MATHENICAL METHODS UNIT 1

Chapter 1 – Reviewing Linear Equations

Chapter 2 – Coordinate geometry & linear relations

Key knowledge

- the equation of a straight line, gradient and axis intercepts, midpoint of a line segment, distance between two points, and parallel and perpendicular lines
- the definition of a function, the concepts of domain, codomain and range, notation for specification of the domain (including the concept of maximal, natural or implied domain), codomain and range and rule of a function

Key skills

- determine by hand the length of a line segment and the coordinates of its midpoint, the equation of a straight line given two points or one point and gradient, and the gradient and equation of lines parallel and perpendicular to a given line through some other point
- substitute integer, simple rational and irrational numbers in exact form into expressions, including rules of functions and relations, and evaluate these by hand
- re-arrange and solve simple algebraic equations and inequalities by hand
- expand and factorise linear expressions with integer coefficients by hand

Exercise Questions

1A LINEAR EQUATIONS
   6aceg, 7aceg, 8, 9
1B CONSTRUCTING LINEAR EQUATIONS
   3, 6, 8, 9, 10, 11, 12, 13, 14
1C SIMULTANEOUS EQUATIONS
   1c, 2c, 3acegi, 4ac
1D CONSTRUCTING SIMULTANEOUS LINEAR EQUATIONS
   1, 3c, 6, 8, 10, 12, 15, 16, 17
1E SOLVING LINEAR INEQUALITIES
   1aceg, 2aceg, 3, 4, 5
1F USING AND TRANSPosing FORMULAS
   4ace, 6, 7, 9, 10

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   1behk, 4bej, 6, 7cd, 8ef, 9cd
2C THE EQUATION OF A STRAIGHT LINE
   1ac, 2ac, 3ac, 4ac, 5b, 6b, 7, 8, 9ac, 10cd, 11ef, 13, 15cd
2D GRAPHING STRAIGHT LINES
   1bd, 2bd, 3bd, 4ac, 5bd, 6bd, 7aceg, 8bd, 9def, 10bd, 11
2E PARALLEL AND PERPENDICULAR LINES
   1cdgh, 2bc, 3cd, 4, 5, 7, 8, 9, 10
2F FAMILIES OF STRAIGHT LINES
   3, 5, 6, 8, 9
2G LINEAR MODELS
   2, 4, 5, 6, 7, 8, 9, 10
2H SIMULTANEOUS LINEAR EQUATIONS
   7, 8, 9, 10, 11, 13, 14, 15

MORE RESOURCES
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1A: Linear Equations

A linear equation (in one unknown) is a particular type of polynomial equation in which the variable is to the first power. The following are examples of linear equations:

\[ 3x - 5 = 11, \quad 7 - 2t = 8t - 11, \quad \frac{x - 3}{4} + \frac{2x - 5}{3} = 11 \]

In each of these equations, the variable is to the first power. The following are examples of non-linear polynomial equations:

\[ x^2 - x - 12 = 0 \text{ (quadratic)}, \quad 2x^3 - x = 0 \text{ (cubic)}, \quad x^4 = 16 \text{ (quartic)} \]

In each of these equations, the highest power of the variable is not the first power (not 1).

**Solving linear equations**

**Linear equations of the form** \( ax + b = c \)

Many linear equations that arise in applications are of the form \( ax + b = c \).

Given an equation, an equivalent equation can be formed by:

- Adding or subtracting the same number on both sides of the equation
- Multiplying or dividing both sides of the equation by the same non-zero number.

**Equations with the unknown on both sides**

Group all the terms containing the variable on one side of the equation and the remaining terms on the other side.

**Example 2**

Solve \( 4x + 3 = 3x - 5 \).

**Equations containing fractions**

A frequently used first step is to multiply both sides of the equation by the lowest common multiple of the denominators of the fractions.

**Example 5**

Solve \( \frac{x - 3}{2} - \frac{2x - 4}{3} = 5 \)
Literal equations

An equation for the variable \( x \) in which all the coefficients of \( x \), including the constants, are pronumerals is known as a literal equation.

