6.1 Overview

Why learn this?
Humans must measure! How much paint or carpet will you need to redecorate your bedroom? How many litres of water will it take to fill the new pool? How far is it to the end of the universe? These are just a few examples of where measurements skills are needed. Measuring tools have advanced significantly in their capacity to measure extremely small and extremely large amounts, leading to many breakthroughs in medicine, engineering, science, architecture and astronomy.

What do you know?
1 THINK List what you know about measurement. Use a thinking tool such as a concept map to show your list.
2 PAIR Share what you know with a partner and then with a small group.
3 SHARE As a class, create a thinking tool such as a large concept map wheel that shows your class’s knowledge of measurement.

Learning sequence
6.1 Overview
6.2 Area
6.3 Total surface area
6.4 Volume
6.5 Review
Watch this video
The story of mathematics: Australian megafauna
SEARCHLIGHT ID: eles-1845
6.2 Area

- The **area** of a figure is the amount of surface covered by the figure.
- The units used for area are mm$^2$, cm$^2$, m$^2$, km$^2$ or ha (hectares), depending upon the size of the figure.

$$1 \text{ ha} = 10000 \text{ (or } 10^4) \text{ m}^2$$

- There are many real-life situations that require an understanding of the area concept. Some are, ‘the area to be painted’, ‘the floor area of a room or house’, ‘how much land one has’ and ‘how many tiles are needed for a wall’.
- It is important that you are familiar with converting units of area.

**Using area formulas**

- The area of many plane figures can be found by using a formula. The table below shows the formula for the area of some common shapes.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Square</td>
<td>$A = l^2$</td>
</tr>
<tr>
<td>2. Rectangle</td>
<td>$A = lw$</td>
</tr>
<tr>
<td>3. Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
</tr>
<tr>
<td>4. Trapezium</td>
<td>$A = \frac{1}{2}(a + b)h$</td>
</tr>
<tr>
<td>5. Circle</td>
<td>$A = \pi r^2$</td>
</tr>
<tr>
<td>6. Parallelogram</td>
<td>$A = bh$</td>
</tr>
</tbody>
</table>
### Shape | Formula
--- | ---
7. Sector | \[ A = \frac{\theta^\circ}{360^\circ} \times \pi r^2 \]
8. Kite (including rhombus) | \[ A = \frac{1}{2}xy, \text{ where } x \text{ and } y \text{ are diagonals.} \]
9. Ellipse | \[ A = \pi ab, \text{ where } a \text{ and } b \text{ are the lengths of the semi-major and semi-minor axes respectively.} \]

**Note:** A calculator uses a stored value for \( \pi \) of approximately 3.141592654. Before calculators were in common usage, \( \pi \) was often taken to be approximately \( \frac{22}{7} \) or 3.14. You are advised to use the \( \pi \) button on your calculator rather than \( \frac{22}{7} \) or 3.14.

### Heron’s formula
- If the lengths of all three sides of a triangle are known, its area, \( A \), can be found by using **Heron’s formula:**
\[
A = \sqrt{s(s-a)(s-b)(s-c)}
\]
where \( a, b \) and \( c \) are the lengths of the three sides and \( s \) is the semi-perimeter where
\[
s = \frac{a + b + c}{2}.
\]

#### WORKED EXAMPLE 1
**Find the areas of the following plane figures, correct to 2 decimal places.**

**THINK**
1. Three side lengths are known, but not the height. In this case apply Heron’s formula.
2. Identify the values of \( a, b \) and \( c \).
3. Calculate the value of \( s \), the semi-perimeter of the triangle.

**WRITE**

- **a**
  \[ A = \sqrt{s(s-a)(s-b)(s-c)} \]
  \[ a = 3, \ b = 5, \ c = 6 \]
  \[ s = \frac{a + b + c}{2} = \frac{3 + 5 + 6}{2} = \frac{14}{2} = 7 \]
4 Substitute the values of \( a, b, c \) and \( s \) into Heron’s formula and evaluate, correct to 2 decimal places.

\[
A = \sqrt{7(7-3)(7-5)(7-6)}
= \sqrt{7 \times 4 \times 2 \times 1}
= \sqrt{56}
= 7.48 \text{ cm}^2
\]

b 1 The shape shown is an ellipse. Write the appropriate area formula.

\[
A = \pi ab
\]

2 Identify the values of \( a \) and \( b \) (the semi-major and semi-minor axes).

3 Substitute the values of \( a \) and \( b \) into the formula and evaluate, correct to 2 decimal places.

\[
a = 5, \; b = 2
A = \pi \times 5 \times 2
= 31.42 \text{ cm}^2
\]

c 1 The shape shown is a sector. Write the formula for finding the area of a sector.

2 Write the value of \( \theta \) and \( r \).

3 Substitute and evaluate the expression, correct to 2 decimal places.

\[
\theta = 40^\circ, \; r = 15
A = \frac{40^\circ}{360^\circ} \times \pi \times 15^2
= 78.54 \text{ cm}^2
\]

Areas of composite figures

- A *composite figure* is a figure made up of a combination of simple figures.
- The area of a composite figure can be calculated by:
  - calculating the sum of the areas of the simple figures that make up the composite figure
  - calculating the area of a larger shape and then subtracting the extra area involved.

**WORKED EXAMPLE 2**

Find the area of each of the following composite shapes.

**THINK**

a 1 ACBD is a quadrilateral that can be split into two triangles: \( \Delta ABC \) and \( \Delta ABD \).

**WRITE**

a Area ACBD = Area \( \Delta ABC \) + Area \( \Delta ABD \)
2 Write the formula for the area of a triangle containing base and height.

\[ A_{\text{triangle}} = \frac{1}{2}bh \]

3 Identify the values of \( b \) and \( h \) for \( \Delta ABC \).

\( \Delta ABC: b = AB = 8, \ h = EC = 6 \)

Area of \( \Delta ABC = \frac{1}{2} \times AB \times EC \)

\[ = \frac{1}{2} \times 8 \times 6 \]

\[ = 24 \text{ cm}^2 \]

4 Substitute the values of the pronumerals into the formula and, hence, calculate the area of \( \Delta ABC \).

5 Identify the values of \( b \) and \( h \) for \( \Delta ABD \).

\( \Delta ABD: b = AB = 8, \ h = FD = 2 \)

Area of \( \Delta ABD = \frac{1}{2}AB \times FD \)

\[ = \frac{1}{2} \times 8 \times 2 \]

\[ = 8 \text{ cm}^2 \]

6 Calculate the area of \( \Delta ABD \).

Area of \( \Delta ABD = 24 \text{ cm}^2 + 8 \text{ cm}^2 \)

\[ = 32 \text{ cm}^2 \]

7 Add the areas of the two triangles together to find the area of the quadrilateral ACBD.

\[ \text{Area of ACBD} = 24 \text{ cm}^2 + 8 \text{ cm}^2 \]

\[ = 32 \text{ cm}^2 \]

One way to find the area of the shape shown is to find the total area of the rectangle ABGH and then subtract the area of the smaller rectangle DEFC.

1 Write the formula for the area of a rectangle.

\[ A_{\text{rectangle}} = l \times w \]

2 Write the formula for the area of a rectangle.

Identify the values of the pronumerals for the rectangle ABGH.

