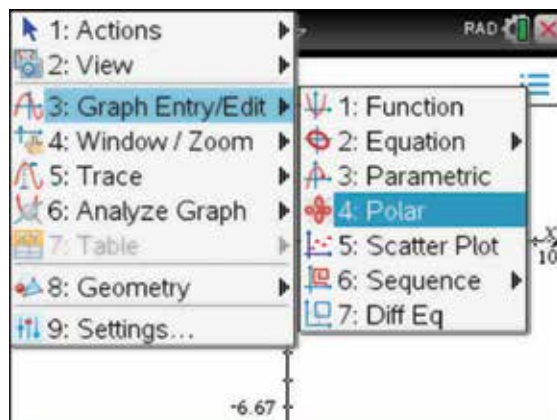
18.3 How to plot polar graphs

Polar graphs are described by $r = f(\theta)$ where $x = r \cos(\theta)$ and $y = r \sin(\theta)$. A point in the plane is described by $[r, \theta] = [r, \theta + 2\pi]$; as both r and θ change, x and y change and the path is traced out.

1. Start with a new document and add a Graphs page.

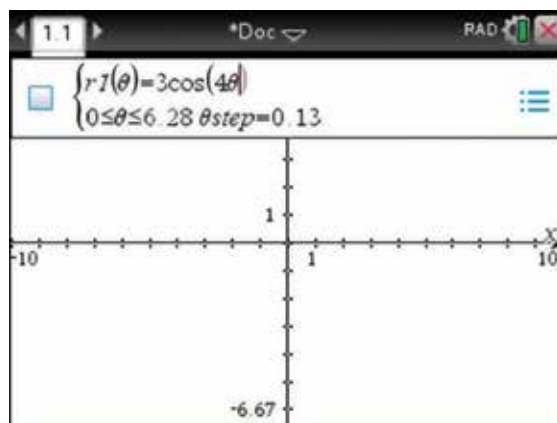
To sketch polar graphs, press:

- MENU **menu**
- 3: Graph Entry/Edit **3**
- 4: Polar **4**.



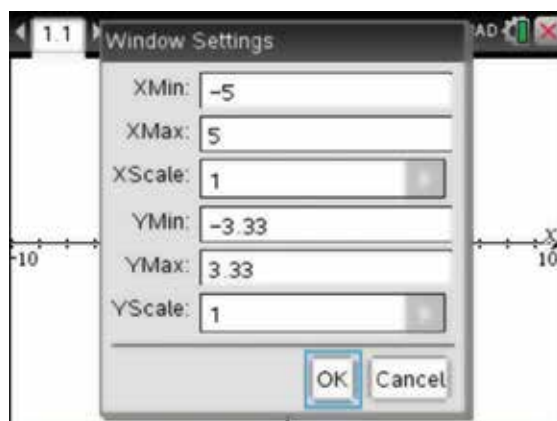
2. As an example, to plot the graph of $r = 3 \cos(4\theta)$, complete the entry line as shown, then press the enter key **enter**.

Note: To insert θ , press **π** and use the Touchpad.

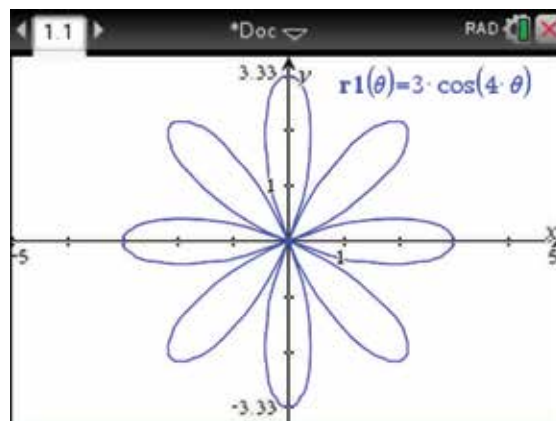


3. Change to the viewing window as shown.

Note that these window settings preserve the squareness and aspect ratio of the graphing window. The Graphing Angle in the Graphs and Geometry Settings must be set to Radians.



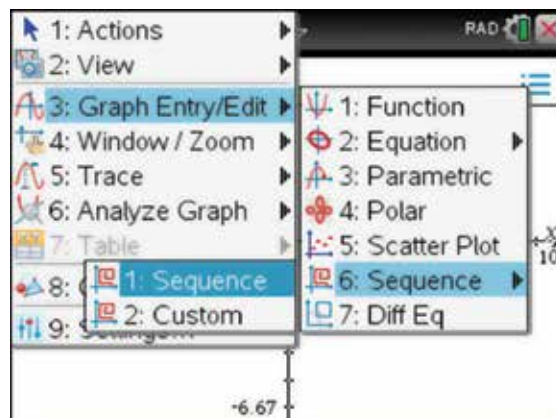
4. The graph is shown. Move the label to a suitable position.



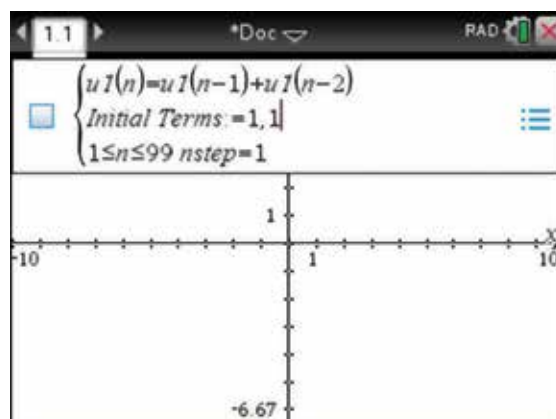
18.4 How to plot sequence graphs

1. Start with a new document and add a Graphs page.
To sketch a sequence graph, press:

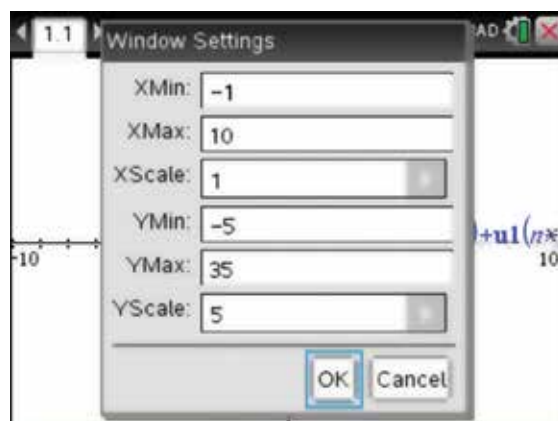
- **MENU** menu
- **3: Graph Entry/Edit** 3
- **6: Sequence** 6
- **1: Sequence** 1.



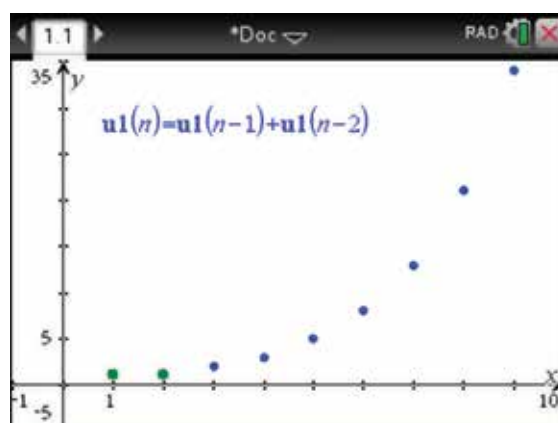
2. As an example, plot the sequence graph of $u(n) = u(n-1) + u(n-2)$ with $u(1) = 1$ and $u(2) = 1$. Complete the entry line as shown, then press the enter key enter.



3. Change to the viewing window as shown.

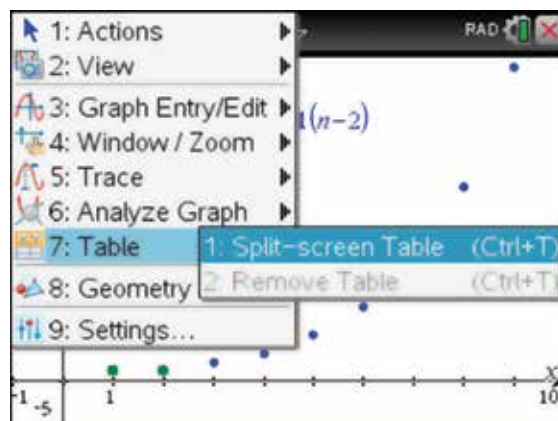


4. The sequence graph is shown.



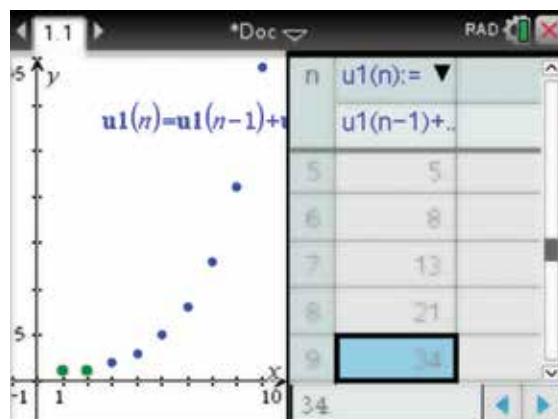
5. To see the table of values, press:

- MENU **menu**
- 7: Table **7**
- 1: Split-screen Table **1**.



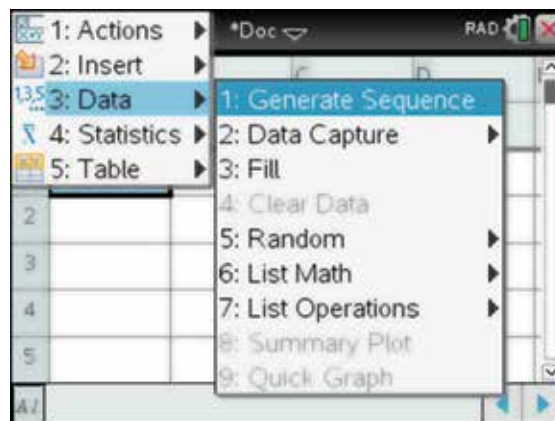
6. A table of values is shown. Note that $u(9) = 34$ and that this sequence is the Fibonacci sequence.