Example 6

Solve \( ax + b = cx + d \) for \( x \)

Using the TI-Nspire

- To find the solution to the linear equation, use a **Calculator** application.
- Select (menu) > **Algebra > Solve**.
- Enter the equation

\[
\frac{x - 3}{2} - \frac{2x - 4}{3} = 5
\]

- Press (enter) to obtain the solution.

**Note:** A template for fractions may be obtained by pressing [ctrl] \(+\)

For more details on the use of the calculator refer to the TI-Nspire appendix in the Interactive Textbook.
1B Constructing linear equations

Example 7
A chef uses the following rule for cooking a turkey: ‘Allow 30 minutes for each kilogram weight of turkey and then add an extra 15 minutes.’ If the chef forgot to weigh a turkey before cooking it, but knew that it had taken 3 hours to cook, calculate how much it weighed.

Example 8
Find the area of a rectangle whose perimeter is 1.08m, if it is 8cm longer than it is wide.

Example 9
Adam normally takes 5 hours to travel between Higett and Logett. One day he increases his speed by 4 km/h and finds the journey from Higett to Logett takes half an hour less than the normal time. Find his normal speed.
1C Simultaneous equations

A linear equation that contains two unknowns, e.g. \(2y + 3x = 10\), does not have a single solution. Such an equation actually expresses a relationship between pairs of numbers, \(x\) and \(y\), that satisfy the equation. If all possible pairs of numbers \((x, y)\) that satisfy the equation are represented graphically, the result is a straight line; hence the name linear relation.

If the graphs of two such equations are drawn on the same set of axes, the lines will intersect at one point only. Hence there is one pair of numbers that will satisfy both equations simultaneously.

Example 10
Solve the equations \(2x - y = 4\) and \(x + 2y = -3\).

Method 1: Substitution

Method 2: Elimination

Using the TI-Nspire

Simultaneous equations can be solved in a Calculator application.

- Use \(\text{menu} > \text{Algebra} > \text{Solve System of Equations} > \text{Solve System of Equations.}\)
- Complete the pop-up screen.

- Enter the equations as shown to give the solution to the simultaneous equations \(2x - y = 4\) and \(x + 2y = -3\).

Note: The solution can also be found with \(\text{solve}(2x - y = 4 \text{ and } x + 2y = -3, x, y)\).
Graphs application

The simultaneous equations can also be solved graphically in a Graphs application.

Entering the equations:

- The equations can be entered directly in the form $a \cdot x + b \cdot y = c$ using menu > Graph Entry/Edit > Equation > Line > $a \cdot x + b \cdot y = c$.
- Enter the equations as shown.

**Hint:** Use ▼ to enter the second equation.

Alternatively:

- The equations can be rearranged to make $y$ the subject. The equations in this form are $f_1(x) = 2x - 4$ and $f_2(x) = \frac{-3 - x}{2}$.
- Enter these in the default function entry line.

**Note:** If the entry line is not visible, press tab or double click in an open area. Pressing enter will hide the entry line.

Finding the intersection point:

- Use menu > Geometry > Points & Lines > Intersection Point(s).
- Use the touchpad to move the cursor to select each of the two graphs.

- The intersection point’s coordinates will appear on the screen. Press esc to exit the Intersection Point(s) tool.

**Note:** You can also find the intersection point using menu > Analyze Graph > Intersection.
The geometry of simultaneous equations

Two distinct straight lines are either parallel or meet at a point.

There are three cases for a system of two linear equations with two variables.

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</tbody>
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1D Constructing simultaneous linear equations

Problems involving two unknowns can often be solved by using simultaneous equations with two variables. The following examples show how this may be done.

Example 11
The sum of two numbers is 24 and their difference is 96. Find the two numbers.

Example 12
3 kg of jam and 2 kg of butter cost $29, and 6 kg of jam and 3 kg of butter cost $54. Find the cost per kilogram of jam and butter.
1E Solving linear inequalities

An inequality is a mathematical statement that contains an inequality symbol rather than an equals sign: for example, \(2x + 1 < 4\). When you solve the inequality \(2x + 1 < 4\), you answer the question:

‘Which numbers \(x\) satisfy the property that \(2x + 1\) is less than 4?’