Rectangle ABGH: \( l = 9 + 2 + 9 \)

\[ = 20 \]

\( w = 10 \)

Area of ABGH = \( 20 \times 10 \)

\[ = 200 \text{ cm}^2 \]

3 Identify the values of the pronumerals for the rectangle DEFC.

Rectangle DEFC: \( l = 5, \ w = 2 \)

Area of DEFC = \( 5 \times 2 \)

\[ = 10 \text{ cm}^2 \]

4 Substitute the values of the pronumerals into the formula to find the area of the rectangle ABGH.

Area = 200 – 10

\[ = 190 \text{ cm}^2 \]
Exercise 6.2 Area

**INDIVIDUAL PATHWAYS**

- **PRACTISE**
  Questions: 1, 3–5, 8, 9, 11, 12, 14

- **CONSOLIDATE**
  Questions: 1–6, 8–10, 12, 14, 16, 18

- **MASTER**
  Questions: 1–9, 12–19

Where appropriate, give answers correct to 2 decimal places.

**FLUENCY**

1. Find the areas of the following shapes.

   - a) 4 cm
   - b) 12 cm
   - c) 15 cm
   - d) 12 cm 8 cm
   - e) 15 cm 10 cm
   - f) 8 mm 13 mm
   - g) 18 cm
   - h) 6 m 7 m
   - i) 15 cm 10 cm

2. Express the area in questions 1e and 1g in terms of \( \pi \).

3. Use Heron’s formula to find the area of the following triangles correct to 2 decimal places.

   - a) 5 cm 16 cm 12 cm
   - b) 8 cm 6 cm 3 cm

**REFLECTION**

How are perimeter and area different but fundamentally related?
4. **WE1b** Find the area of the following ellipses. Answer correct to 1 decimal place.

   a. [Ellipse with major axis 9 mm and minor axis 4 mm]
   b. [Ellipse with major axis 12 mm and minor axis 5 mm]

5. **WE1c** Find the area of the following shapes, i stating the answer exactly; that is, in terms of \( \pi \) and ii correct to 2 decimal places.

   a. [Triangle with base 12 cm and angle 30°]
   b. [Circle with radius 6 mm and angle 345°]
   c. [Sector of a circle with central angle 70° and radius 18 cm]

6. **MC** A figure has an area of about 64 cm\(^2\). Which of the following cannot possibly represent the figure?

   A. A triangle with base length 16 cm and height 8 cm
   B. A circle with radius 4.51 cm
   C. A rectangle with dimensions 16 cm and 4 cm
   D. A square with side length 8 cm
   E. A rhombus with diagonals 16 cm and 4 cm

7. **MC** The area of the quadrilateral shown below right is to be calculated. Which of the following lists all the lengths required to calculate the area?

   A. AB, BC, CD and AD
   B. AB, BE, AC and CD
   C. BC, BE, AD and CD
   D. AC, BE and FD
   E. AC, CD and AB

8. **WE2** Find the area of the following composite shapes.

   a. [Composite shape with a circle and a rectangle]
   b. [Composite shape with a rectangle and a semi-circle]

---

*Note: The diagrams are not provided in the text.*
Find the shaded area in each of the following.

a

b

c

d

e

f

**UNDERSTANDING**

10 A sheet of cardboard is 1.6 m by 0.8 m. The following shapes are cut from the cardboard:
- a circular piece with radius 12 cm
- a rectangular piece 20 cm by 15 cm
- 2 triangular pieces with base 30 cm and height 10 cm
- a triangular piece with side length 12 cm, 10 cm and 8 cm.
What is the area of the remaining piece of cardboard?

11 A rectangular block of land, 12 m by 8 m, is surrounded by a concrete path 0.5 m wide. Find the area of the path.

12 Concrete slabs 1 m by 0.5 m are used to cover a footpath 20 m by 1.5 m. How many slabs are needed?

13 A city council builds a 0.5 m wide concrete path around the garden as shown below.

Find the cost of the job if the workman charges $40.00 per m².

14 A tennis court used for doubles is 10.97 m wide, but a singles court is only 8.23 m wide, as shown in the diagram.

   - What is the area of the doubles tennis court?
   - What is the area of the singles court?
   - What percentage of the doubles court is used for singles? Give your answer to the nearest whole number.

15 Ron the excavator operator has 100 metres of barricade mesh and needs to enclose an area to work in safely. He chooses to make a rectangular region with dimensions $x$ and $y$.
   - Write an equation that connects $x$, $y$ and the perimeter.
   - Write $y$ in terms of $x$.
   - Write an equation for the area of the region in terms of $x$.
   - Fill in the table for different values of $x$.

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>5</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
<th>30</th>
<th>35</th>
<th>40</th>
<th>45</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area (m²)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   - Can $x$ have a value more than 50? Why?
   - Sketch a graph of area against $x$.
   - Determine the value of $x$ that makes the area a maximum.
h What is the value of \( y \) for maximum area?
i What shape encloses the maximum area?
j Calculate the maximum area.
Ron decides to choose to make a circular area with the barricade mesh.
k What is the radius of this circular region?
l What area is enclosed in this circular region?
m How much extra area does Ron now have compared to his rectangular region?

REASONING
16 Dan has purchased a country property with layout and dimensions as shown in the diagram.
a Show that the property has a total area of 987.5 ha.
b Dan wants to split the property in half (in terms of area) by building a straight-lined fence running either north–south or east–west through the property. Assuming the cost of the fencing is a fixed amount per linear metre, justify where the fence should be built (that is, how many metres from the top left-hand corner and in which direction), to minimise the cost.

17 In question 15, Ron the excavator operator could choose to enclose a rectangular or circular area with 100 m of barricade mesh. In this case, the circular region resulted in a larger safe work area.
a Show that for 150 m of barricade mesh, a circular region again results in a larger safe work area as opposed to a rectangular region.
b Show that for \( n \) metres of barricade mesh, a circular region will result in a larger safe work area as opposed to a rectangular region.

PROBLEM SOLVING
18 ABC is a scalene triangle with a base length of 80 cm and a perpendicular height of 40 cm. A right-angled triangle, AED, is nestled within ABC such that DE is 10 cm to the left of the perpendicular height, as shown. Find the lengths of the sides labelled \( x \) and \( y \) if the shorter side of the two is 20 cm less than the longer side and the areas of the two shaded regions are the same.
19 Proving the segment formula
Prove the formula for the area of a segment using the fact that area of the segment = area of sector ABC − 2 × area of triangle ACD.

a Using trigonometry, show that \( \frac{AD}{r} = \sin\left(\frac{\theta}{2}\right) \).

b Show that \( \frac{CD}{r} = \cos\left(\frac{\theta}{2}\right) \).

c Show that the area of triangle ACD is \( \frac{r^2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2} \).

Note that this formula is the same if \( \theta \) is in degrees or radians.

d Finally, show that the area of the segment (in purple) is \( r^2\left(\pi \times \frac{\theta}{360°} - \sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)\right) \) if \( \theta \) is in degrees.

6.3 Total surface area

- The total surface area (TSA) of a solid is the sum of the areas of all the faces of that solid.

TSA of rectangular prisms and cubes
- The formula for finding the TSA of a rectangular prism (cuboid) is:

\[
TSA = 2(lh + lw + wh)
\]

- A special case of the rectangular prism is the cube, where all sides are equal \( (l = w = h) \).

\[
TSA = 6l^2
\]

TSA of spheres and cylinders
Sphere:
\[
TSA = 4\pi r^2
\]

Note: The mathematics required to prove the formula for the total surface area of a sphere is beyond the scope of Year 10.
Cylinder:
TSA = 2πr(r + h) or 2πr² + 2πrh

• The formula for the TSA of a cylinder is found from the area of the net as shown.
TSA = πr² + πr² + 2πrh
= 2πr² + 2πrh
= 2πr(r + h)

WORKED EXAMPLE 3

Find the total surface area of the solids, correct to the nearest cm².

a  
\[ r = 7 \text{ cm} \]

THINK
1 Write the formula for the TSA of a sphere.
2 Identify the value for \( r \).
3 Substitute and evaluate.