{ 1, 1, 2, 3, 5, 8, 13, 21, 34, ... }

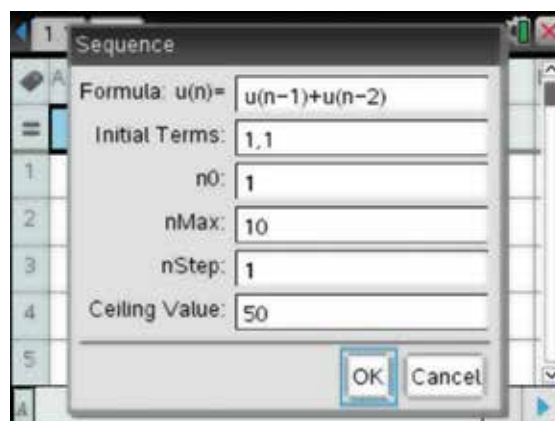


7. Add a Lists and Spreadsheet page to the document.
With the cursor in the first column, press:

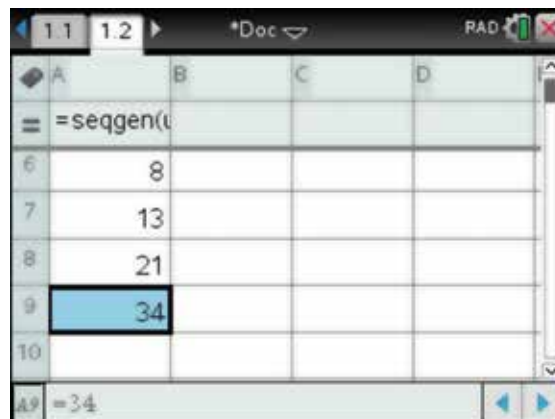
- MENU **menu**
- 3: Data **3**
- 1: Generate Sequence **1**.



8. Complete as shown, then tab **tab** to OK and press the enter key **enter**.



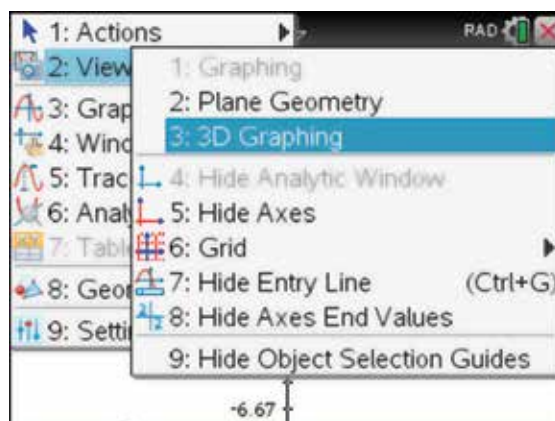
9. The values of the sequence are displayed in the column. Again note that $u(9) = 34$.



18.5 How to plot 3-dimensional surfaces

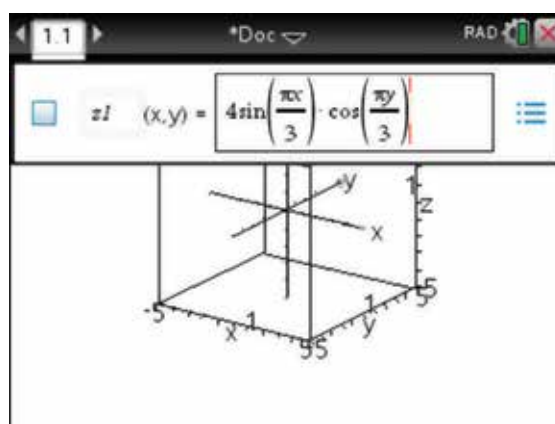
1. In 3 dimensions, the equation $z = f(x, y)$ represents a surface. Start with a new document and add a Graphs page. To plot 3-dimensional graphs, press:

- MENU **menu**
- 2: View **2**
- 3: 3D Graphing **3**.

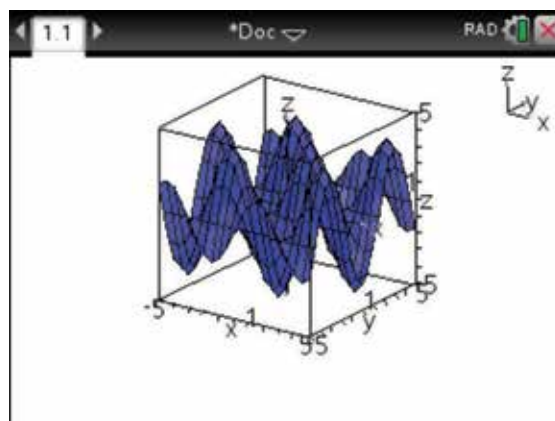


2. As an example, plot the 3 dimensional surface for $z = 4 \sin\left(\frac{\pi x}{3}\right) \cos\left(\frac{\pi y}{3}\right)$, complete the entry line as shown, then press the enter key **enter**.

Note that the Graphing Angle in the Graphs and Geometry Settings must be set to Radians.

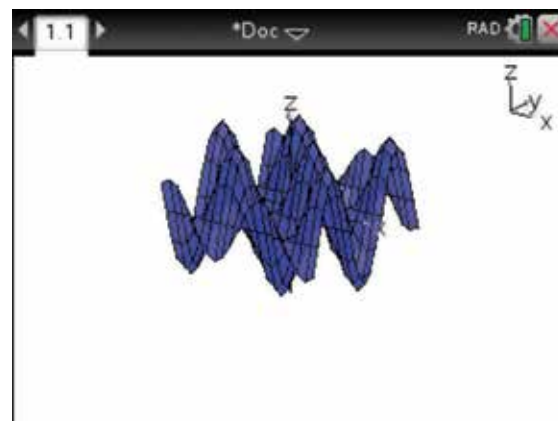


3. There are many options to choose. We can view the surface as surface only, surface with wire, or wire only. We can change the fill colour or line colour and change the levels of transparency. We can view different orientations such as x-y, y-z or z-y, magnify or shrink the box, and choose range or window settings. Pressing the arrow keys will rotate the views.



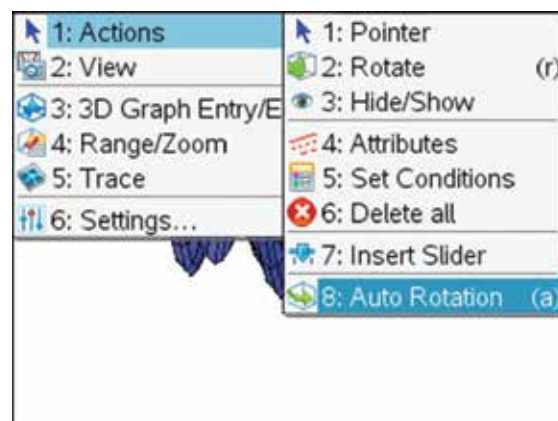
4. Hide the box and press:

- MENU **menu**
- 2: View **2**
- 4: Hide Box **4**.

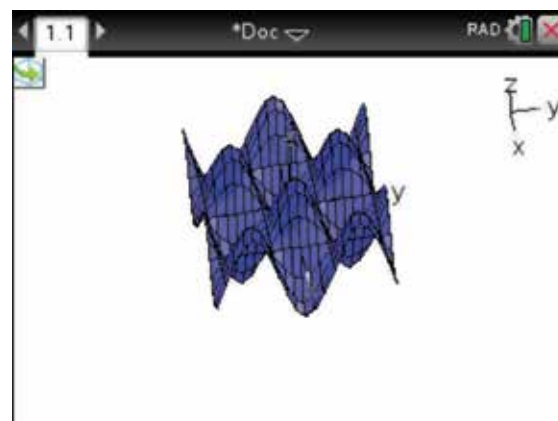


5. Now press:

- MENU **menu**
- 1: Actions **1**
- 8: Auto Rotation **8**.



6. Watch as the surface automatically rotates; note the position of the x - y - z axes in the top right corner of the screen. Press the escape key **esc** to stop the rotation.



18.6 How to insert images

- Images may be inserted into Graphs, Geometry, Data & Statistics, Notes or Question pages. The image file type may be .jpg (Joint Photographic Experts Group), .bmp (Device-independent Bitmap), or .png (Portable Network Graphics). Although it is not possible to insert an image directly onto a document on a handheld, it is possible to insert an image onto a document on the computer software, save the document, and then transfer the saved document onto the handheld or transfer this file to other handhelds. Then it is possible to open the document containing the image. The image shown has been inserted onto a Geometry page.



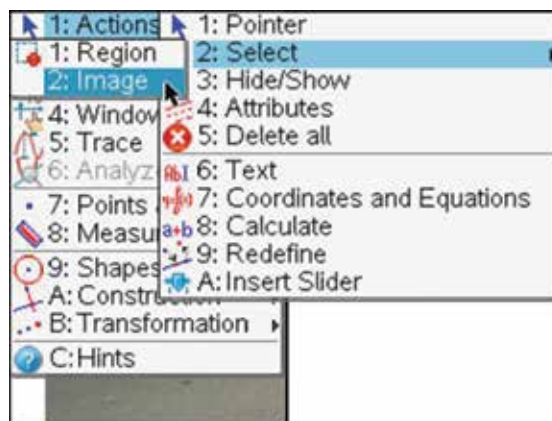
- Images can be viewed in split screens. To do this, press:

- **doc** **doc**
- **5: Page Layout** **5**
- **2: Select Layout** **2**
- **2: Layout 2** **2**.

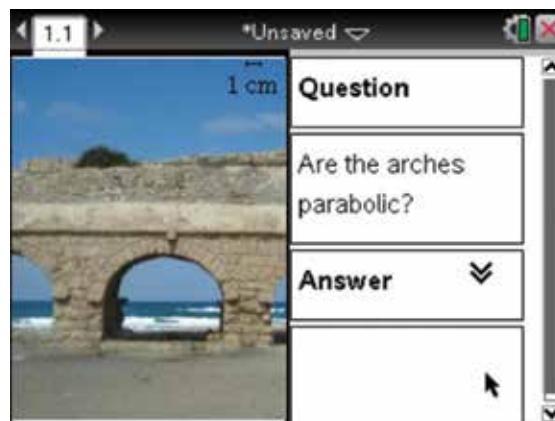


- Although there can only be one image per page, images can be resized, moved or deleted. To do this, press:

- **MENU** **menu**
- **1: Actions** **1**
- **2: Select** **2**
- **2: Image** **2**.



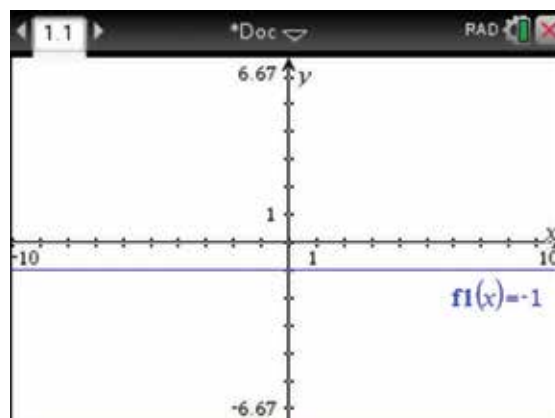
4. Images may be used with Notes or Question and Answer templates for further mathematical investigations.