You will find that your answers can be described using a number line. This is a good way to represent the solution, as there are infinitely many numbers that satisfy an inequality such as \(2x + 1 < 4\). For example:

\[
2(1) + 1 = 3 < 4, \quad 2(0) + 1 = 1 < 4, \quad 2\left(\frac{1}{2}\right) + 1 = 2 < 4, \quad 2(-1) + 1 = -1 < 4
\]

To solve linear inequalities, proceed exactly as for equations with the following exception:

- When multiplying or dividing both sides by a negative number, the ‘direction’ of the inequality symbol is reversed.

Example 13
Solve the inequality \(2x + 1 < 4\).

Example 14
Solve the inequality \(3 - 2x \leq 4\).
Example 15
Solve the inequality \( \frac{2x+3}{5} \geq \frac{3-4x}{3} + 2 \)

Using the TI-Nspire
The inequality can be solved in a **Calculator** application.

- Choose \( \text{solve(} \) from the **Algebra** menu to give the solution to

\[
\frac{2x + 3}{5} \geq \frac{3 - 4x}{3} + 2
\]

**Note:** For the inequality signs template, press \( \text{ctrl}[\text{=}] \).

1F Using and transposing formulas

An equation containing symbols that states a relationship between two or more quantities is called a formula. An example of a formula is \( A = lw \) (area = length \( \times \) width). The value of \( A \), called the subject of the formula, can be found by substituting in given values of \( l \) and \( w \).

Example 16
Find the area of a rectangle with length \( (l) \) 10 cm and width \( (w) \) 4 cm.
Sometimes we wish to rewrite a formula to make a different symbol the subject of the formula. This process is called transposing the formula. The techniques for transposing formulas include those used for solving linear equations detailed in Section 1A.

**Example 17**

Transpose the formula \( v = u + at \) to make \( a \) the subject.

If we wish to evaluate an unknown that is not the subject of the formula, we can either substitute the given values for the other variables and then solve the resulting equation, or we can first transpose the formula and then substitute the given values.

**Example 19**

A path \( x \) metres wide surrounds a rectangular lawn. The lawn is \( l \) metres long and \( b \) metres wide. The total area of the path is \( A \) m\(^2\).

a) Find \( A \) in terms of \( l \), \( b \) and \( x \).

b) Find \( b \) in terms of \( l \), \( A \) and \( x \).

**Example 20**

For each of the following, make \( c \) the subject of the formula:

a) \( e = \sqrt{3c - 7a} \)

b) \( \frac{1}{a} - \frac{1}{b} = \frac{1}{c - 2} \)
Chapter 1 - Summary

- A linear equation is one in which the variable is only to the first power, i.e. \( x^1 \) or \( x \).
- It is often helpful to look at how the equation has been constructed so that the steps necessary to ‘undo’ the equation can be identified. It is most important that the steps taken to solve the equation are done in the correct order.
- An equation for the variable \( x \) in which all the coefficients of \( x \), including the constants, are pronumerals is known as a literal equation: for example, \( ax + b = c \).
- The two methods for solving simultaneous linear equations are substitution and elimination.
- An inequality is a mathematical statement that contains an inequality symbol rather than an equals sign: for example, \( 2x + 1 < 4 \).
- To solve linear inequalities, proceed exactly as for equations except that, when multiplying or dividing both sides by a negative number, the ‘direction’ of the inequality symbol is reversed.
- An equation containing symbols that states a relationship between two or more quantities is called a formula. An example of a formula is \( A = lw \) (area = length \( \times \) width). The subject of this formula is \( A \).
- If we wish to evaluate an unknown that is not the subject of the formula, we can either substitute the given values for the other variables and then solve the resulting equation, or we can first transpose the formula and then substitute the given values.