WRITE
a  
TSA = 4\pi r²
\[ r = 7 \]
TSA = 4 \times \pi \times 7²
\approx 615.8 \text{ cm}²
\approx 616 \text{ cm}²

b  
\[ r = 50 \text{ cm}, h = 1.5 \text{ m} = 150 \text{ cm} \]

THINK
1 Write the formula for the TSA of a cylinder.
2 Identify the values for \( r \) and \( h \). Note that the units will need to be the same.
3 Substitute and evaluate.

WRITE
b  
TSA = 2\pi(r + h)
TSA = 2 \times \pi \times 50 \times (50 + 150)
\approx 62831.9 \text{ cm}²
\approx 62832 \text{ cm}²

TSA of cones

• The total surface area of a cone can be found by considering its net, which is comprised of a small circle and a sector of a larger circle.

\[ r = \text{radius of the cone} \]
\[ l = \text{slant height of the cone} \]
The sector is a fraction of the full circle of radius \( l \) with circumference \( 2\pi l \).

The sector has an arc length equivalent to the circumference of the base of the cone, \( 2\pi r \).

The fraction of the full circle represented by the sector can be found by writing the arc length as a fraction of the circumference of the full circle, \( \frac{2\pi r}{2\pi l} = \frac{r}{l} \).

Area of a sector = fraction of the circle \( \times \pi l^2 \)
\[
= \frac{r}{l} \times \pi l^2 = \pi rl
\]

Therefore, \( SA = A_{\text{circular base}} + A_{\text{curved surface}} \)

\[
= \pi r^2 + \pi rl
= \pi r(r + l)
\]

Cone: \( TSA = \pi r(r + l) \) or \( \pi l^2 + \pi rl \)

**WORKED EXAMPLE 4**

**Find the total surface area of the cone shown.**

**THINK**

1. Write the formula for the TSA of a cone.

2. State the values of \( r \) and \( l \).

3. Substitute and evaluate.

**WRITE**

\[
TSA = \pi r(r + l)
\]

\[
r = 12, \quad l = 15
\]

\[
TSA = \pi \times 12 \times (12 + 15)
= 1017.9 \text{ cm}^2
\]

**TSA of other solids**

- TSA can be found by summing the areas of each face.
- The areas of each face may need to be calculated separately.
- Check the total number of faces to ensure that none are left out.

**WORKED EXAMPLE 5**

**Find the total surface area of the square-based pyramid shown.**

**THINK**

1. There are five faces: The square base and four identical triangles.

2. Find the area of the square base.

3. Draw and label one triangular face and write the formula for finding its area.

**WRITE/DRAW**

\[
TSA = \text{Area of square base} + \text{area of four triangular faces}
\]

Area of base = \( l^2 \), where \( l = 6 \)
Area of base = \( 6^2 \)
\[
= 36 \text{ cm}^2
\]

Area of a triangular face = \( \frac{1}{2}bh \), \( b = 6 \)

\[
 \text{Area of a triangular face} = \frac{1}{2} \times 5 \times 3 = 7.5 \text{ cm}^2
\]

\[
TSA = 36 + 4 \times 7.5
= 102 \text{ cm}^2
\]
Find the height of the triangle, $h$, using Pythagoras’ theorem.

Given:
- $a^2 = c^2 - b^2$, where $a = h$, $b = 3$, $c = 5$
- $h^2 = 5^2 - 3^2$
- $h^2 = 25 - 9$
- $h^2 = 16$
- $h = 4 \text{ cm}$

Calculate the area of the triangular face by substituting $b = 6$ and $h = 4$.

Area of triangular face $= \frac{1}{2} \times 6 \times 4 = 12 \text{ cm}^2$

Calculate the TSA by adding the area of the square base and the area of four identical triangular faces together.

TSA $= 36 + 4 \times 12$
- $= 36 + 48$
- $= 84 \text{ cm}^2$

**TSA of composite solids**
- Composite solids are formed when two or more simple solids are joined together.
- The TSA of a composite solid is calculated by summing the areas of the solid’s external faces.

**WORKED EXAMPLE 6**

Find the total surface area of the solid shown correct to 1 decimal place.

**THINK**
1. The solid shown has 9 faces — five identical squares and four identical triangles.
2. Find the area of one square face with the side length 10 cm.
3. Draw a triangular face and work out its height using Pythagoras’ theorem.

**WRITE/DRAW**

TSA $= 5 \times \text{area of a square}$
+ $4 \times \text{area of a triangle}$

$A_{\text{square}} = l^2$, where $l = 10$
- $A = 10^2$
- $A = 100 \text{ cm}^2$

$a^2 = c^2 - b^2$, where $a = h$, $b = 5$, $c = 6$
- $h^2 = 6^2 - 5^2$
- $h^2 = 36 - 25$
- $h^2 = 11$
- $h = 3.3$ cm (or with rounding, $h = 3.3$)
Find the area of one triangular face.

\[ A_{\text{triangle}} = \frac{1}{2}bh, \text{ where } b = 10, h = 3.31662 \]
\[ = \frac{1}{2} \times 10 \times 3.31662 \ldots \]
\[ = 16.5831 \ldots \text{ cm}^2 \text{ (or, with rounding, } A_{\text{triangle}} = 16.6 \text{ cm}^2) \]

Find the TSA of the solid by adding the area of 5 squares and 4 triangles together.

\[ \text{TSA} = 5 \times 100 + 4 \times 16.5831 \ldots \]
\[ = 500 + 66.3324 \ldots \]
\[ \approx 566.3 \text{ cm}^2 \text{ (or } \approx 566 \text{ using the previously rounded value)} \]

Note: Rounding is not done until the final step. If \( h \) had been rounded to 3.3 in step 3 and this value used in steps 4 and 5, the decimal place value of the TSA would have been lost. It is important to realise that rounding too early can affect the accuracy of results.

### Worked Example 7

The silo shown at right is to be built from metal. The top portion of the silo is a cylinder of diameter 4 m and height 8 m. The bottom part of the silo is a cone of slant height 3 m. The silo has a circular opening of radius 30 cm on the top.

a What area of metal (to the nearest \( \text{m}^2 \)) is required to build the silo?

b If it costs \$12.50 per \( \text{m}^2 \) to cover the surface with an anti-rust material, how much will it cost to cover the silo completely?

**Think**

1. The surface area of the silo consists of an annulus, the curved part of the cylinder and the curved section of the cone.
2. To find the area of the annulus, subtract the area of the small circle from the area of the larger circle. Let \( R = \) radius of small circle.
3. The middle part of the silo is the curved part of a cylinder. Find its area. (Note that in the formula \( \text{TSA}_{\text{cylinder}} = 2\pi r^2 + 2\pi rh \), the curved part is represented by \( 2\pi rh \).)

**Write**

a. \[ \text{TSA} = \text{area of annulus} \]
   + \[ \text{area of curved section of a cylinder} \]
   + \[ \text{area of curved section of a cone} \]

Area of annulus = \( A_{\text{large circle}} - A_{\text{small circle}} \)
\[ = \pi r^2 - \pi R^2 \]
where \( r = \frac{4}{2} = 2 \text{ m} \) and \( R = 30 \text{ cm} = 0.3 \text{ m} \).
Area of annulus = \[ \pi \times 2^2 - \pi \times 0.3^2 \]
\[ = 12.28 \text{ m}^2 \]

Area of curved section of cylinder = \( 2\pi rh \)
where \( r = 2, \ h = 8 \).
Area of curved section of cylinder = \[ 2 \times \pi \times 2 \times 8 \]
\[ = 100.53 \text{ m}^2 \]
4. The bottom part of the silo is the curved section of a cone. Find its area. (Note that in the formula $TSA_{cone} = \pi r^2 + \pi rl$, the curved part is given by $\pi rl$.)