18.7 How to create a locus of points

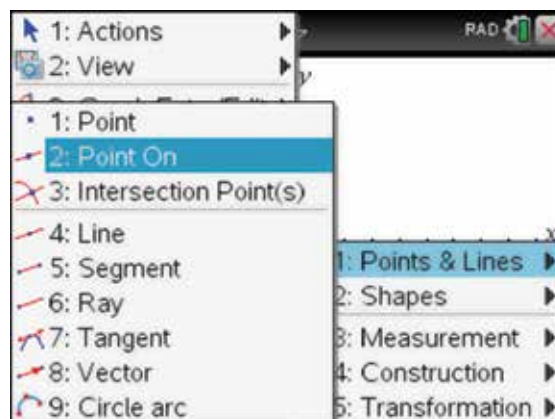
Given a point F and a line d , follow the steps below to construct the locus of a point P . To do this, place a point D on the line d , construct the perpendicular to d through D , and then construct the perpendicular bisector of FD . The point P is the intersection of these two lines. The ideas below build upon the geometry aspects introduced in Chapter 8.


1. Open a new document and insert a Graphs page.
Draw the line $y = -1$ by completing the function entry line as $f1(x) = -1$.



2. Now place a point on this line. To do this, press:

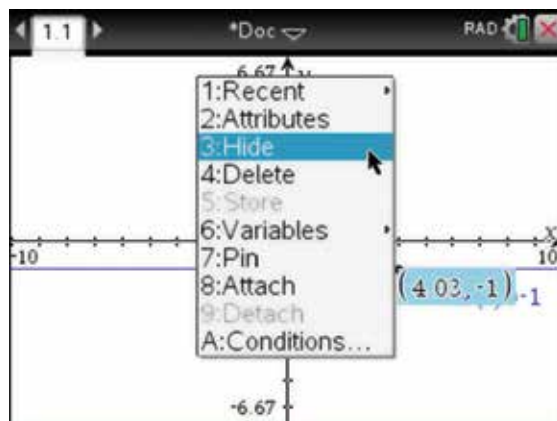
- MENU **menu**
- 8: Geometry **8**
- 1: Points & lines **1**
- 2: Point On **2**.



3. Move over the line and press the click key  or the enter key **enter** to place the point on the line. Hover over the coordinates, then press:

- **ctrl** **ctrl**
- **MENU** **menu**
- **3: Hide** **3**.

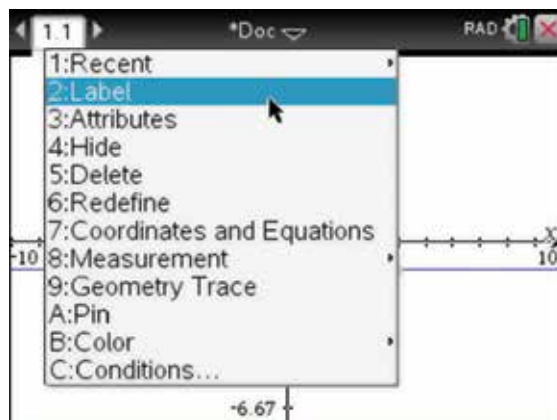
To hide the coordinates of the point, also hide the equation for the line.



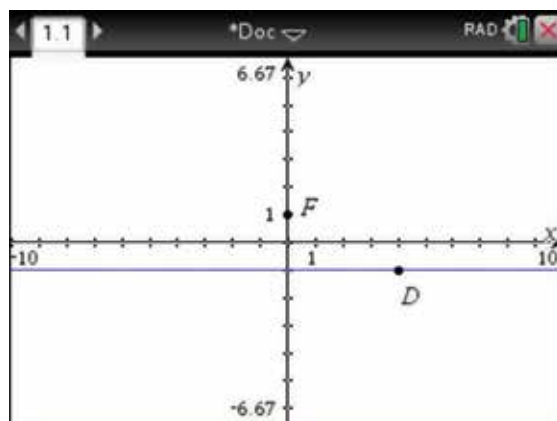
4. Press:

- **ctrl** **ctrl**
- **MENU** **menu**
- **2: Label** **2**.

In the text box that appears, label the point as *D*.



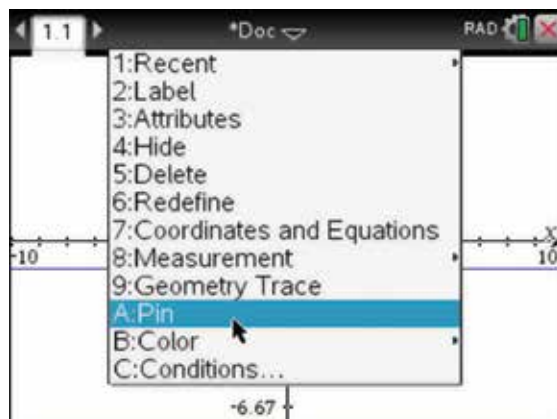
5. Using the above steps, place another point on the *y*-axis, this time at the exact point (0, 1). Hide the coordinates and label this point as *F*.



6. Press:

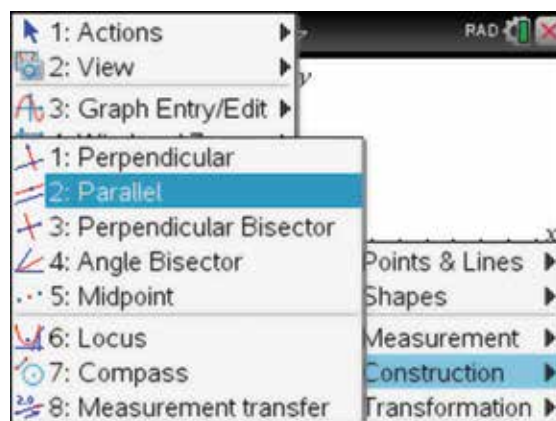
- **ctrl** **ctrl**
- **MENU** **menu**
- **A: Pin** **A**.

Then select the point *F* and repeat for the line. Doing this will ensure that the point *F* and the line cannot be moved.

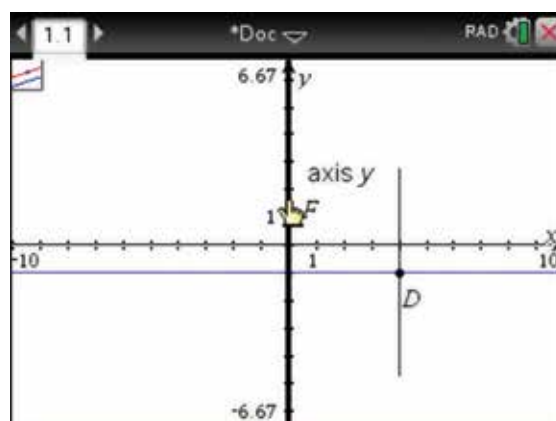


7. Now draw a line through D perpendicular to the line $y = -1$ or parallel to the y -axis. To do this press:

- **MENU** (menu)
- **8: Geometry** (8)
- **4: Construction** (4)
- **2: Parallel** (2).



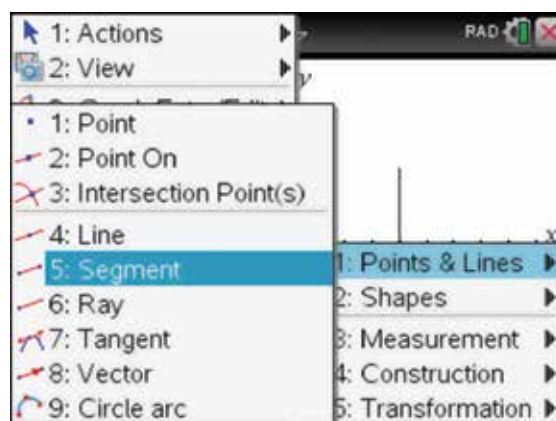
8. Click on the point D , then click on the y -axis. Press enter (enter), then press escape (esc) to exit this tool.



9. Next we need to construct a line segment from F to D . To do this press:

- **MENU** (menu)
- **8: Geometry** (8)
- **1: Points & Lines** (1)
- **5: Segment** (5).

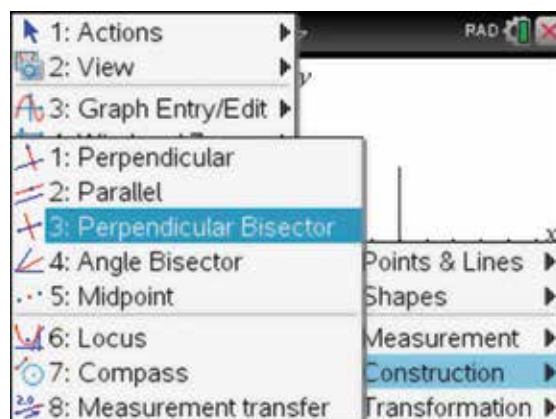
Click on the point F , then click on the point D , and then press escape (esc).



10. Now press:

- **MENU** (menu)
- **8: Geometry** (8)
- **4: Construction** (4)
- **3: Perpendicular Bisector** (3).

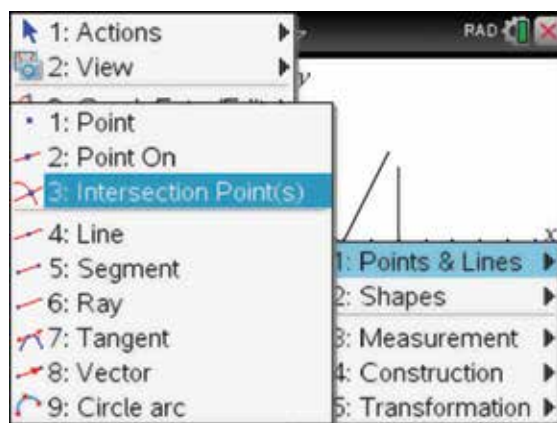
Click on the point F , then click on the point D , and then press escape (esc).