Chapter 2 – Coordinate geometry & linear relations

Introduction

The number plane (Cartesian plane) is divided into four quadrants by two perpendicular axes. These axes intersect at a point called the origin. The position of any point in the plane can be represented by an ordered pair of numbers \((x, y)\), called the coordinates of the point. Given the coordinates of two points, we can find the equation of the straight line through the two points, the distance between the two points and the midpoint of the line segment joining the points. These are the beginning ideas of coordinate geometry. The topic of calculus, which is introduced later in this book, builds on these ideas.

A relation is defined as a set of ordered pairs in the form \((x, y)\). Sometimes we can give a rule relating the \( x \)-value to the \( y \)-value of each ordered pair, such as \( y = 2x + 1 \), and this is a more convenient way of describing the relation. A relation may also be represented graphically on a set of axes. If the resulting graph is a straight line, then the relation is called a linear relation.
2A Distance and midpoints

**Midpoint of a line segment**

A line segment parallel to an axis

![Diagram of a line segment with midpoint](image)

A line segment not parallel to one of the axes

Let \( P(x, y) \) be the midpoint of the line segment joining \( A(x_1, y_1) \) and \( B(x_2, y_2) \), where the line through \( A \) and \( B \) is not parallel to either axis.

Let points \( C \) and \( D \) be chosen so that \( AC \) and \( PD \) are parallel to the \( x \)-axis, and \( PC \) and \( BD \) are parallel to the \( y \)-axis.

The coordinates of the midpoint \( P \) of the line segment \( AB \) joining \( A(x_1, y_1) \) and \( B(x_2, y_2) \) are

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)
\]

That is, we take the average of the \( x \)-coordinates and the average of the \( y \)-coordinates.

**Example 1**

Find the midpoint of the line segment joining \( A(2, 6) \) with \( B(-3, -4) \).

**The distance between two points**

The distance between given points \( A(x_1, y_1) \) and \( B(x_2, y_2) \) can be found by applying Pythagoras’ theorem to the triangle \( ABC \):

Therefore, the distance between the two points \( A(x_1, y_1) \) and \( B(x_2, y_2) \) is

\[
AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

**Example 2**

Calculate the distance \( E-F \) if \( E \) is \((-3, 2)\) and \( F \) is \((4, -2)\).
2B The Gradient of a straight line

The gradient of a straight line is defined as the ratio of the change in the vertical direction (rise) to the change in the horizontal direction (run). Mathematically, it is given by:

\[ m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \]

**Example 3**
Find the gradient of each line:

- **Graph a**
  - Gradient: \( y_2 - y_1 / x_2 - x_1 \)

- **Graph b**
  - Gradient: \( y_2 - y_1 / x_2 - x_1 \)

**Notes:**
- The gradient of a line that slopes **upwards** from left to right is positive, as illustrated in Example 3a.
- The gradient of a line that slopes **downwards** from left to right is negative, as illustrated in Example 3b.
- The gradient of a **horizontal line** (parallel to the x-axis) is zero, since \( y_2 - y_1 = 0 \).
- The gradient of a **vertical line** (parallel to the y-axis) is undefined, since \( x_2 - x_1 = 0 \).

**Example 4**
Find the gradient of the line that passes through the points (1, 6) and (−3, 7).

**The tangent of the angle of a slope**

**Positive gradient**

From the diagram, it follows that:

\[ m = \tan \theta \]

where \( \theta \) is the angle that the line makes with the positive direction of the x-axis.
Negative gradient
From the diagram, it follows that:
\[ m = -\tan \alpha = \tan \theta \]

Example 6
Determine the gradient of the line passing through the points (5, -3) and (-1, 5) and the angle \( \theta \) that the line makes with the positive direction of the \( x \)-axis.

2C The equation of a straight line

In this section we discuss different ways of determining the equation of a straight line. In general two ‘independent pieces of information’ are required. The following given information is considered:

- Gradient and \( y \)-axis intercept
- Gradient and a point
- Two points.

Gradient–intercept form of the equation of a straight line

The line \( y = mx+c \)

- The line with gradient \( m \) and \( y \)-axis intercept \( c \) has equation \( y = mx + c \).
- Conversely, the line with equation \( y = mx + c \) has gradient \( m \) and \( y \)-axis intercept \( c \).