Area of curved section of cone = $\pi rl$
where $r = 2$, $l = 3$.
Area of curved section of cone = $\pi \times 2 \times 3 = 18.85 \text{ m}^2$

5. Find the total surface area of the silo by finding the sum of the surface areas calculated above.

$TSA = 12.28 + 100.53 + 18.85 = 131.66 \text{ m}^2$

6. Write the answer in words.

The area of metal required is 132 m$^2$, correct to the nearest square metre.

b To find the total cost, multiply the total surface area of the silo by the cost of the anti-rust material per m$^2$ ($12.50$).

Cost = 132 $\times$ $12.50 = $1650.00

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**Exercise 6.3 Total surface area**

**INDIVIDUAL PATHWAYS**

**PRACTISE**
Questions: 1–4, 6a–e, 7, 10, 12

**CONSOLIDATE**
Questions: 1–4, 6, 7, 9–12, 15, 18

**MASTER**
Questions: 1–8, 10–18

**Individual pathway interactivity**

---

**FLUENCY**

*Note: Where appropriate, give the answers correct to 1 decimal place.*

1. Find the total surface areas of the solids shown.

a

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>10 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

b

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Height</th>
</tr>
</thead>
<tbody>
<tr>
<td>8 cm</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

2. Find the total surface area of the solids shown below.

---

a

$\tau = 3 \text{ m}$

b

$21 \text{ cm}$

---

Note: Where appropriate, give the answers correct to 1 decimal place.
3 **WE4** Find the total surface area of the cones below.

![Cone A with dimensions 14 cm, 20 cm, and 14 cm]

![Cone B with dimensions 8 cm, 12 cm, and 8 cm]

4 **WE5** Find the total surface area of the solids below.

![Pyramid A with dimensions 12 cm, 15 cm, and 9.1 cm]

![Pyramid B with dimensions 2.5 m, 1.5 m, and 1.5 m]

![Solid C with dimensions 14 cm, 15 cm, and 14 cm]

![Solid D with dimensions 10 cm, 7 cm, and 10 cm]

5 Find the surface areas of the following.

a A cube of side length 1.5 m
b A rectangular prism 6 m × 4 m × 2.1 m
c A cylinder of radius 30 cm and height 45 cm, open at one end
d A sphere of radius 28 mm
e An open cone of radius 4 cm and slant height 10 cm
f A square pyramid of base length 20 cm and slant edge 30 cm

6 **WE6** Find the total surface area of the objects shown.

![Object A with dimensions 10 cm, 8 cm, 5 cm, 12 cm, and 20 cm]

![Object B with dimensions 20 cm, 35 cm, 12 cm, and 20 cm]

![Object C with dimensions 5 cm, 3 cm, and 3 cm]

![Object D with dimensions 2 cm, 3 cm, and 2.5 cm]
7 MC A cube has a total surface area of 384 cm$^2$. The length of the edge of the cube is:
A 9 cm  B 8 cm  C 7 cm  D 6 cm  E 5 cm

UNDERSTANDING
8 Open cones are made from nets cut from a large sheet of paper 1.2 m × 1.0 m. If a cone has a radius of 6 cm and a slant height of 10 cm, how many cones can be made from the sheet? (Assume there is 5% wastage of paper.)
9 A steel girder is to be painted. Calculate the area of the surface to be painted.

10 WE7 The greenhouse shown below is to be built using shade cloth. It has a wooden door of dimensions 1.2 m × 0.5 m.
   a Find the total area of shade cloth needed to complete the greenhouse.
   b Find the cost of the shade cloth at $6.50 per m$^2$.

11 A cylinder is joined to a hemisphere to make a cake holder, as shown below. The surface of the cake holder is to be chromed at 5.5 cents per cm$^2$.
   a Find the total surface area to be chromed.
   b Find the cost of chroming the cake holder.
12 A soccer ball is made up of a number of hexagons sewn together on its surface. Each hexagon can be considered to have dimensions as shown in the diagram.
   a Calculate $\theta^\circ$.
   b Calculate the values of $x$ and $y$ exactly.
   c Calculate the area of the trapezium in the diagram.
   d Hence, determine the area of the hexagon.
   e If the total surface area of the soccer ball is $192\sqrt{3}$ cm$^2$, how many hexagons are on the surface of the soccer ball?

13 a Determine the exact total surface area of a sphere with radius $\sqrt{2}$ metres.
   An inverted cone with side length 4 metres is placed on top of the sphere so that the centre of its base is 0.5 metres above the centre of the sphere.
   b Find the radius of the cone exactly.
   c Find the area of the curved surface of the cone exactly.
   d What are the exact dimensions of a box that could precisely fit the cone connected to the sphere?

REASONING

Complete the following question without the aid of a calculator.

14 The table shown below is to be varnished (including the base of each leg). The table top has a thickness of 180 mm and the cross-sectional dimension of the legs is 50 mm by 50 mm.

A friend completes the calculation as shown. Assume there are no simple calculating errors. Analyse the working presented and justify if the TSA calculated is correct.

<table>
<thead>
<tr>
<th>Table top (inc. leg bases)</th>
<th>0.96</th>
<th>2 \times (0.8 \times 0.6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Legs</td>
<td>0.416</td>
<td>16 \times (0.52 \times 0.05)</td>
</tr>
<tr>
<td>Table top edging</td>
<td>0.504</td>
<td>0.18 \times (2(0.8 + 0.6))</td>
</tr>
<tr>
<td>TSA</td>
<td>1.88 m$^2$</td>
<td></td>
</tr>
</tbody>
</table>

15 A shower recess with dimensions 1500 mm (back wall) by 900 mm (side wall) needs to have the back and two side walls tiled to a height of 2 m.
   a Calculate the area to be tiled in m$^2$.
   b Justify that 180 tiles (including those that need to be cut) of dimension 20 cm by 20 cm will be required. Disregard the grout and assume that once a tile is cut, only one piece of the tile can be used.
   c Evaluate the cheapest option of tiling; $1.50/tile or $39.50/box, where a box covers 1 m$^2$, or tiles of dimension 30 cm by 30 cm costing $3.50/tile.
16 If the surface area of a sphere to that of a cylinder is in the ratio 4 : 3 and the sphere has a radius of \(3a\), show that if the radius of the cylinder is equal to its height, then the radius of the cylinder is \(\frac{3\sqrt{3}a}{2}\).

PROBLEM SOLVING

Frustum of a cone

17 A frustum of a cone is a cone with the top sliced off (see the drawing on the right). When the curved side is ‘opened up’, it creates a shape, ABYX, as shown in the diagram.

a Write an expression for the arc length XY in terms of the angle \(\theta\). Write another expression for the arc length AB in terms of the same angle \(\theta\). Show that, in radians, \(\theta = \frac{2\pi(r - t)}{s}\).

b i Using the above formula for \(\theta\), show that \(x = \frac{sl}{(r - t)}\).

ii Use similar triangles to confirm this formula.

c Determine the area of sectors AVB and XGY and hence determine the area of ABYX. Add the areas of the 2 circles to the area of ABYX to determine the TSA of a frustum.