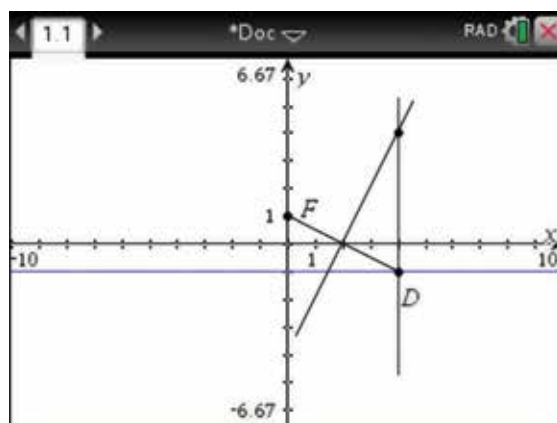
11. To find the intersection point of this perpendicular bisector and the line through D parallel to the y -axis or perpendicular to the line $y = -1$, press:

- **MENU** **(menu)**
- **8: Geometry** **(8)**
- **1: Points & Lines** **(1)**
- **3: Intersection Point(s)** **(1)**.

Click on both these lines, then press enter **(enter)** and then escape **(esc)**.

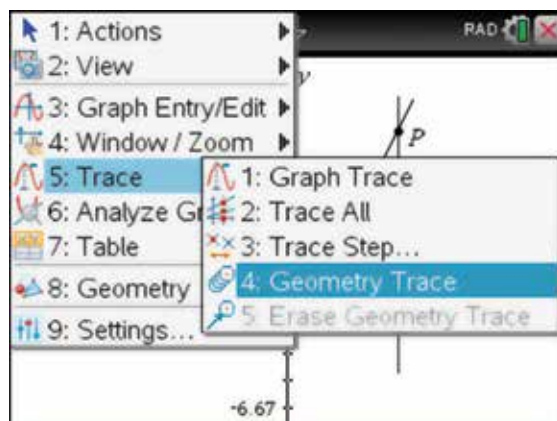


12. The point of intersection will be shown.



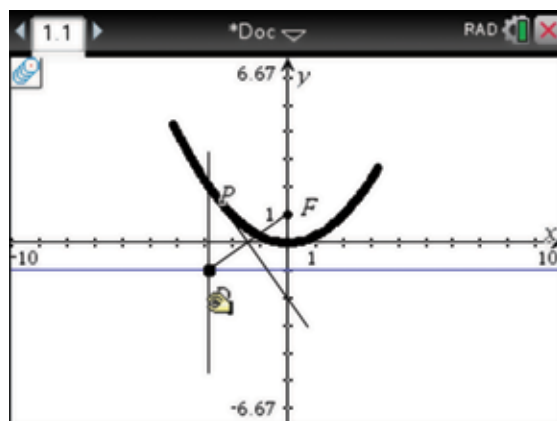
13. Now label this newly found intersection point P , as described as above. Note that we can drag and move the point D along the line and watch as the point P moves. We can trace the locus of the point P as we move the point D . To do this, press:

- **MENU** **(menu)**
- **5: Trace** **(5)**
- **4: Geometry Trace** **(4)**.

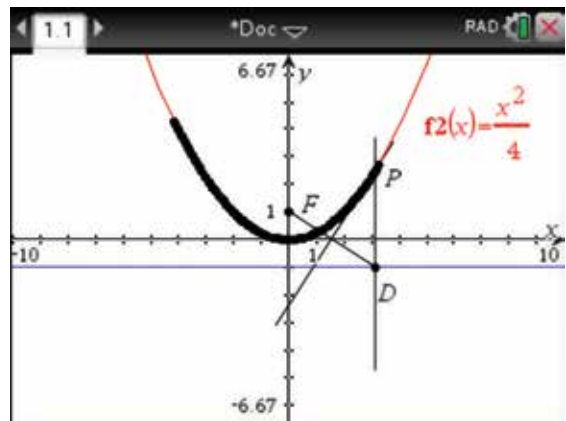


14. Click on the point P and then drag and move the point D along the red line $y = -1$. The black set of points is traced out.

This locus is in fact a parabola, and the construction used above is one definition of a parabola. That is, the distance from F to P is always equal to the distance from D to P . We could verify that these distances are always equal using the measurement and length tool. The point F is called the focus, and the line through D is called the directrix. Save this document and give it the name 'Parabola Locus'.

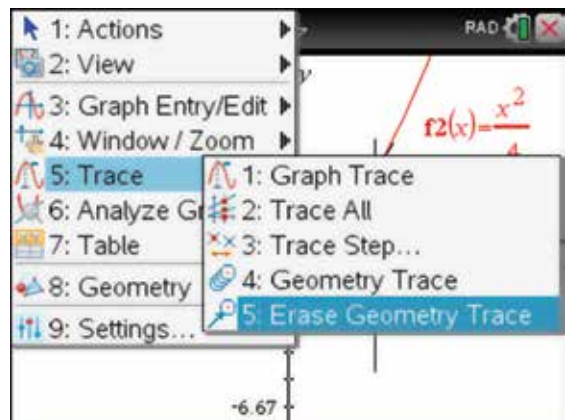


15. To verify that the set of points is indeed a parabola and that its equation is $y = \frac{x^2}{4}$, press tab **(tab)** and enter this function on the entry line as $f2(x) = \frac{x^2}{4}$.



16. To erase the geometry trace, press:

- **MENU** **(menu)**
- **5: Trace** **(5)**
- **5: Erase Geometry Trace** **(5)**.






CHAPTER 19

Programming with the TI-Nspire CAS calculator

The TI-Nspire can create programs or functions. A function must return something like a value, a list or an expression, and can be used in any application. A created function can therefore be used just like any intrinsic function, such as $\cos(x)$. A program can also output a value, a list or an expression, but it can be used only within the Calculator page.

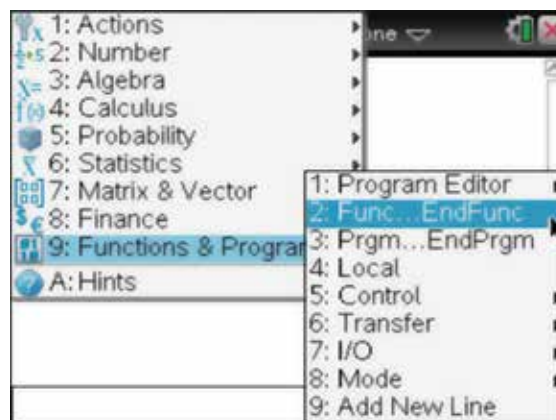
19.1 How to write a user-defined function

1. To write a function to find the volume of cone of height h and radius r , open a new document and add a Calculator page. Then press:

- MENU 
- 1: Actions 
- 1: Define 

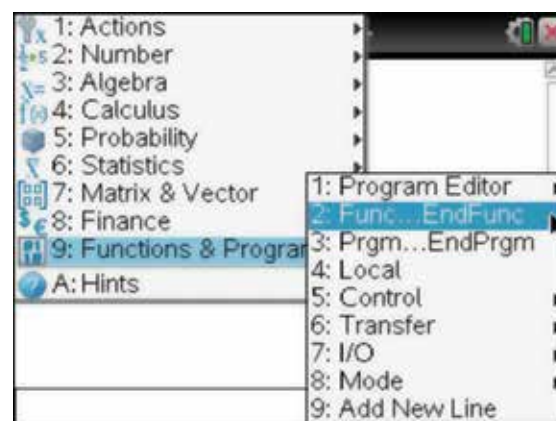
Complete the entry line as:

Define $volcone(r,h)=$

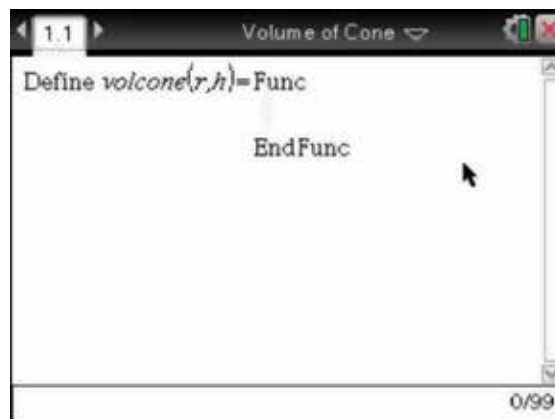


2. Press:

- MENU 
- 9: Functions & Programs 
- 2: Func...EndFunc 

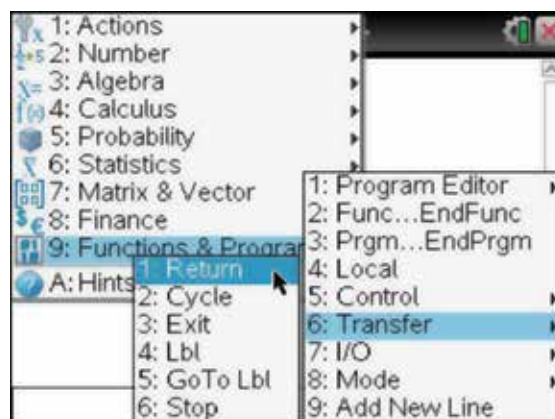


3. The basic template for the function has been entered. Save the document and give it the file name 'Volume of Cone'.



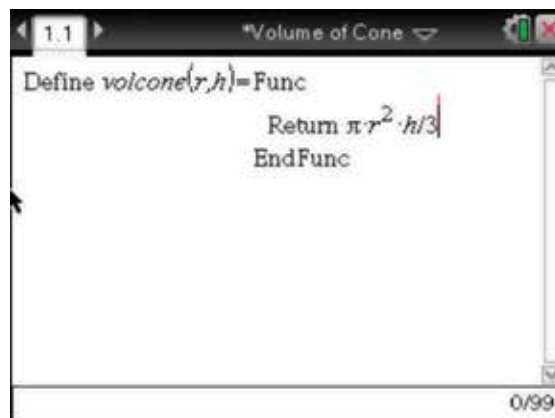
4. With the cursor under the word Func, press:

- **MENU** **menu**
- **9: Functions & Programs** **9**
- **6: Transfer** **6**
- **1: Return** **1**.



5. Type in $\pi r^2 h / 3$.

Note: Do not use the division template. Remember to include the multiplication signs between variables.



6. Press:

- ENTER (enter)
- var (var)
- ENTER (enter).

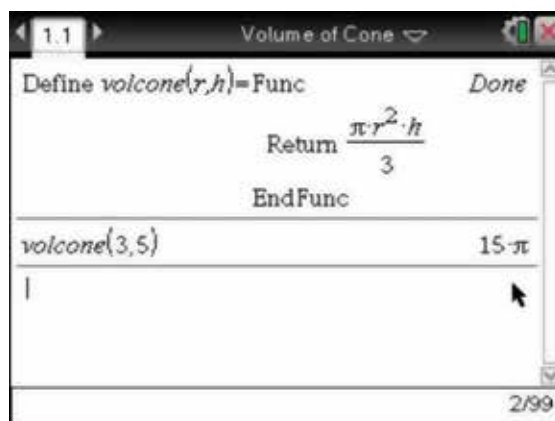
Complete the entry line as:

$\text{volcone}(3,5)$

Save the document by pressing:

- ctrl (ctrl)
- S (S).