Example 8
Find the equation of the line with gradient \(-3\) and \( y \)-axis intercept 5.

Example 9
State the gradient and \( y \)-axis intercept of the line \( 3y + 6x = 9 \).
Point–gradient form of the equation of a straight line

The point–gradient form of the equation of a straight line is

\[ y - y_1 = m(x - x_1) \]

where \((x_1, y_1)\) is a point on the line and \(m\) is the gradient.

Example 11
Find the equation of the line that passes through the point \((3, 2)\) and has a gradient of \(-2\).

A line through two points
To find the equation of the line through two given points \((x_1, y_1)\) and \((x_2, y_2)\), first find the gradient

\[ m = \frac{y_2 - y_1}{x_2 - x_1} \]
then use the point-gradient form

\[ y - y_1 = m(x - x_1) \]

We can also find the equation directly by taking the point \(P(x, y)\) and noting that:

\[ m = \frac{y - y_1}{x - x_1} \]

Example 13
Find the equation of the straight line with \(y\)-axis intercept \(-3\) which passes through the point with coordinates \((1, 10)\).
Two intercepts

A special case is when the line intercepts both axes.

The intercept form of the equation of a straight line is

\[ \frac{x}{a} + \frac{y}{b} = 1 \]

where \( a \) and \( b \) are the \( x \)-axis intercept and \( y \)-axis intercept respectively.

**Vertical and horizontal lines**

- If a line is horizontal, then its gradient \( m = 0 \) and its equation is simply \( y = c \), where \( c \) is the \( y \)-axis intercept.
- If a line is vertical, then its gradient is undefined and its equation is \( x = a \), where \( a \) is the \( x \)-axis intercept.

**General form of the equation of a straight line**

We have seen that all points on the line through two given points satisfy an equation of the form \( mx + ny + p = 0 \), with \( m \) and \( n \) not both 0. Conversely, any ‘linear equation’ \( mx + ny + p = 0 \) is the equation of a (straight) line. This is called the general form of the equation of a line.
Example 16
Sketch the graph of \( y = 2x - 6 \) by first finding the intercepts

**Using the TI-Nspire**
To sketch the graph of \( 6x + 3y = 9 \):

- Open a **Graphs** application: press \( \text{menu} \) and select the **Graphs** icon, or use \( \text{ctrl} \) and select **Add Graphs**.
- Equations of the form \( ax + by = c \) can be entered directly using \( \text{menu} \) > **Graph Entry/Edit** > **Equation** > **Line**. Enter as \( 6x + 3y = 9 \).

**Note:** The window settings \( \text{menu} \) > **Window/Zoom** > **Window Settings** will have to be changed if the axis intercepts do not appear on the screen.

- The axis intercepts can be found using \( \text{menu} \) > **Geometry** > **Points & Lines** > **Intersection Point(s)**. Select the \( x \)-axis and the graph to display the \( x \)-axis intercept. Select the \( y \)-axis and the graph to display the \( y \)-axis intercept.
- To show the coordinates of these points, use \( \text{menu} \) > **Actions** > **Coordinates and Equations** and double click on each of the points.
- Press \( \text{esc} \) to exit the **Coordinates and Equations** tool.
Example 17
For each of the following lines, find the magnitude of the angle \( \theta \) (correct to two decimal places) that the line makes with the positive direction of the \( x \)-axis:

a) \( y=2x+3 \)

b) \( 3y=3x-6 \)

---

2E Parallel and perpendicular lines

**Parallel lines**
- Two non-vertical lines are parallel if they have the same gradient.
- Conversely, if two non-vertical lines are parallel, then they have the same gradient.

**Perpendicular lines**
- Two lines with gradients \( m_1 \) and \( m_2 \) (both non-zero) are perpendicular if and only if \( m_1m_2 = -1 \).