18 Tina is re-covering a footstool in the shape of a cylinder with diameter 50 cm and height 30 cm. She also intends to cover the base of the cushion.

She has 1 m\(^2\) of fabric to make this footstool. When calculating the area of fabric required, allow an extra 20% of the total surface area to cater for seams and pattern placings. Explain whether Tina has enough material to cover the footstool.

6.4 Volume

- The volume of a 3-dimensional object is the amount of space it takes up.
- The volume is measured in units of mm\(^3\), cm\(^3\) and m\(^3\).

Volume of a prism

- The volume of any solid with a uniform cross-sectional area is given by the formula: \(V = AH\), where \(A\) is the cross-sectional (or base) area and \(H\) is the height of the solid.

Cube

\[
\text{Volume} = l^3
\]
Rectangular prism
Volume = \( AH \)
= area of a rectangle \( \times \) height
= \( lwh \)

Cylinder
Volume = \( AH \)
= area of a circle \( \times \) height
= \( \pi r^2 h \)

Triangular prism
Volume = \( AH \)
= area of a triangle \( \times \) height
= \( \frac{1}{2}bh \times H \)

Find the volumes of the following shapes.

**a**

![Cylinder diagram]

- 14 cm
- 20 cm

**WRITE**

\[
V = \pi r^2 h
\]

\[
r = 14, \ h = 20
\]

\[
V = \pi \times 14^2 \times 20
\]

\[
\approx 12315.04 \text{ cm}^3
\]

**b**

![Triangular prism diagram]

- 5 cm
- 4 cm
- 10 cm

**WRITE**

\[
V = \frac{1}{2}bh \times H
\]

\[
b = 4, \ h = 5, \ H = 10
\]

\[
V = \frac{1}{2} \times 4 \times 5 \times 10
\]

\[
= 100 \text{ cm}^3
\]
### WORKED EXAMPLE 9

**a** What effect will doubling each of the side lengths of a cube have on its volume?

**b** What effect will halving the radius and doubling the height of a cylinder have on its volume?

#### THINK

<table>
<thead>
<tr>
<th>WRITE</th>
<th>THINK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>a</strong> Write the formula for the volume of the cube.</td>
<td>1 Write the formula for the volume of the cube.</td>
</tr>
<tr>
<td>$V = l^3$</td>
<td></td>
</tr>
<tr>
<td>Identify the value of the pronumeral. <em>Note: Doubling is the same as multiplying by 2.</em></td>
<td>2 Identify the value of the pronumeral. <em>Note: Doubling is the same as multiplying by 2.</em></td>
</tr>
<tr>
<td>$l_{\text{new}} = 2l$</td>
<td></td>
</tr>
<tr>
<td>Substitute and evaluate.</td>
<td>3 Substitute and evaluate.</td>
</tr>
<tr>
<td>$V_{\text{new}} = (2l)^3$</td>
<td></td>
</tr>
<tr>
<td>$= 8l^3$</td>
<td></td>
</tr>
<tr>
<td>Compare the answer obtained in step 3 with the volume of the original shape.</td>
<td>4 Compare the answer obtained in step 3 with the volume of the original shape.</td>
</tr>
<tr>
<td>Doubling each side length of a cube increases the volume by a factor of 8; that is, the new volume will be 8 times as large as the original volume.</td>
<td></td>
</tr>
<tr>
<td>Write your answer.</td>
<td>5 Write your answer.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WRITE</th>
<th>THINK</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>b</strong> Write the formula for the volume of the cylinder.</td>
<td>1 Write the formula for the volume of the cylinder.</td>
</tr>
<tr>
<td>$V = \pi r^2 h$</td>
<td></td>
</tr>
<tr>
<td>Identify the value of the pronumerals. <em>Note: Halving is the same as dividing by 2.</em></td>
<td>2 Identify the value of the pronumerals. <em>Note: Halving is the same as dividing by 2.</em></td>
</tr>
<tr>
<td>$r_{\text{new}} = \frac{r}{2}, h_{\text{new}} = 2h$</td>
<td></td>
</tr>
<tr>
<td>Substitute and evaluate.</td>
<td>3 Substitute and evaluate.</td>
</tr>
<tr>
<td>$V_{\text{new}} = \pi \left(\frac{r}{2}\right)^2 2h$</td>
<td></td>
</tr>
<tr>
<td>$= \pi \times \frac{r^2}{4} \times 2h$</td>
<td></td>
</tr>
<tr>
<td>$= \frac{\pi r^2 h}{2}$</td>
<td></td>
</tr>
<tr>
<td>$= \frac{1}{2} \pi r^2 h$</td>
<td></td>
</tr>
<tr>
<td>Compare the answer obtained in step 3 with the volume of the original shape.</td>
<td>4 Compare the answer obtained in step 3 with the volume of the original shape.</td>
</tr>
<tr>
<td>Halving the radius and doubling the height of a cylinder decreases the volume by a factor of 2; that is, the new volume will be half the original volume.</td>
<td></td>
</tr>
<tr>
<td>Write your answer.</td>
<td>5 Write your answer.</td>
</tr>
</tbody>
</table>
Volume of spheres

- The volume of a sphere of radius, \( r \), can be calculated using the formula \( V = \frac{4}{3} \pi r^3 \).

WORKED EXAMPLE 10

Find the volume of a sphere of radius 9 cm. Answer correct to 1 decimal place.

THINK

1. Write the formula for the volume of a sphere. 
2. Identify the value of \( r \).
3. Substitute and evaluate.

WRITE

\[
V = \frac{4}{3} \pi r^3 \\
= \frac{4}{3} \times \pi \times 9^3 \\
= 3053.6 \text{ cm}^3
\]

Volume of pyramids

- Pyramids are not prisms as the cross-section changes from the base upwards.

- The volume of a pyramid is one-third the volume of an equivalent prism with the same base area and height.

\[
\text{Volume of a pyramid} = \frac{1}{3} A H
\]

- Since a cone is a pyramid with a circular cross-section, the volume of a cone is one-third the volume of a cylinder with the same base area and height.

\[
\text{Volume of a cone} = \frac{1}{3} A H \\
= \frac{1}{3} \pi r^2 h
\]

WORKED EXAMPLE 11

Find the volume of each of the following solids.

a

b
**THINK**

a 1. Write the formula for the volume of a cone.
   2. Identify the values of \( r \) and \( h \).
   3. Substitute and evaluate.

b 1. Write the formula for the volume of a pyramid.
   2. Find the area of the square base.
   3. Identify the value of \( H \).
   4. Substitute and evaluate.

**WRITE**

a \[ V = \frac{1}{3} \pi r^2 h \]
   \( r = 8, \ h = 10 \)
   \[ V = \frac{1}{3} \times \pi \times 8^2 \times 10 \]
   \[ = 670.21 \text{ cm}^3 \]

b \[ V = \frac{1}{3} A H \]
   \[ A = l^2 \text{ where } l = 8 \]
   \[ A = 8^2 \]
   \[ = 64 \text{ cm}^2 \]
   \[ H = 12 \]
   \[ V = \frac{1}{3} \times 64 \times 12 \]
   \[ = 256 \text{ cm}^3 \]

**Volume of composite solids**

- A composite solid is a combination of a number of solids.
- The volume of each smaller solid component can be calculated separately.
- The volume of a composite solid is calculated by summing the volumes of each of the smaller solid components.

**WORKED EXAMPLE 12**

**THINK**

1. The given solid is a composite figure, made up of a cube and a square-based pyramid.
2. Find the volume of the cube.
3. Write the formula for finding the volume of a square-based pyramid.