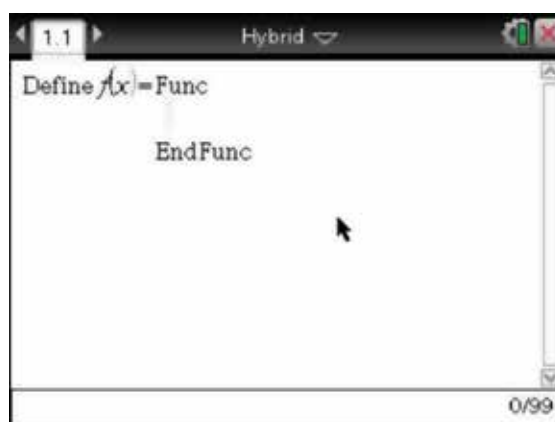
This finds the volume of a cone of radius 3 and height 5.



7. In section 7.2, we defined the hybrid function

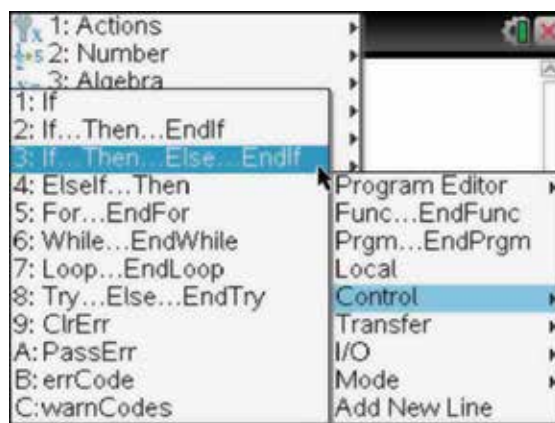
$$f(x) = \begin{cases} x & \text{for } x < 0 \\ x+1 & \text{for } 0 \leq x < 2 \\ 5-x & \text{for } x \geq 2 \end{cases}$$

We will now write a user-defined function for this function. Open a new document and add a Calculator page. Open the basic template for the Defined function $f(x)$ and save the document, giving it the file name 'Hybrid'.

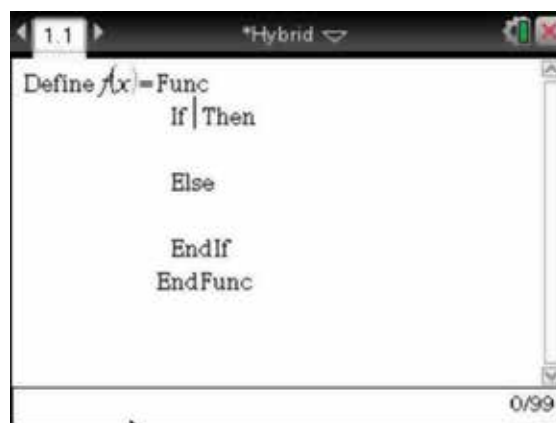


8. Press:


- MENU (menu)
- 9: Functions & Programs (9)
- 5: Control (5)
- 3: If...Then...Else...EndIf (3).

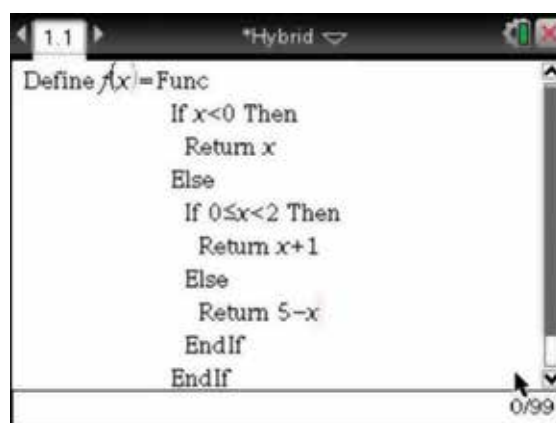


9. The screen opposite is shown. Now move the cursor to the position shown.



10. Use the given templates to type in the keywords to complete the function as shown.

Note: To make a new line in the function, press the new line character  located to the right of the letter **U**.



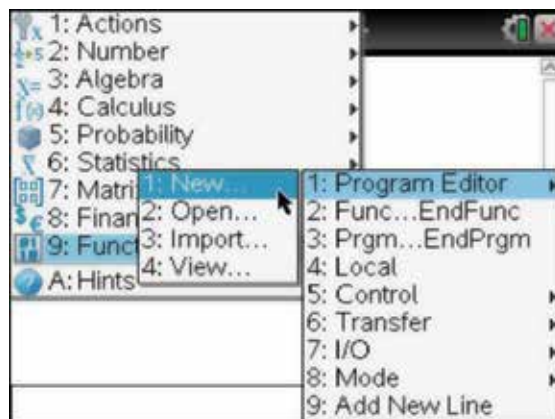
11. We can test our function as shown and use it like any other intrinsic function. We could differentiate the function or include it in the function entry line in a Graphs page.



19.2 How to create, write, edit and run a program

1. To create a program, open a new document and add a Calculator page, then press:

- **MENU** **(menu)**
- **9: Functions & Programs** **(9)**
- **1: Program Editor** **(1)**
- **1: New** **(1)**.



2. To write a program that simulates throwing two dice a number of times and calculating the probability of doubles appearing, complete the following steps.

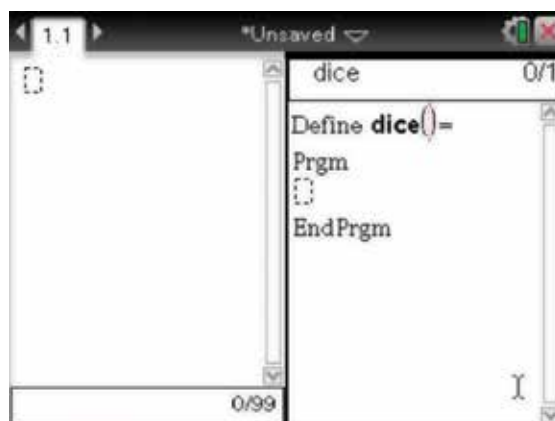
Name the new program 'Dice'. Leave Type as Program and leave Library Access as None.

Use the tab key **(tab)** to select OK, then press the enter key **(enter)**.






3. The screen will split into two, with the Calculator page on the left and the Program Editor on the right.



Note: The basic template for the program is already entered.



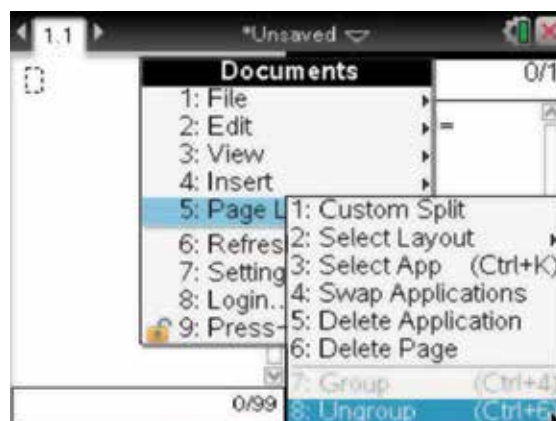
4. To write and edit the program in a full screen, ungroup the pages by pressing:

- **doc** 
- **5: Page Layout** 
- **8: Ungroup** .




Then press:

- **ctrl** 
- 

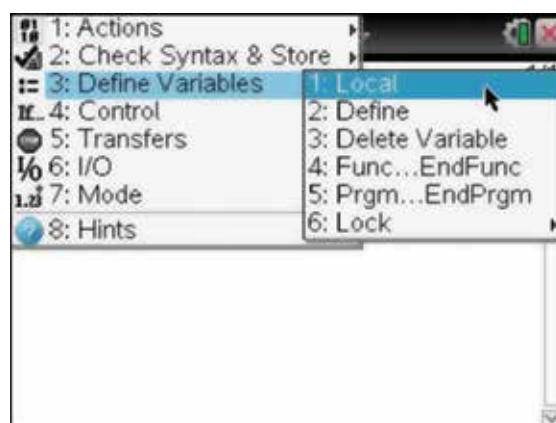
to return to the program on the Calculator page.



5. Now, move the cursor to the line under Prgm and press:

- **MENU** 
- **3: Define Variables** 
- **1: Local** .

This is where we will type in the variables that will be used in the program. Variables can be global, that is, their values will be known outside the program. Local variables exist only while the program is being run, and their values are deleted when the program has finished running.



6. One variable that we need is *maxthrows*, which will be entered as the number of times to throw the dice when the program is being run. Other variables are *die1* and *die2*; these will be random numbers on a single die. We will use *n* as the loop counter, and *doubles* as a counter that will count the number of times that doubles appears. Type these variables, separated by commas, after the word 'Local' as shown opposite.




Under the local variables, enter:

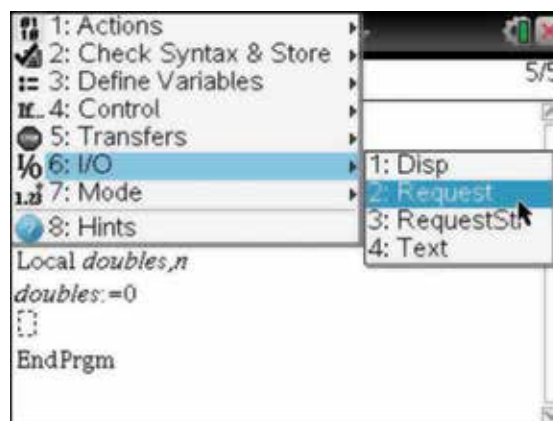
doubles:= 0

to initialise the value of the variable *doubles* to zero when the program is run.



7. To enter the value of *maxthrows* while the program is being run, press:

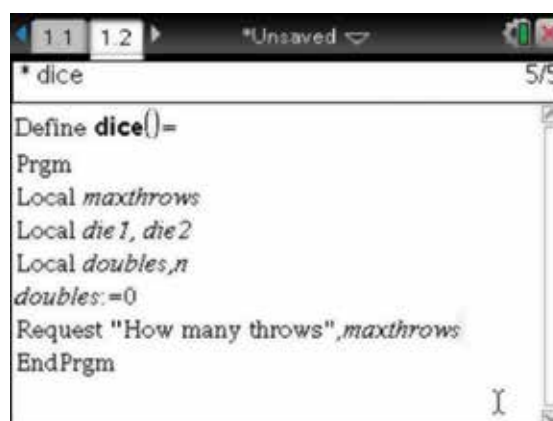
- MENU 
- 6: I/O 
- 2: Request .