Example 18
Find the equation of the straight line which passes through \( (1, 2) \) and is:

a) parallel to the line with equation \( 2x - y = 4 \)

b) perpendicular to the line with equation \( 2x - y = 4 \).
Example 19
The coordinates of the vertices of a triangle $ABC$ are $A(0, -1)$, $B(2, 3)$ and $C(3, -2 \frac{1}{2})$. **Show that** the side $AB$ is perpendicular to the side $AC$.

2F Families of straight lines

3 families of straight lines:
- $y = mx$, where the gradient $m$ of the lines varies – the graphs are the straight lines through the origin.
- $y = 3x + c$, where the $y$-axis intercept $c$ of the lines varies – the graphs are the straight lines with gradient 3.
- $y = mx + 2$, where the gradient $m$ of the lines varies – the graphs are the straight lines with $y$-axis intercept 2.

The variable $m$ is called a parameter. Some graphs in this family are illustrated.

**Example 21**

A family of lines have equations of the form $y = mx + 2$, where $m$ is a negative number.

a) Find the $x$-axis intercept of a line in this family in terms of $m$.

b) For which values of $m$ is the $x$-axis intercept greater than 3?

c) Find the equation of the line perpendicular to the line $y = mx + 2$ at the point $(0,2)$.
2G Linear Models

There are many practical situations where a linear relation can be used.

**Example 22**
Austcom’s rates for local calls from private telephones consist of a quarterly rental fee of $50 plus 25c for every call. Construct a cost function that describes the quarterly telephone bill and sketch the linear graph for this.

**Example 23**
A car starts from point $A$ on a highway 10 kilometres past the Wangaratta post office. The car travels at a constant speed of 90 km/h towards picnic stop $B$, which is 120 kilometres further on from $A$. Let $t$ hours be the time after the car leaves point $A$.

a) Find an expression for the distance $d_1$ of the car from the post office at time $t$ hours.

b) Find an expression for the distance $d_2$ of the car from point $B$ at time $t$ hours.

c) On separate sets of axes sketch the graphs of $d_1$ against $t$ and $d_2$ against $t$ and state the gradient of each graph.
2H Simultaneous linear equations

The geometry of simultaneous equations
There are three possible outcomes when considering a system of two simultaneous linear equations in two unknowns:

- There is a unique solution. (Lines intersect at a point.)
- There are infinitely many solutions. (Lines coincide.)
- There is no solution. (Lines are parallel.)

Example 24
Explain why the simultaneous equations $2x + 3y = 6$ and $4x + 6y = 24$ have no solution.

Example 26
The family of lines $y = mx + 2$ with varying gradient $m$ all pass through the point $(0, 2)$.

a For what values of $m$ does the line $y = mx + 2$ not intersect the line $y = 5x - 3$?

b For what values of $m$ does the line $y = mx + 2$ intersect the line $y = 5x - 3$?

c If the line $y = mx + 2$ intersects the line $y = 5x - 3$ at the point $(5,22)$, find the value of $m$. 

Using the TI-Nspire
Simultaneous equations can be solved in a Calculator application.

- Use (menu) > Algebra > Solve System of Equations > Solve System of Equations.
- Complete the pop-up screen.
Example 29
Consider the simultaneous linear equations \((m - 2)x + y = 2\) and \(mx + 2y = k\). Find the values of \(m\) and \(k\) such that the system of equations has:

a) no solution

b) infinitely many solutions

c) a unique solution.

Applications of Simultaneous equations
Example 30
There are two possible methods for paying gas bills:

- **Method A**: A fixed charge of $25 per quarter + 50c per unit of gas used
- **Method B**: A fixed charge of $50 per quarter + 25c per unit of gas used

Determine the number of units which must be used before method B becomes cheaper than method A.
Example 31

Robyn and Cheryl race over 100 metres. Robyn runs so that it takes \( a \) seconds to run 1 metre, and Cheryl runs so that it takes \( b \) seconds to run 1 metre. Cheryl wins the race by 1 second. The next day they again race over 100 metres but Cheryl gives Robyn a 5-metre start so that Robyn runs 95 metres. Cheryl wins this race by 0.4 seconds. Find the values of \( a \) and \( b \) and the speed at which Robyn runs.