**WRITE**

\[ V = \text{Volume of cube} + \text{Volume of pyramid} \]

\[ V_{\text{cube}} = l^3 \text{ where } l = 3 \]
\[ V_{\text{cube}} = 3^3 \]
\[ = 27 \text{ m}^3 \]

\[ V_{\text{square-based pyramid}} = \frac{1}{3} A H \]
4 Find the area of the square base. 
\[ A = l^2 = 3^2 = 9 \text{ m}^2 \]

5 Identify the value of \( H \). 
\[ H = 1.5 \]

6 Substitute and evaluate the volume of the pyramid. 
\[ V_{\text{square-based pyramid}} = \frac{1}{3} \times 9 \times 1.5 = 4.5 \text{ m}^3 \]

7 Find the total volume by adding the volume of the cube and pyramid. 
\[ V = 27 + 4.5 = 31.5 \text{ m}^3 \]

**Capacity**

- Some 3-dimensional objects are hollow and can be filled with liquid or some other substance.
- The amount of substance which a container can hold is called its capacity.
- **Capacity** is essentially the same as volume but is usually measured in \( \text{mL}\), \( \text{L} \) and \( \text{kL} \) where
  - \( 1 \text{ mL} = 1 \text{ cm}^3 \)
  - \( 1 \text{ L} = 1000 \text{ cm}^3 \)
  - \( 1 \text{ kL} = 1 \text{ m}^3 \).

**WORKED EXAMPLE 13**

Find the capacity (in litres) of a cuboidal aquarium, which is 50 cm long, 30 cm wide and 40 cm high.

**THINK**

1 Write the formula for the volume of a rectangular prism.

2 Identify the values of the pronumerals.

3 Substitute and evaluate.

4 State the capacity of the container in millilitres, using \( 1 \text{ cm}^3 = 1 \text{ mL} \).

5 Since \( 1 \text{ L} = 1000 \text{ mL} \), to convert millilitres to litres divide by 1000.

6 Give a worded answer.

**WRITE**

- \( V = lwh \)
- \( l = 50, \ w = 30, \ h = 40 \)
- \( V = 50 \times 30 \times 40 = 60000 \text{ cm}^3 = 60000 \text{ mL} \)
- \( = 60 \text{ L} \)
- The capacity of the fish tank is 60 L.
Exercise 6.4 Volume

INDIVIDUAL PATHWAYS

PRACTISE
Questions: 1–4, 6–8, 9a, 10, 13, 14, 20

CONSOLIDATE
Questions: 1–8, 10–12, 14, 16, 19, 20, 22, 25

MASTER
Questions: 1–18, 20–26

REFLECTION
Volume is measured in cubic units. How is this reflected in the volume formula?

FLUENCY

1 Find the volumes of the following prisms.

![Prism A](image)

![Prism B](image)

![Prism C](image)

![Prism D](image)

2 Calculate the volume of each of these solids.

- **a**
  - Base area: 25 mm²
  - Height: 18 mm

- **b**
  - Base area: 24 cm²
  - Height: 15 cm

3 Find the volume of each of the following. Give each answer correct to 1 decimal place where appropriate.

- **a**
  - Radius: 6 mm
  - Height: 12 cm

- **b**
  - Radius: 5 mm
  - Height: 2.7 m

- **c**
  - Base area: 25 mm²
  - Height: 10 cm

- **d**
  - Base area: 12 mm²
  - Height: 8 mm
4. **WE10** Find the volume of a sphere (correct to 1 decimal place) with a radius of:
   a. 1.2 m  
   b. 15 cm  
   c. 7 mm  
   d. 50 cm.

5. Find the volume of each of these figures, correct to 2 decimal places.
   a.  
   b.  
   c.  
   d.  

6. **WE11a** Find the volume of each of the following cones, correct to 1 decimal place.
   a.  
   b.  

7. **WE11b** Find the volume of each of the following pyramids.
   a.  
   b.  
8. Calculate the volume of each of the following composite solids correct to 2 decimal places where appropriate.

a.  
\[
\begin{array}{c}
10 \text{ cm} & 8 \text{ cm} & 5 \text{ cm} & 12 \text{ cm} & 5 \text{ cm} \\
20 \text{ cm}
\end{array}
\]

b.  
\[
\begin{array}{c}
20 \text{ cm} & 12 \text{ cm} & 35 \text{ cm}
\end{array}
\]

c.  
\[
\begin{array}{c}
5 \text{ cm} & \ \\
3 \text{ cm}
\end{array}
\]

d.  
\[
\begin{array}{c}
2 \text{ cm} & 3 \text{ cm} & 2.5 \text{ cm}
\end{array}
\]

e.  
\[
\begin{array}{c}
3.5 \text{ cm} & 10 \text{ cm}
\end{array}
\]

f.  
\[
\begin{array}{c}
12 \text{ cm} & 15 \text{ cm} & 5 \text{ cm} & 20 \text{ cm}
\end{array}
\]

UNDERSTANDING

9. a. What effect will tripling each of the side lengths of a cube have on its volume?

b. What effect will halving each of the side lengths of a cube have on its volume?

c. What effect will doubling the radius and halving the height of a cylinder have on its volume?

d. What effect will doubling the radius and dividing the height of a cylinder by 4 have on its volume?

e. What effect will doubling the length, halving the width and tripling the height of a rectangular prism have on its volume?

10. A hemispherical bowl has a thickness of 2 cm and an outer diameter of 25 cm. If the bowl is filled with water the capacity of the water will be closest to:

A 1.526 L  B 1.30833 L  C 3.05208 L
D 2.61666 L  E 2.42452 L
11 Tennis balls of diameter 8 cm are packed in a box 40 cm × 32 cm × 10 cm, as shown. How much space is left unfilled?

12 A cylindrical water tank has a diameter of 1.5 m and a height of 2.5 m. What is the capacity (in litres) of the tank?

13 A monument in the shape of a rectangular pyramid (base length of 10 cm, base width of 6 cm, height of 8 cm), a spherical glass ball (diameter of 17 cm) and conical glassware (radius of 14 cm, height of 10 cm) are packed in a rectangular prism of dimensions 30 cm by 25 cm by 20 cm. The extra space in the box is filled up by a packing material. What volume of packing material is required?

14 A swimming pool is being constructed so that it is the upper part of an inverted square-based pyramid.
   a Calculate $H$.
   b Calculate the volume of the pool.
   c How many 6 m$^3$ bins will be required to take the dirt away?
   d How many litres of water are required to fill this pool?
   e How deep is the pool when it is half-filled?

15 A soft drink manufacturer is looking to repackage cans of soft drink to minimise the cost of packaging while keeping the volume constant.
   Consider a can of soft drink with a capacity of 400 mL.
   a If the soft drink was packaged in a spherical can:
      i find the radius of the sphere
      ii find the total surface area of this can.
   b If the soft drink was packaged in a cylindrical can with a radius of 3 cm:
      i find the height of the cylinder
      ii find the total surface area of this can.
   c If the soft drink was packaged in a square-based pyramid with a base side length of 6 cm:
      i find the height of the pyramid
      ii find the total surface area of this can.
   d Which can would you recommend the soft drink manufacturer use for its repackaging? Why?
16. The volume of a cylinder is given by the formula \( V = \pi r^2 h \).
   a. Transpose the formula to make \( h \) the subject.
   b. A given cylinder has a volume of 1600 cm\(^3\). Find its height if it has a radius of:
      i. 4 cm
      ii. 8 cm.
   c. Transpose the formula to make \( r \) the subject.
   d. What restrictions must be placed on \( r \)? Why?
   e. A given cylinder has a volume of 1800 cm\(^3\). Find its radius if it has a height of:
      i. 10 cm
      ii. 15 cm.