8. Complete the entry line as:

Request "How many throws", *maxthrows*

Note: You can use the Request command in a program but not in a function.



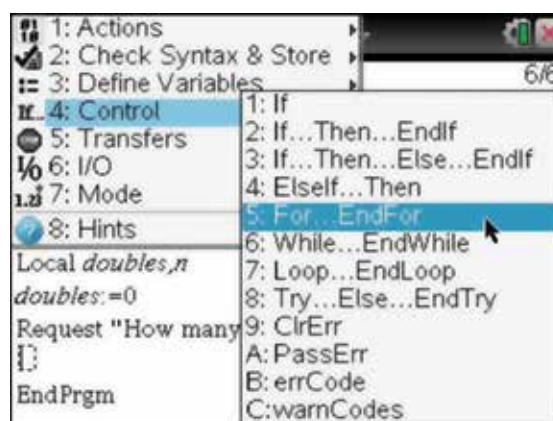
9. To insert a loop into our program, press:

- MENU 
- 4: Control 
- 5: For...EndFor .




Complete the entry line as:


For *n*,1,*maxthrows*

Notice that the commas and blank lines are already inserted.



10. Now, press:

- **catalog** 
- **1** 
- **R** 

Scroll using the Touchpad to the keyword `randInt(`. The help at the bottom of the screen shows that this will produce a random integer between `lowBound` and `upBound`. Press the enter key  to paste the command into the program and move the cursor to type in:

`die1:=randInt(1,6)`

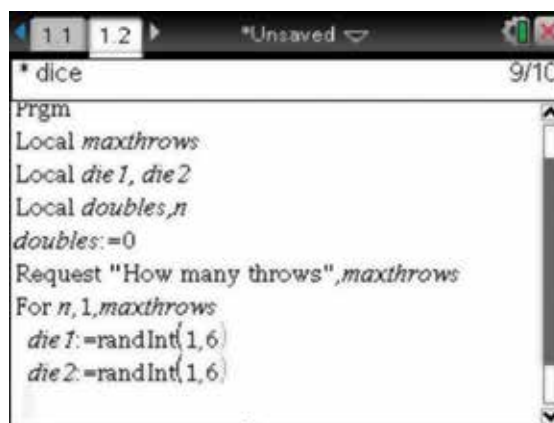
This will simulate rolling our first die and produce a random number between 1 and 6 inclusive.






11. Repeat the procedure, or copy and edit the line as:

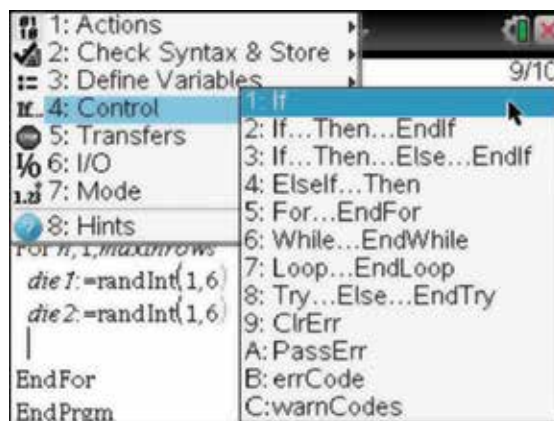
`die2:=randInt(1,6)`

for the second die.



12. Press:

- **MENU** 
- **4: Control** 
- **1: If** 

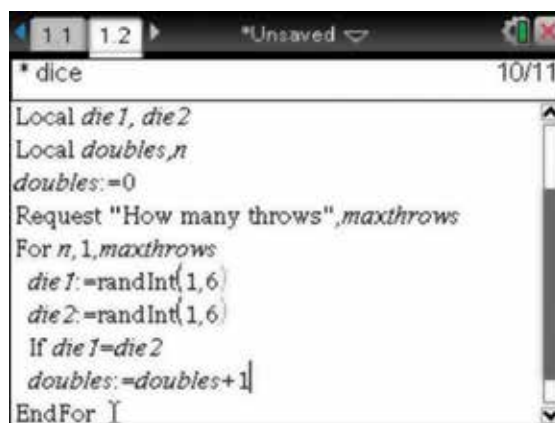


13. Complete the entry line as:




If $die1=die2$

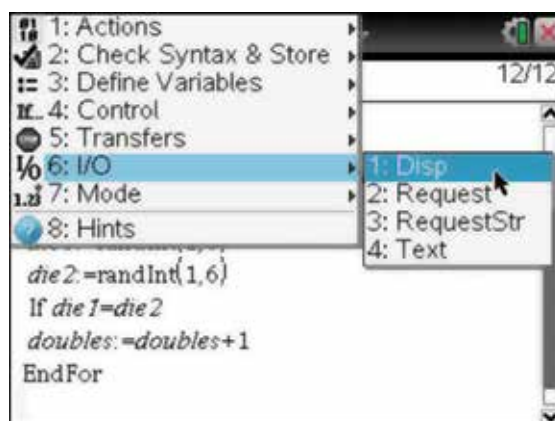
$doubles:=doubles+1$

Note: There is a difference between the stored or assigned equals to and the check in the single equals, which is used as a comparison.



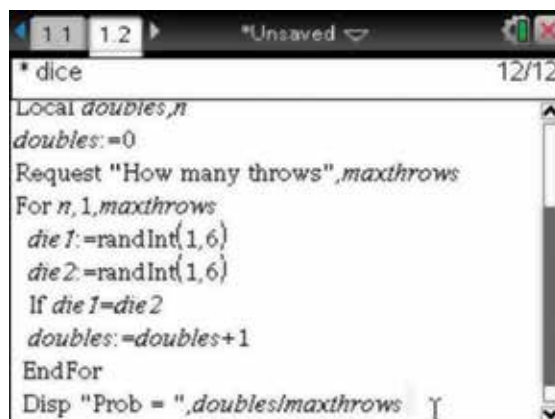
14. Now, move the cursor after EndFor and insert a new blank line. This is where we will display the output (the probability of doubles). Press:

- MENU 
- 6: I/O 
- 1: Disp 





15. After Disp, type in:

"Prob = ", $doubles/maxthrows$

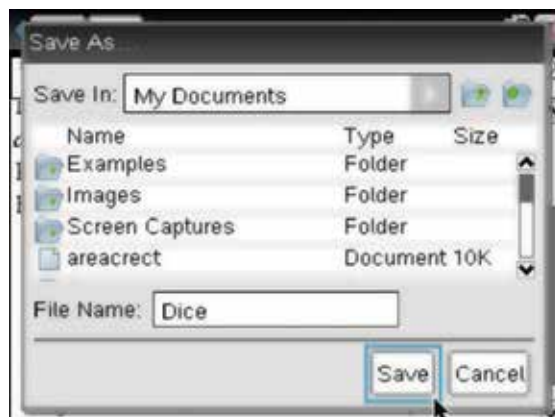


16. Save the document by pressing:




- ctrl 
- Save 

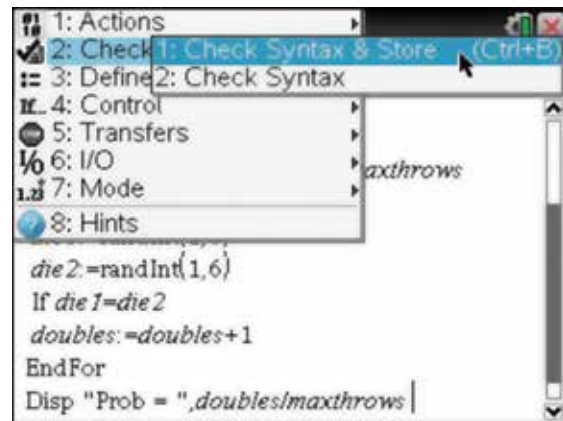
This saves the document in the default folder, 'My Documents'. Give the document the name 'Dice'.

Note that the document and program name do not have to be the same. Our program is now complete.



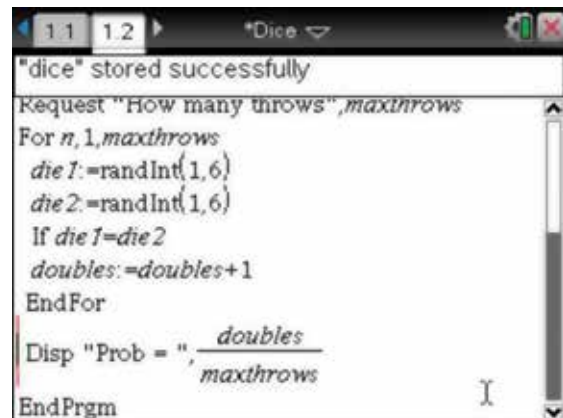
17. We should check our syntax often. To do this, press:

- **MENU** 
- **2: Check Syntax & Store** 
- **1: Check Syntax & Store** .





18. The program has been stored successfully. We have also resaved the program; notice that the asterisk in front of the document name has disappeared.

Now, provided you have typed the program in exactly as shown and there are no syntax errors, the program can be run. However, if you have syntax errors, you will be notified by an error message. You will have to identify the errors and correct them, then save the document.



19. To run our saved and stored program, go to the Calculator page by pressing:

- **ctrl** 
- .



Now press:

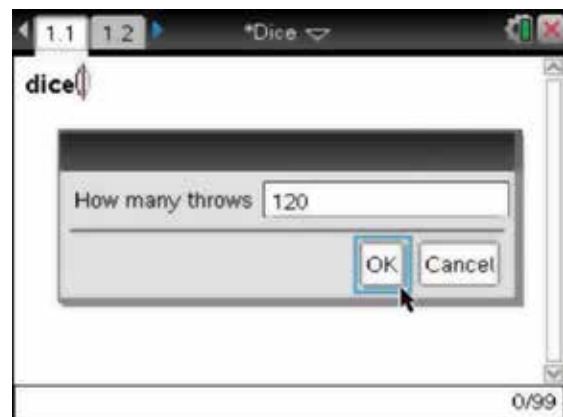
- **var** 
- **ENTER** .

This will paste **dice()** onto the Calculator page.

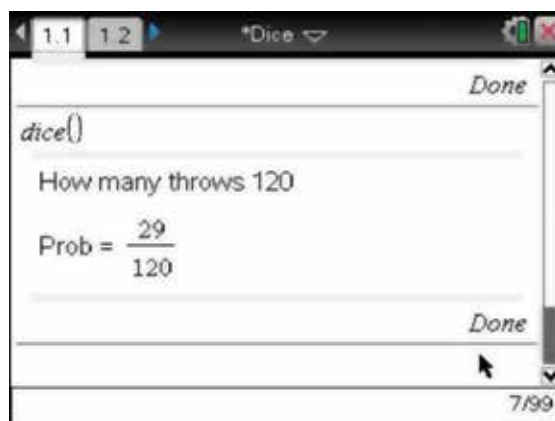
Press the enter key .



20. While the program is running, we are prompted by the Request statement for the value of the variable *maxthrows*. Type 120 into the dialog box. Use the tab key  to select OK, then press the enter key .



21. The output of the program is shown. The output could be different every time the program is run. Rerun the program again and again, typing *dice()* into the entry line. Use different values for *maxthrows*, such as 600 and 6000. What value is the answer for Prob approaching?

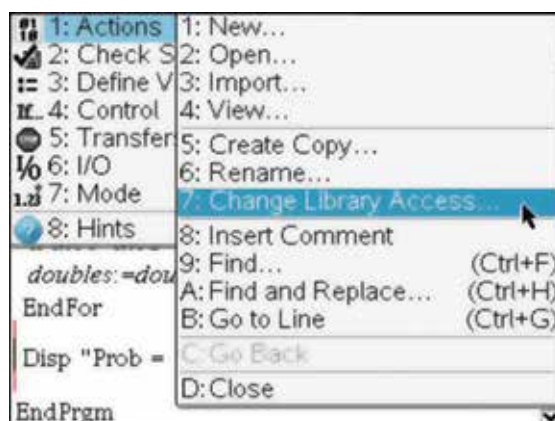


22. The program is available to be run only from within the saved document. Make the Program page the active page by pressing:

- **ctrl** (ctrl)
- **▶**.

We can change the library access by pressing:

- **MENU** (menu)
- **1: Actions** (1)
- **7: Change Library Access** (7).

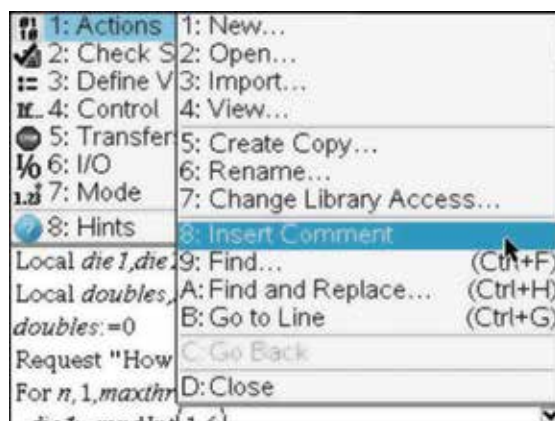


23. Using the Touchpad, choose LibPub (Show in Catalog). Press the enter key (enter), then select OK and press the enter key (enter) again.



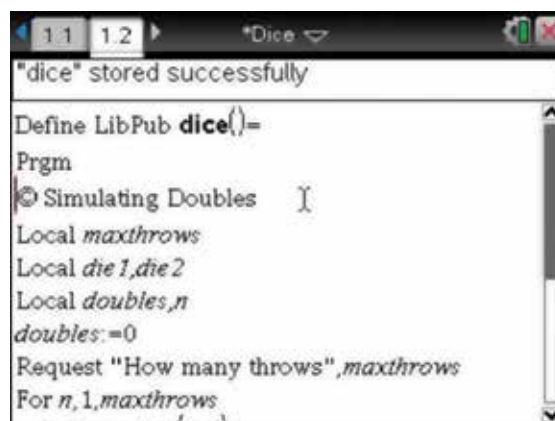
24. Move the cursor to before the first Local, press the enter key (enter) to insert a blank line, then press:

- **MENU** (menu)
- **1: Actions** (1)
- **8: Insert Comment** (8).



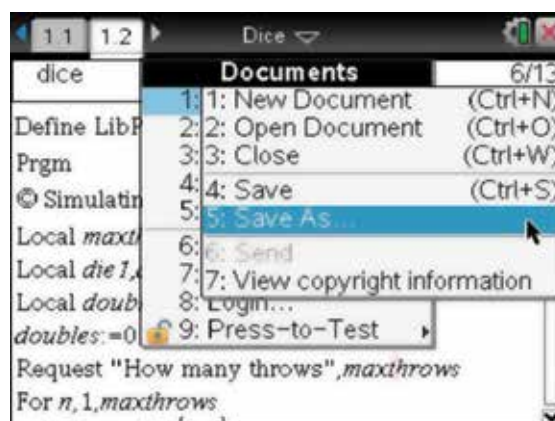
25. Type a comment on that line, such as:

© Simulating Doubles

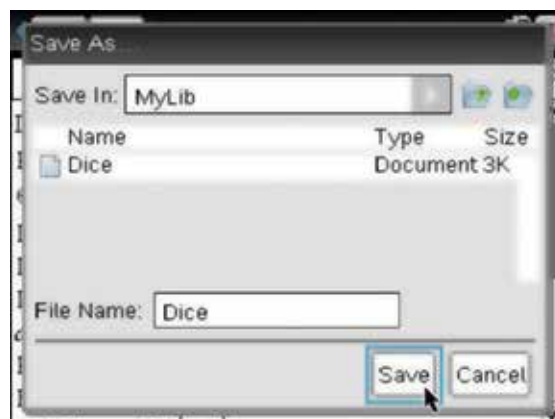


26. We need to store the document in the folder MyLib by pressing:

- **doc** (doc)
- **1: File** (1)
- **5: Save As** (5).

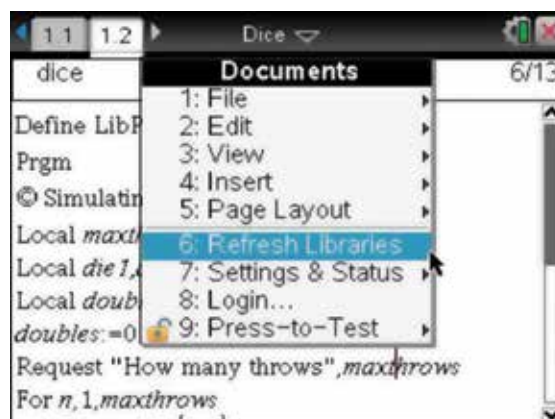


27. Save as the document as 'Dice', as before, but note that it must be stored in the MyLib folder.





28. Then refresh the libraries by pressing:

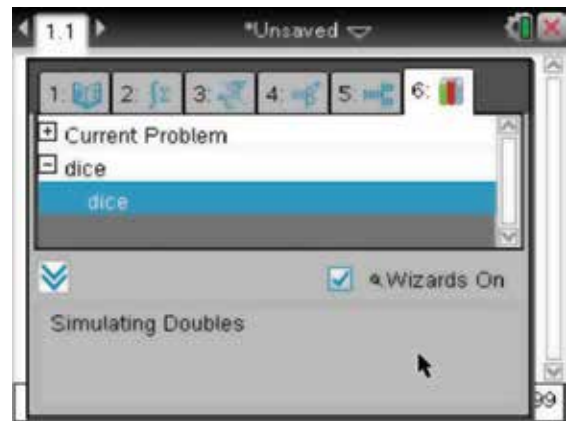
- **doc** (doc)
- **6: Refresh Libraries** (6).



29. Now, when we open any document and add a Calculator page, we can access the program Dice from the catalog by pressing:

- **catalog** 
- **6: 6**.

Press the enter key  to expand Current Problem.



30. We see that our comment is included as a help, and the program now behaves as any intrinsic application that can be run from any document.

The program runs correctly, but what happens if we enter 10.5 or a negative number in the dialog box for *maxthrows*? The final full program has been modified slightly and is listed below. It now has a loop around the Request statement and will accept only a positive integer as input for the variable *maxthrows*. However, if you enter an alphabetic character, the program will crash and terminate with error messages.

Define LibPub **dice()**=Prgm

© Simulating doubles

Local *maxthrows*

Local *die1, die2*

Local *doubles, n*

doubles:=0

Loop

Request "How many throws",*maxthrows*

If iPart(*maxthrows*)=*maxthrows* and *maxthrows*>0

Goto *validthrows*

EndLoopLbl *validthrows*

For *n, 1, maxthrows*

die1:=randInt(1,6)

die2:=randInt(1,6)

If *die1*=*die2*

doubles:=*doubles*+1

EndFor

Disp "Prob=",((*doubles*)/(*maxthrows*))

EndPrgm



CHAPTER 20

Problem solving

20.1 Matrices and simultaneous equations

Question 1

When the polynomial $x^3 + bx^2 + cx - 12$ is divided by $x + 3$, the remainder is 3. When it is divided by $x - 2$, the remainder is -2 . Find the values of b and c .

Question 2

Determine the values of p and q for which the simultaneous equations

$$-8x + 5y = 7$$

$$px - 3y = q$$

have:

- (a) a unique solution
- (b) an infinite number of solutions
- (c) no solutions.

Question 3

Given the system of simultaneous equations

$$x - 2y + 3z = 4$$

$$2x - 3y + 2z = 1$$

$$-3x + k^2y - z = k$$

find the values of k for which the equations have:

- (a) a unique solution
- (b) no solution
- (c) an infinite number of solutions (in this case, find the solution).

Question 4

A cubic polynomial $y = ax^3 + bx^2 + cx + d$ passes through the points $(1, 64)$, $(2, 45)$, $(3, 24)$ and $(4, 7)$.

Set up simultaneous equations for a , b , c and d .

Find the inverse of an appropriate 4×4 matrix; hence, find the exact values of a , b , c and d .

Sketch the graph of $y = ax^3 + bx^2 + cx + d$, stating the coordinates of all the turning points.

20.2 Curve sketching

Question 1

- (a) Find the values of k for which the graphs of $y = 2k - kx$ and $y = x^2 - x - 1$:
 - (i) intersect at two points
 - (ii) intersect at one point
 - (iii) do not intersect.
- (b) Find the values of b and k for which the graphs of $y = kx$ and $y = x^2 + bx + k^2$:
 - (i) intersect at two points
 - (ii) intersect at one point
 - (iii) do not intersect.

Question 2

Given that the function $f(x) = x^3 + bx^2 + cx + d$ crosses the y-axis at $y = 2$ and has a stationary point at $(1, 3)$, find the values of b , c and d , and sketch the curve, finding all the turning points, axial cuts and the point of inflection.

Question 3

Sketch the graph of the function

$$f(x) = \begin{cases} -2, & x > 2 \\ 4 - 3x, & -1 \leq x \leq 2 \\ 9 + 2x, & x < -1 \end{cases}$$

At what points is the function: (i) continuous? (ii) differentiable?