17. A toy maker has enough rubber to make one super-ball of radius 30 cm. How many balls of radius 3 cm can he make from this rubber?

18. A manufacturer plans to make a cylindrical water tank to hold 2000 L of water.
   a. What must the height be if he uses a radius of 500 cm?
   b. What must the radius be if he uses a height of 500 cm?
   c. What will be the surface area of each of the two tanks? Assume the tank is a closed cylinder and give your answer in square metres.

19. The ancient Egyptians knew that the volume of the frustum of a square-based pyramid was given by the formula \( V = \frac{1}{3}h(x^2 + xy + y^2) \), although how they discovered this is unclear. (A frustum is the part of a cone or pyramid that is left when the top is cut off.)
   a. Find the volume of the frustum shown below.

   ![Diagram of a frustum]

   b. What would be the volume of the missing portion of the square-based pyramid shown?
REASONING

20 Archimedes is considered to be one of the three greatest mathematicians of all time (along with Newton and Gauss). He discovered several of the formulas used in this chapter. Inscribed on his tombstone was a diagram of his proudest discovery. It shows a sphere inscribed (fitting exactly) into a cylinder.

Show that \[
\frac{\text{volume of the cylinder}}{\text{volume of the sphere}} = \frac{\text{surface area of the cylinder}}{\text{surface area of the sphere}}.
\]

21 Marion has mixed together ingredients for a cake. The recipe requires a baking tin that is cylindrical in shape with a diameter of 20 cm and a height of 5 cm. Marion only has a tin as shown below left and a muffin tray consisting of 24 muffin cups. Each of the muffin cups in the tray is a portion of a cone as shown in the diagram below.

Should Marion use the tin or the muffin tray? Explain.

22 Nathaniel and Andrew are going to the snow for survival camp. They plan to construct an igloo, consisting of an entrance and hemispherical structure, as shown. Nathaniel and Andrew are asked to redraw their plans and increase the size of the liveable region (hemispherical structure) so that the total volume (including the entrance) is doubled. How can this be achieved?

23 Sam is having his 16th birthday party and wants to make an ice trough to keep drinks cold. He has found a square piece of sheet metal with a side length of 2 metres. He cuts squares of side length \(x\) metres from each corner, then bends the sides of the remaining sheet.

When four squares of the appropriate side length are cut from the corners the capacity of the trough can be maximised at 588 litres. Explain how Sam should proceed to maximise the capacity of the trough.

24 The Hastings family house has a rectangular roof with dimensions 17 m \(\times\) 10 m providing water to three cylindrical water tanks, each with a radius of 1.25 m and a height of 2.1 m. Show that approximately 182 millimetres of rain must fall on the roof to fill the tanks.
PROBLEM SOLVING

25 Six tennis balls are just contained in a cylinder as the balls touch the sides and the end sections of the cylinder. Each tennis ball has a radius of $R$ cm.
   a. Express the height of the cylinder in terms of $R$.
   b. Find the total volume of the tennis balls.
   c. Find the volume of the cylinder in terms of $R$.
   d. Show that the ratio of the volume of the tennis balls to the volume of the cylinder is $2 : 3$.

26 A frustum of a square-based pyramid is a square pyramid with the top sliced off. $H$ is the height of the full pyramid and $h$ is the height of the frustum.

a. Find the volume of the large pyramid which has a square base side of $X$ cm.
   b. Find the volume of the small pyramid which has a square base side of $x$ cm.
   c. Show that the relationship between $H$ and $h$ is given by $H = \frac{Xh}{X - x}$.
   d. Show that the volume of the frustum is given by $\frac{1}{3}h(X^2 + x^2 + Xx)$.

CHALLENGE 6.2

A large container is five-eighths full of ice-cream. After removing 27 identical scoops it is one-quarter full. How many scoops of ice-cream are left in the container?
6.5 Review

The Maths Quest Review is available in a customisable format for students to demonstrate their knowledge of this topic.

The Review contains:

- **Fluency** questions — allowing students to demonstrate the skills they have developed to efficiently answer questions using the most appropriate methods
- **Problem Solving** questions — allowing students to demonstrate their ability to make smart choices, to model and investigate problems, and to communicate solutions effectively.

A summary of the key points covered and a concept map summary of this topic are available as digital documents.

---

**Language**

It is important to learn and be able to use correct mathematical language in order to communicate effectively. Create a summary of the topic using the key terms below. You can present your summary in writing or using a concept map, a poster or technology.

<table>
<thead>
<tr>
<th>area</th>
<th>ellipse</th>
<th>sector</th>
</tr>
</thead>
<tbody>
<tr>
<td>capacity</td>
<td>face</td>
<td>semi-perimeter</td>
</tr>
<tr>
<td>circle</td>
<td>hemisphere</td>
<td>sphere</td>
</tr>
<tr>
<td>composite figure</td>
<td>parallelogram</td>
<td>square</td>
</tr>
<tr>
<td>cone</td>
<td>prism</td>
<td>surface</td>
</tr>
<tr>
<td>cross-section</td>
<td>pyramid</td>
<td>trapezium</td>
</tr>
<tr>
<td>cube</td>
<td>rectangle</td>
<td>triangle</td>
</tr>
<tr>
<td>cylinder</td>
<td>rhombus</td>
<td>volume</td>
</tr>
</tbody>
</table>

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The **story of mathematics** is an exclusive Jacaranda video series that explores the history of mathematics and how it helped shape the world we live in today.

*Australian megafauna* (eles-1845) tells the story of some of the largest and most impressive animals to ever walk the Earth. Several of these animals are introduced as we question what led to the extinction of Australian megafauna.
INVESTIGATION

RICH TASK

So close!

Humans must measure! Imagine what a chaotic world it would be if we didn’t measure anything. Some of the things we measure are time, length, weight and temperature; we also use other measures derived from these such as area, volume, speed.

Accurate measurement is important. The accuracy of a measurement depends on the instrument being used to measure and the interpretation of the measurement. There is no such thing as a perfectly accurate measurement. The best we can do is learn how to make meaningful use of the numbers we read off our devices. It is also important to use appropriate units of measurement.

Measurement errors
When we measure a quantity by using a scale, the accuracy of our measurement depends on the markings on the scale. For example, the ruler shown can measure both in centimetres and millimetres.

Measurements made with this ruler would have $\pm 0.5$ mm added to the measurement. The quantity $\pm 0.5$ is called the tolerance of measurement or measurement error.

**Tolerance of measurement** $= \frac{1}{2} \times$ size of smallest marked unit

For a measurement of $5.6 \pm 0.5$ mm, the largest possible value is $5.6 \text{ cm} + 0.5 \text{ mm} = 5.65 \text{ cm}$, and the smallest value is $5.6 \text{ cm} - 0.5 \text{ mm} = 5.55 \text{ cm}$. 
1 For the thermometer scale at right:
   a determine the temperature
   b state the measurement with its tolerance
   c determine the largest and smallest possible values.

2 Calculate the largest and smallest values for:
   a \((56.2 \pm 0.1) - (19.07 \pm 0.05)\)
   b \((78.4 \pm 0.25) \times (34 \pm 0.1)\).

**Significant figures in measurement**
A significant figure is any non zero-digit, any zero appearing between two non-zero digits, any trailing zeros in a number containing a decimal point, and any digits in the decimal places. For example, the number 345.6054 has 7 significant figures, whereas 300 has 1 significant figure.