20.3 Calculus and function notation

Question 1

(a) Given that $y = f(x) \sin(2x)$ and $\frac{dy}{dx} = 2e^{2x}(\sin(2x) + \cos(2x))$, find the function $f(x)$.

(b) If $f(x) = e^{h(x)}$ and $f'(x) = 3\cos(3x)e^{h(x)}$, find the function $h(x)$.

Question 2

Which of the functions A to D below satisfy each of the following?

(a) $f(a + b) = f(a) + f(b)$

(b) $f(a + b) = f(a)f(b)$

(c) $f(ab) = f(a) + f(b)$

(d) $f(ab) = f(a)f(b)$

A. $f(x) = x$

B. $f(x) = e^x$

C. $f(x) = \log_e(x)$

D. $f(x) = x^2$

Question 3

Given the function

$$f(x) = \begin{cases} \frac{A}{(2x-1)^2}, & x \geq 2 \\ mx + 11, & x < 2 \end{cases}$$

find the values of A and m if f is a differentiable function.

Question 4

The tangent to the curve $y = \frac{1}{x}$ at the point where $x = a$ and $a > 0$ crosses the x -axis at R and crosses the y -axis at S .

Show that the area of the triangle ROS , where O is the origin, is independent of a and always is equal to 2.

Question 5

(a) Sketch the graph of $y = x^3 - 8x^2 + 4x + 48$, clearly showing the axial intercepts and the turning points. Determine the equation of the tangent to the curve $y = x^3 - 8x^2 + 4x + 48$ at the point where $x = 5$, and show that this tangent intersects the curve at one of the x -intercepts.

(b) Show that the tangent to the cubic $y = (x - \alpha)(x - \beta)(x - \gamma)$, where $\alpha < \beta < \gamma$, at the point where $x = \frac{\alpha + \beta}{2}$ crosses the x -axis at the point where $x = \gamma$.

Question 6

The area bounded by the curve $y = ax + b - x^2$, the x -axis, $x = 1$ and $x = 2$ is $2\frac{1}{6}$.

The area bounded by the curve $y = ax + b - x^2$, the x -axis, $x = 3$ and $x = 5$ is $11\frac{1}{3}$.

Find the values of a and b .

Question 7

The curve $y = x^3 + bx^2 + cx + d$ has a turning point at $(2, 34)$.

The area bounded by the curve, the coordinate axes and the line $x = 6$ is 120 square units.

Find the values of b , c and d .

Question 8

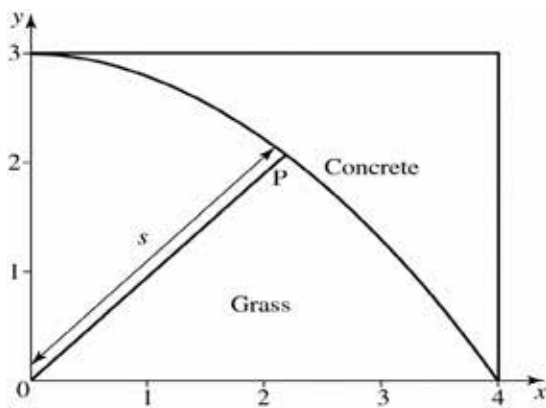
The curve $y = x^4 + ax^3 + bx^2 + cx + d$ has a turning point at $(2, 1)$ and crosses the y -axis at $y = 17$.

The area bounded by the curve, the coordinate axes and the line $x = 2$ is $8\frac{2}{3}$ square units.

Find the values of a , b , c and d .

Question 9

A rectangular garden patch has a length of 4 metres and a width of 3 metres as shown in the diagram below.



Coordinate axes have been placed with the origin O at the bottom left-hand corner. The lower section is to contain grass, and the upper section is to contain concrete. The patch is divided by a curve having the form $y = f(x)$. A straight-line path of distance s is required from the origin to a point P on the curve with coordinates $(u, f(u))$, where $0 \leq u \leq 4$. For each of the following models for $f(x)$, determine:

- (i) the values of a and b
- (ii) the distance s as a function of u
- (iii) the minimum distance s and the value of u for which it occurs
- (iv) the area of the grassed region.

- (a) $f(x) = ax^2 + b$
- (b) $f(x) = \sqrt{ax^2 + b}$
- (c) $f(x) = a + be^x$
- (d) $f(x) = a \log_e(bx + 5)$
- (e) $f(x) = a + \frac{b}{x-5}$

Which model gives the minimum distance for s ? Which model gives the largest value for the grassed region?

Question 10

(a) Find and sketch the area bounded by the curve $y = 8x - x^2 - 12$ and the x -axis.

Draw the rectangle that passes through the points $(2, 0)$, $(6, 0)$, $(6, 4)$ and $(2, 4)$.

Show that the area of the parabolic region is $\frac{2}{3}$ the area of the rectangle.

- (b) Consider the function $f(x) = -(x - \alpha)(x - \beta)$, where $\alpha > \beta$.
 Find the area bounded by the graph of $f(x)$ and the x -axis.
 Given the points $A(\alpha, 0)$ and $B(\beta, 0)$, let M be the coordinate of the maximum point on the graph of $y = f(x)$.
 The point C has the x -coordinate of A and the y -coordinate of M . The point D has the x -coordinate of B and the y -coordinate of M .
 Show that the area of the parabolic region is $\frac{2}{3}$ the area of the rectangle $ABCD$.

20.4 Complex numbers

Question 1

- (a) Sketch the graphs of each of the following pairs of quadratics on the one set of axes, stating the range and coordinates of the turning point and any axial cuts.

What do you observe about the pairs of quadratic functions?

Investigate the exact complex roots and compare them with the roots of the other quadratic.

- (i) $y_1 = x^2 - 4x + 13$ and $y_2 = -x^2 + 4x + 5$
 (ii) $y_1 = x^2 + 6x + 16$ and $y_2 = -x^2 - 6x - 2$
 (iii) $y_1 = 2x^2 - 5x + 9$ and $y_2 = -2x^2 + 5x + \frac{11}{4}$
 (b) If $y_1 = ax^2 + bx + c$ and $y_2 = -ax^2 - bx + k$ satisfy the same properties as above, assuming that $a > 0$, express c in terms of k , a and b . Using your relationships, verify your results to (i), (ii) and (iii) in part (a).

Question 2

Given that $4 + 5i$ is a root of the equation $z^3 + bz^2 + cz + 82 = 0$, find the values of the real constants b and c , and find all the roots.

Question 3

Solve for z if $z^3 + 64i = 0$.

20.5 Binomial theorem

Question 1

Find the value of d if the coefficient of x^3 in the expansion of $\left(2x^2 + \frac{d}{x}\right)^9$ is $-6\,300\,000$.

Question 2

In the expansion of $\left(ax + \frac{b}{x}\right)^8$, the term independent of x is $3\,543\,750$, and the coefficient of x^2 is $1\,701\,000$. Find the values of a and b .

20.6 Probability

Question 1

In an examination, it is found that 20% of the students scored more than 75 marks, while 30% scored less than 40 marks. Assuming the test marks are normally distributed, find the mean and standard deviation of the scores, correct to 1 decimal place.

Question 2

The probability density function of a continuous random variable X is given by

$$f(x) = \begin{cases} kxe^{\frac{-x}{2}}, & x \geq 0 \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Show that $k = \frac{1}{4}$.
 (b) Sketch the graph of $f(x)$.

- (c) Find the mode.
- (d) Find $E(X)$.
- (e) Find $\text{var}(X)$.
- (f) Find $\Pr(X > 4)$.
- (g) Find the median.

20.7 Linear regression

Question 1

The prices and sizes of computer hard disks are given in the following table.

Size (GB)	160	320	500	640	750	1000
Price \$	57	73	93	103	127	155

- (a) Plot the price versus size, and find the linear equation of best fit.
- (b) Does a quadratic or cubic function fit the data better?

Question 2

The following data gives the temperature, T °C, of a cup of coffee after t minutes.

Time (minutes)	0	1	2	3	4	5	6	7	8	9	10
Temperature (°C)	75.5	75.0	73.0	71.5	70.0	68.5	67.0	65.5	64.5	63.0	62.0

Plot the data and model the temperature versus time as:

- (a) a linear function, $T = at + b$
- (b) a quadratic function, $T = at^2 + bt + c$
- (c) an exponential function, $T = ab^t$.

20.8 Differential equations

Question 1

- (a) If $y = Cx^2e^{-3x}$ satisfies the differential equation $\frac{d^2y}{dx^2} + 6\frac{dy}{dx} + 9y = 6e^{-3x}$, find the value of C .
- (b) If $y = Ax^2e^{-kx}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 2k\frac{dy}{dx} + k^2y = Be^{-kx}$, show that $B = 2A$.

Question 2

A simple model is given by $\frac{dn}{dt} = kn^\alpha$, $n(0) = n_0$, where n_0 is the initial number of bacteria in the culture, and k and α are constants.

- (a) Find general expressions for the number of bacteria in the sample after a time t hours in terms of k and n_0 for the cases when:
 - (i) $\alpha = -1$
 - (ii) $\alpha = \frac{1}{2}$
 - (iii) $\alpha = 1$.
- (b) Initially, the number of bacteria in the sample was found to be 16, and this number grows to 100 after 4 hours. For each of the cases in part (a), find the value of k and sketch the graphs of n versus t using an appropriate domain. For each model, find and tabulate the number of bacteria present after 2, 4, 6, 8, 10 and 12 hours. Realistically, the number of bacteria cannot rise beyond 200. Discuss your results and compare the limitations of each model.
- (c) A more sophisticated mathematical model for the rate at which the number of bacteria increases is given by $\frac{dn}{dt} = r\left(1 - \frac{n}{k}\right)n$, $n(0) = n_0$, where n_0 is the initial number of bacteria in the culture, and k and r are constants.
 - (i) Using this model, show that the number of bacteria in the sample after a time t hours is given by $n = n(t) = \frac{n_0k}{n_0 + (k - n_0)e^{-rt}}$.

- (ii) Given that $k = 200$, the number of bacteria in the sample initially was 16 and this number grows to 100 after 4 hours, show that $r = \frac{1}{4} \log_e \left(\frac{23}{2} \right)$.
- (iii) Sketch the graph of n versus t , stating an upper limit for the number of bacteria. Using this model, find and tabulate the number of bacteria present after 2, 4, 6, 8, 10 and 12 hours. Discuss your results.
- (iv) Using this model, find at what time the number of bacteria is increasing the most rapidly. Explain this result with reference to the graph.



Digital doc

Worked solutions

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