The number of significant figures is an expression of the accuracy of a measurement. The greater the number of significant figures, the more accurate the measurement. For example, a fast food chain claims it has sold 6,000,000,000 hamburgers, not 6,453,456,102. The first measurement has only 1 significant figure and is a very rough approximation of the actual number sold, which has 10 significant figures.

Reducing the number of significant figures is a process that is similar to rounding.

**Rounding and measurement error in calculations**
When you perform calculations, it is important to keep as many significant digits as practical, and to perform any rounding as the final step. For example, calculating \(5.34 \times 341\) by rounding to 2 significant figures before multiplying gives \(5.30 \times 340 = 1802\), compared with 1820 if the rounding is carried out after the multiplication.

Calculations that involve numbers from measurements containing errors can result in answers with even larger errors. The smaller the tolerances, the more accurate the answers will be.

3 a Calculate \(45.943.450 \times 86.765 \times 303\) by:
   i first rounding each number to 2 significant figures
   ii rounding only the answer to 2 significant figures.
   b Compare the two results.

**Error in area and volume resulting from an error in a length measurement**
The side length of a cube is measured and incorrectly recorded as 5 cm. The actual size is 6 cm. The effect of the length measurement error used on calculations of the surface area is shown below. Complete the calculations for volume.

Error used in length measurement = 1 cm
Surface area calculated with incorrectly recorded value = \(5^2 \times 6 = 150\) cm\(^2\)
Surface area calculated with actual value = \(6^2 \times 6 = 216\) cm\(^2\)

Percentage error = \(\frac{216 - 150}{6} \times 100\% \approx 30.5\%\)

4 a Complete a similar calculation for the volume of the cube using the incorrectly recorded length. What conclusion can you make regarding errors when the number of dimensions increase?
   b Give three examples of a practical situation where an error in measuring or recording would have a potentially disastrous impact.
Australian inventions!

The answers to the measurement problems give the puzzle’s code.
Activities

6.1 Overview
Video
• The story of mathematics (eles-1845)

6.2 Area
eLesson
• Heron’s formula (eles-0177)
Interactivity
• IP interactivity 6.2 (int-4593): Area
Digital docs
• SkillSHEET (doc-5236): Conversion of area units
• SkillSHEET (doc-5237): Using a formula to find the area of a common shape
• WorkSHEET 6.1 (doc-5241): Area

6.3 Total surface area
Interactivities
• TSA — sphere (int-2782)
• IP interactivity 6.3 (int-4594): Total surface area
Digital docs
• SkillSHEET (doc-5238): Total surface area of cubes and rectangular prisms
• WorkSHEET 6.2 (doc-5242): Surface area

6.4 Volume
Interactivities
• Maximising the volume of a cuboid (int-1150)
• IP interactivity 6.4 (int-4595): Volume
Digital docs
• SkillSHEET (doc-5239): Conversion of volume units
• SkillSHEET (doc-5240): Volume of cubes and rectangular prisms
• WorkSHEET 6.3 (doc-6733): Volume

6.5 Review
Interactivities
• Word search (int-2841)
• Crossword (int-2842)
• Sudoku (int-3593)
Digital docs
• Topic summary (doc-13721)
• Concept map (doc-13722)

To access eBookPLUS activities, log on to www.jacplus.com.au
Answers

TOPIC 6 Surface area and volume

Exercise 6.2 — Area

1. a) 16 cm²; b) 48 cm²; c) 75 cm²
d) 120 cm²; e) 706.86 cm²; f) 73.5 mm²
g) 254.47 cm²; h) 21 m²; i) 75 cm²
2. Part e = 225π cm²; part g = 81π cm²
3. a) 20.66 cm²; b) 7.64 cm²
c) 113.1 mm²; d) 188.5 mm²
e) 12π cm²; f) 37.7 cm²
g) 69π mm²; h) 108.38 mm²

Exercise 6.3 — Total surface area

1. a) 600 cm²; b) 384 cm²; c) 1440 cm²
d) 27 m²
2. a) 113.1 m²; b) 6729.3 cm²; c) 8.2 m²
d) 452.4 cm²
3. a) 1495.4 cm²; b) 502.7 cm²; c) 170.77 m²
e) 506.0 cm²; f) 9.4 m²; g) 340.4 cm²
h) 224.1 cm²

Exercise 6.4 — Volume

1. a) 27 cm³; b) 740.88 cm³
c) 3600 cm³; d) 94.5 cm³
e) 450 mm³; f) 360 cm³
g) 633.5 cm³; h) 19.1 m³
i) 280 cm³; j) 288 mm³
k) 91.6 m³; l) 21470.8 cm³

Challenge 6.1

20 tiles

Exercise 6.3 — Total surface area

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d) 27 m²
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Exercise 6.4 — Volume

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Challenge 6.1

20 tiles
10. **E**

11. 7438.35 cm³

12. 4417.9 L

13. 10 215.05 cm³

14. a) \( V_{\text{new}} = \pi r^2 h \), the volume will remain the same.
   b) \( V_{\text{new}} = 3\pi h \), the volume will be 3 times as large as the original value.
   c) 19 bins
   d) 112 000 L
   e) 1.95 m from floor

15. a) i) 4.57 cm  
       ii) 262.5 cm²
   b) i) 14.15 cm  
       ii) 323.27 cm²
   c) i) 33.33 cm  
       ii) 437.62 cm²
   d) Sphere. Costs less for a smaller surface area.

16. a) \( h = \frac{V}{\pi r^2} \)
   b) i) 31.8 cm  
       ii) 8.0 cm
   c) \( r \geq 0 \), since \( r \) is a length
   d) i) 7.6 cm  
       ii) 6.2 cm

17. 1000

18. a) 2.55 cm
   b) 35.68 cm
   c) \( A_a = 157.88 \text{ m}^2 \), \( A_b = 12.01 \text{ m}^2 \)

19. a) 126.67 m³
   b) 53.33 m³

20. Answers will vary.

21. Required volume = 1570.80 cm³; tin volume = 1500 cm³; muffin tray volume = 2814.72 cm³. Marion could fill the tin and have a small amount of mixture left over, or she could almost fill 14 of the muffin cups and leave the remaining cups empty.

22. Increase radius of hemispherical section to 1.92 m.

23. Cut squares of side length, \( s = 0.3 \) m or 0.368 m from the corners.

24. Volume of water needed; 30.9 m³.

25. a) \( H = 12R \)
   b) \( 8\pi R^3 \)
   c) \( 12\pi R^3 \)
   d) \( 8 : 12 = 2 : 3 \)

26. a) \( \frac{1}{3}X^2H \)
   b) \( \frac{1}{3}X^2(h - H) \)
   c, d Check with your teacher.

**Challenge 6.2**

18 scoops

**Investigation — Rich task**

1. a) The temperature reading is 26.5°C.
   b) The smallest unit mark is 1°C, so the tolerance is 0.5.
   c) Largest possible value = 27°C, smallest possible value = 26°C

2. a) Largest value = 37.28, smallest value = 36.98
   b) Largest value = 2681.965, smallest value = 2649.285

3. a) i) 4 002 000  
       ii) 4 000 000
   b) The result for i has 4 significant figures, whereas ii has only 1 significant figure after rounding. However, ii is closer to the actual value (3 986 297.386 144 940.9).

4. a) Volume using the incorrectly recorded value = 125 cm³
   Volume using the actual value = 216 cm³
   The percentage error is 42.1%, which shows that the error compounds as the number of dimensions increases.
   b) Check with your teacher.

**Code puzzle**

Bionic ear implant
Black box flight memory recorder